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Investigation of bandwidth in multimode graded-index plastic optical fibers

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Abstract: A new method is proposed for investigating the bandwidth in multimode graded-index plastic optical fibers (GI POFs). By numerically solving the time-dependent power flow equation, bandwidth is reported for a varied launch conditions (radial offsets) of multimode GI POF. Our theoretical results are supported by the experimental results which show that bandwidth decreases with increasing radial offset. This decrease is more pronounced at short fiber lengths. At fiber length close to the coupling length \(L_c\) at which an equilibrium mode distribution (EMD) is achieved, this decrease becomes slower, indicating that mode coupling improves bandwidth at larger fiber lengths. With further increase of fiber length, bandwidth becomes nearly independent of the radial offset, indicating that a steady-state distribution (SSD) is achieved. Such a fiber characterization can be applied to optimize fiber performance in POF links.

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1. Introduction

Multimode POFs are often used in local area networks, sensors, power delivery systems, and automobile communication systems. Multimode POFs have large numerical aperture which simplifies employment of various light sources (laser, LED and VCSEL). POFs offer a low-cost installation with low-precision plastic components without precision couplers. Their flexibility is especially useful when installing them in tight spaces such as in homes, offices or vehicles. GI POFs exhibit far less modal dispersion than step-index (SI) POFs. This is due to their gradual decreasing of the refractive index of core with radial distance from the fiber axis. A bandwidth-length product of SI POF of \(\sim 50 \text{MHz} \cdot 100\text{m}\) has been reported [1], while a bandwidth of around 5.2 GHz at a distance of 30 m, which leads to a bandwidth-length product of 0.156 GHz \(\cdot\) km for GI POF has been reported [2].

Propagation characteristics of GI POFs are influenced by differential mode attenuation, mode coupling and modal dispersion [3–10]. Modal attenuation originates from conventional loss mechanisms, such as absorption, Rayleigh scattering and loss on reflection at the core-cladding interface. Mode coupling is mainly caused by scattering of light that transfers power from one mode to another due to intrinsic perturbations in multimode optical fibers. These effects can be attributed to various irregularities such as microscopic bends, voids and cracks, diameter variation, and density and refractive index fluctuations. Mode coupling effects results in the length-dependence variations of pulse dispersion and bandwidth [5]. It has been shown that the shorter the fiber length at which equilibrium mode distribution is achieved, the faster bandwidth improvement (slower bandwidth decrease) occurs [9]. Since GI POFs are primarily used within short distances (few tens of meters up to 100 m), it is important to characterize transmission in GI POFs as a function of fiber length for various launch conditions (radial offset and tilt angle).
Until recently, commercial simulation software packages were either designed specifically for single mode optical fibers or individual guided modes in few mode optical fibers. This is not adequate for multimode optical fibers with thousands or even millions of propagation modes and strong mode coupling. There is a clear need for a new and effective simulation tool for modeling of transmission in multimode optical fibers. The most effective way to describe mode coupling, modal attenuation and modal delay is provided by the time-dependent power flow equation to describe light transmission in multimode fibers. Since for this equation an analytical solution has been reported only for the steady state in GI optical fiber [11], in a more general cases this equation has to be solved by applying an appropriate numerical method. Along the way, one has to overcome many challenges to ensure the accuracy of the numerical model as well as a stability of the proposed numerical schemes. The model proposed in this work provides the space-time evolution of the modal power distribution when it is transmitted along the GI optical fiber. Following this path, in this article, in our best knowledge for the first time, by numerical solving the time-dependent power flow equation using explicit finite difference method (EFDM), the bandwidth in GI POF for various launch conditions (radial offsets) is obtained. Such obtained theoretical results for bandwidth are compared to our experimental findings.

2. Time-dependent power flow equation for GI optical fibers

The index profile of GI optical fibers may be expressed as:

\[
n(r, \lambda) = \begin{cases} 
n_1(\lambda) \left[ 1 - 2\Delta(\lambda) \left( \frac{r}{a} \right)^g \right]^{1/2} & (0 \leq r \leq a) \\
n_1(\lambda) \left( 1 - 2\Delta(\lambda) \right)^{1/2} = n_2(\lambda) & (r > a)
\end{cases}
\]  

where \(g\) is the core index exponent, \(a\) is the core radius, \(n_1(\lambda)\) is the maximum index of the core (measured at the fiber axis), \(n_2(\lambda)\) is the index of the cladding and \(\Delta = \left[ n_1(\lambda) - n_2(\lambda) \right] / n_1(\lambda)\) is the relative index difference (Fig. 1). The optimum value of the core index exponent \(g\) to obtain maximum bandwidth depends on the wavelength \(\lambda\) (in free-space) of the source.

![Fig. 1. Refractive index distribution of graded-index optical fiber, where \(a\) is core radius and \(b\) is fiber diameter.](image)

Time-dependent power flow equation for multimode GI optical fibers is:

\[
\frac{\partial P(m, \lambda, z, \omega)}{\partial z} + j\omega \tau(m, \lambda) P(m, \lambda, z, \omega) = -\alpha(m, \lambda) P(m, \lambda, z, \omega) + \frac{\partial P(m, \lambda, z, \omega)}{\partial m} \frac{\partial d(m, \lambda)}{\partial m} + d(m, \lambda) \frac{\partial P^2(m, \lambda, z, \omega)}{\partial m^2}
\]  

where \(P(m, \lambda, z, \omega)\) is the power in the \(m\)-the principal mode (modal group) (the principal mode number \(m\) is treated as a continuous variable [11]), \(z\) is coordinate along the fiber axis from the input fiber end, \(\alpha(m, \lambda)\) is the attenuation of the mode \(m\), \(d(m, \lambda)\) is the coupling coefficient of...
the mode \( m, \omega = 2\pi f \) is the baseband angular frequency, is delay time per unit length of mode \( m \), which can be determined as:

\[
\tau(m, \lambda) = \frac{n_1(\lambda)}{c} \left[ 1 + \frac{g - 2}{g + 2} \Delta(\lambda) \left( \frac{m}{M(\lambda)} \right)^{2g/(g+2)} + \frac{1}{2} \frac{3g - 2}{g + 2} \Delta(\lambda)^2 \left( \frac{m}{M(\lambda)} \right)^{4g/(g+2)} \right] \tag{3}
\]

where \( c \) is the free-space velocity of light and:

\[
P(m, \lambda, z, \omega) = \int_{-\infty}^{+\infty} P(m, \lambda, z, \omega) \exp(-j\omega t) dt \tag{4}
\]

The first and second term on the left-hand side of Eq. (2) describe the space and time evolution of the modal power distribution along the fiber, respectively. The first term on the right-hand side of Eq. (2) describes modal attenuation, the second and third term describe drift of the power distribution towards \( m = 0 \), and the fourth term describes diffusivity (broadening) of the modal power distribution. The maximum principal mode number is \([10]\):

\[
M(\lambda) = \sqrt{\frac{g \Delta(\lambda)}{g + 2}} \frac{\rho k n_1(\lambda)}{c} \tag{5}
\]

Where \( k = 2\pi / \lambda \). Gaussian launch-beam distribution \( P_0(\theta, \lambda, z = 0) \) can be transformed into \( P_0(m, \lambda, z = 0) \) one needs \( P_0(\lambda, \lambda, z = 0) \) to numerically solve the time-dependent power flow Eq. (2), using the following relationship \([12]\):

\[
\frac{m}{M(\lambda)} = \left[ \left( \frac{r_0}{a} \right)^{g} + \frac{\theta^2}{2 \Delta(\lambda)} \right]^{(g+2)/2g} \tag{6}
\]

where \( r_0 \) is radial distance (radial offset) between the launch beam position and the core center and \( \theta \) is tilt angle measured in respect to the fiber axis.

It is apparent that \( P(m, \lambda, z, \omega) \) is complex. By separating \( P \) into the real part \( P^r \) and imaginary part \( P^i \), Eq. (2) can be rewritten as the following simultaneous partial differential equations:

\[
\frac{\partial P^r}{\partial z} = -\alpha P^r + \frac{d}{m} \frac{\partial P^r}{\partial m} + \frac{d}{m} \frac{\partial P^i}{\partial m} + \frac{d^2 P^r}{\partial m^2} + \omega \tau P^i \tag{7}
\]

\[
\frac{\partial P^i}{\partial z} = -\alpha P^i + \frac{d}{m} \frac{\partial P^i}{\partial m} + \frac{d}{m} \frac{\partial P^r}{\partial m} + \frac{d^2 P^i}{\partial m^2} - \omega \tau P^i \tag{8}
\]

where \( P = P^r + jP^i \). Assuming a constant coupling coefficient \( d \equiv D \) \([10]\), Eqs. (7) and (8) can be written as:

\[
\frac{\partial P^r}{\partial z} = -\alpha P^r + \frac{D}{m} \frac{\partial P^r}{\partial m} + D \frac{\partial^2 P^r}{\partial m^2} + \omega \tau P^i \tag{9}
\]

\[
\frac{\partial P^i}{\partial z} = -\alpha P^i + \frac{D}{m} \frac{\partial P^i}{\partial m} + D \frac{\partial^2 P^i}{\partial m^2} - \omega \tau P^r \tag{10}
\]

Using the EFDM, discretization of Eqs. (9) and (10) leads to:

\[
P^r_{k+1,j} = \frac{\Delta z D}{\Delta m} \frac{\Delta z D}{2m_k \Delta m} n_{0,1} P^r_{k-1,j} + (1 - \frac{2 \Delta z D}{\Delta m^2} \alpha_k \Delta z) P^r_{k,j} + \frac{\Delta z D}{\Delta m^2} \frac{\Delta z D}{2m_k \Delta m} \omega n_0 P^r_{k+1,j} + \frac{\omega n_0 \Delta z}{2c} m_k^2 P^i_{k,j} \tag{11}
\]

\[
P^i_{k+1,j} = \frac{\Delta z D}{\Delta m} \frac{\Delta z D}{2m_k \Delta m} n_{0,1} P^i_{k-1,j} + (1 - \frac{2 \Delta z D}{\Delta m^2} \alpha_k \Delta z) P^i_{k,j} + \frac{\Delta z D}{\Delta m^2} \frac{\Delta z D}{2m_k \Delta m} \omega n_0 P^i_{k+1,j} + \frac{\omega n_0 \Delta z}{2c} m_k^2 P^r_{k,j} \tag{12}
\]
where $k$ and $l$ refer to the discretization step lengths $\Delta m$ and $\Delta z$ for the mode $m$ and length $z$, respectively, i.e. $P_{k,l}^r \equiv P^r(m_k,z_l,\lambda)$ and $P_{k,l}^i \equiv P^i(m_k,z_l,\lambda)$.

If $P^r$ and $P^i$ are obtained by solving Eqs. (11) and (12), the transmission characteristics can be calculated. Thus, the frequency response of fiber at length $z$ is:

$$H(\lambda, z, \omega) = \frac{\int_1^M 2m[P_r(m, \lambda, z, \omega) + jP_i(m, \lambda, z, \omega)] dm}{\int_1^M 2m[P_r(m, \lambda, z, 0) + jP_i(m, \lambda, z, 0)] dm}$$

(13)

where the factor $2m$ denotes degeneracy of modal group $m$. The frequency responses for a given fiber can be obtained at a range of lengths providing information on the bandwidth dependence with distance. The modal power distribution $P(m, \lambda, z, \omega)$ and the spatial transient of power $P_L(\lambda, z, \omega)$ can be obtained by:

$$P(m, \lambda, z, \omega) = [P^r(m, \lambda, z, \omega)^2 + P^i(m, \lambda, z, \omega)^2]^{1/2}$$

(14)

$$P_L(\lambda, z, \omega) = 2\pi \int_0^M m P(m, \lambda, z, \omega) dm$$

(15)

This enable us to determine the relationship between the transmission length and bandwidth in GI POF for various launch conditions.

3. Results and discussion

We applied our method to the GI POF (OM Giga, Fiber FinTM), which we previously investigated experimentally [10]. The core diameter of the fiber was $2a=0.9$ mm (fiber diameter was $b=1$ mm). The refractive index of the core measured at the fiber axis was $n_1=1.522$ and the refractive index of the cladding was $n_2=1.492$ (measured at $\lambda=633$ nm). For the GI POF under investigation, the maximum principal mode number is $M=656$ (for $\lambda=633$ nm), $g=1.80101$ and $\Delta=(n_1-n_2)n_1=0.019711$. The constant coupling coefficient for the investigated GI POF is $D=1482$ $1/m$ [10], which we adopted in this work. One should note that assumption of constant coupling coefficient for this GI POF led to the correct prediction of the output angular power distribution [10] (constant coupling coefficient is commonly used in modeling GI POF, e.g. in Ref. [13]). One can observe from Fig. 2, that except near $m=0$, measured mode-dependent attenuation can be assumed constant $\alpha(m, \lambda) \equiv \alpha_0=0.0122$ $1/m$. In the numerical calculations, a Gaussian beam $P(\theta,z)$ is assumed to be launched with $\langle \theta \rangle=0^\circ$ and standard deviation $\sigma_\theta=1.3^\circ$ (FWTM =3.06°) [10]. The bandwidth of the GI POF was evaluated by the time domain measurement method in which the bandwidth was estimated by measuring the output pulse waveform. The optical signal was injected through a single-mode fiber which was butt-coupled to the GI POF being tested and aligned parallel to the fiber axis with controlled lateral displacement to the axis [10]. The output pulse from the fiber was measured by a sampling optical oscilloscope (Hamamatsu Photonics Co. C8188-03).

Figures 3 and 4 show the numerically calculated and measured bandwidth, respectively, for five radial offsets $\Delta r=0, 100, 200, 300$ and 400 $\mu$m at different fiber lengths. One can see that our theoretical and experimental results for bandwidth are in good agreement. One can see from
**Fig. 2.** Measured mode-dependent attenuation $\alpha(m)$ for GI POF.

**Fig. 3.** Numerically calculated bandwidth as a function of transmission length of GI POF for a various radial offsets.

**Fig. 4.** Measured bandwidth as a function of transmission length of GI POF for a various radial offset (lines are drawn to guide the eye).
Figs. 3 and 4 that bandwidth decreases with increasing radial offset, which is a consequence of larger modal dispersion in case of excitation of higher guided modes.

This decrease is more pronounced at short fiber lengths. At fiber length \( z \approx 30 \text{ m} \), which is close to the experimentally obtained coupling length \( L_c = 31 \text{ m} \) (Fig. 5) [10] at which an EMD is achieved, this decrease becomes slower, indicating that strong mode coupling causes bandwidth improvement in GI POFs. Thus, we obtain a bandwidth of around 4.6 GHz at a distance of 100 m, which leads to a bandwidth-length product of 0.46 GHz·km for radial offset of \( \Delta r = 0 \mu\text{m} \) for the investigated GI POF. This bandwidth-length product is greater than a bandwidth-length product of 0.156 GHz·km for GI POF investigated by Chun-Yu Lin et al. [2].

**Fig. 5.** Normalized output modal power distribution \( P(m) \) in SC GI POF over a range of radial offsets \( \Delta r \), obtained from the measured angular power distributions \( P(\theta) \), at different fiber lengths (a) \( z = 1 \text{ m} \), (b) \( z = 5 \text{ m} \), (c) \( z = 10 \text{ m} \), (d) \( z = 20 \text{ m} \) and (e) \( z = 31 \text{ m} \) [10].

One should note that \( L_c \) marks the length at which the distribution of the highest guiding mode (excited by largest radial offset launch) becomes centrally located (Fig. 5(e)). The shorter the length at which EMD is achieved, the earlier the bandwidth would switch from steep to slower
bandwidth decrease. With further increase of fiber length, bandwidth becomes nearly independent of the radial offset, indicating that a SSD is established. Finally, such a fiber characterization can be applied to optimize fiber performance in POF links, particularly in in-home networks [14]. This optimization can be performed by comparing a performance properties and numerical simulation results of various GI POFs with different refractive index distributions (different \( g \)), coupling coefficient \( D \) and attenuation \( \alpha \). One should note that the proposed method for investigating the bandwidth in multimode GI POF can be employed for any multimode optical fiber with GI core distribution, such as silica optical fibers and plastic clad silica fibers.

4. Conclusion

By numerically solving the time-dependent power flow equation we calculated bandwidth for various launch conditions (radial offsets) of multimode GI POF. We found that our theoretical and experimental results are in good agreement, showing that bandwidth decreases with increasing radial offset. This decrease is more pronounced at short fiber lengths. At coupling length \( L_c \) at which an EMD is achieved, this decrease becomes slower, indicating a strong influence of mode coupling on transmission in GI POFs. The shorter the length at which EMD is achieved, the earlier the bandwidth would switch from steep to slower bandwidth decrease. With further increase of fiber length, bandwidth becomes nearly independent of the radial offset, indicating that a SSD is established. Thus, we obtain a bandwidth of around 4.6 GHz at a distance of 100 m for radial offset of \( \Delta r = 0 \) µm for the investigated GI POF.

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