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The phase sensitivity of an SU(1,1) interferometer with coherent and squeezed-vacuum light

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Abstract

We theoretically study the phase sensitivity of an SU(1,1) interferometer with a coherent state in one input port and a squeezed-vacuum state in the other input port using the method of homodyne detection. In this interferometer, beam splitting and recombination are generated by the parametric amplifiers instead of the beam splitters. Compared with the traditional Mach–Zehnder interferometer, the phase sensitivity of this interferometer can be improved due to the amplification process of the parametric amplifiers. Combined with the squeezed state input, the sensitivity can be improved further due to the noise reduction. The phase sensitivity of our scheme can approach the Heisenberg limit and the associated optimal condition is analyzed. The scheme can be implemented with current experimental technology.

Keywords: phase sensitivity, interferometers, quantum measure limits

1. Introduction

Quantum metrology (or quantum parameter estimation), which is the use of quantum measurement techniques to obtain higher statistical precision than purely classical approaches, has been receiving a lot of attention in recent years [1–10] because essentially all measurements



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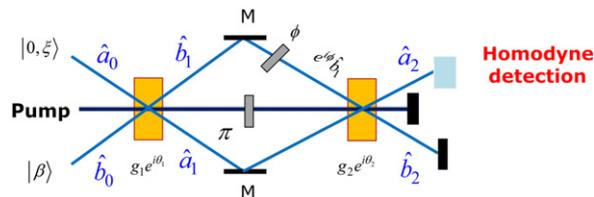


Figure 1. The schematic diagram of the SU(1,1) interferometer. Two OPAs take the place of two beam splitters in the traditional Mach–Zehnder interferometer. g_1 (g_2) and θ_1 (θ_2) describe the strength and phase shift in the OPA process 1 (2), respectively. a_i and b_i ($i = 0, 1, 2$) denote two light beams in the different processes. A coherent state $|\beta\rangle$ is in one input port and a squeezed-vacuum state $|0, \xi\rangle$ in the other input port of the SU(1,1) interferometer. Homodyne detection is at the dark port. The pump field between the two OPAs has a π phase difference. ϕ : phase shift; M : mirrors.

are affected by statistical errors. These arise from experimental imperfections or, more fundamentally, from the Heisenberg uncertainty relations. To reduce the effect of the errors, we can repeat the measurement under the exact same conditions, averaging the outcomes. The average has an uncertainty that is reduced by a scaling factor of $1/\sqrt{N}$, where N is the number of repetitions. This scaling defines the so-called ‘standard quantum limit’ (SQL) or ‘shot noise limit’ (SNL)—it has been confirmed that no classical procedure can do better than that [7]. Readout of a continuous variable always has an intrinsic precision. For a continuous parameter ϕ , if it is not associated with a Hermitian operator. The uncertainty can be inferred from the variance of some observable A via the relation $\delta\phi = \Delta A |d\langle A\rangle/d\phi|^{-1}$, where $\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$. One can obtain high measurement precision by decreasing the ΔA or increasing the slope $|d\langle A\rangle/d\phi|$, or operating by doing at the same time. Therefore, recent research has focused on beating the SQL, and this can be summarized as follows. (1) For decreasing the ΔA , some research work has considered how to improve the input states to reduce the noise below vacuum noise, such as by using squeezed states [3, 11, 12] or two-mode squeezed states [13, 14]. Thus the sensitivity can be improved to beat the SQL. (2) The mean value $\langle A\rangle$ can be written as $\langle A\rangle = Ap \cos(\phi_0 + \phi)$, where ϕ is a phase shift.

To make the slope $|d\langle A\rangle/d\phi|$ larger, some research work has also considered ways to optimize the input states, such as by using NOON states [15, 16], which to the mean value can be written as $\langle A\rangle = Ap \cos(\phi_0 + N\phi)$. The slope gets larger by a factor of N . Other research has considered how to change the structure or hardware of the interferometer to enhance the amplitude Ap to improve slope $|d\langle A\rangle/d\phi|$ [17–22]. For example, nonlinear elements were introduced in the linear interferometers. A class of such interferometers introduced by Yurke *et al* [17] is described by the group SU(1,1), as opposed to SU(2), where the 50-50 beam splitters in a traditional Mach–Zehnder interferometer (MZI) were replaced by optical parameter amplifiers (OPAs) (see figure 1), and they showed that the sensitivity of this device exhibits Heisenberg scaling [17].

Recently, an improved theoretical scheme was presented by Plick *et al* [23], who proposed to inject a strong coherent beam to ‘boost’ the photon number. Their scheme circumvents the low photon number problem encountered by Leibfried *et al* [24] in their experiments. Experimental realization of this SU(1,1) interferometer was reported recently [25]. The maximum output intensity of this interferometer can be much higher than the input intensity as well as the intensity inside the interferometer (the phase-sensing intensity).

More recently, the noise performance of this interferometer was analyzed and under the same phase-sensing intensity condition an improvement of 4.1 dB in signal-to-noise ratio was observed [26]. Due to the improved phase measurement sensitivity of this interferometer, it was proposed for gravitational wave detection; however, it needs strong coherent light input [23]. The very strong coherent light will generate the high-order nonlinear effect and the radiation pressure noise.

To get the same measurement sensitivity and reduce the required intensities of the input states, we consider a squeezed state to replace one of the two input coherent states, which can eliminate the disadvantages due to input strong coherent states. In addition, we use homodyne detection instead of intensity detection, which is convenient for experimental detection of squeezing operation. Our scheme is the case that decreases the ΔA and increases the slope $|d\langle A \rangle/d\phi|$ at the same time to improve phase sensitivity approaching the Heisenberg limit (HL).

In this paper, we study the phase sensitivity of the SU(1,1) interferometer for coherent light combined with squeezed vacuum light as input using homodyne detection. We give the requirement to approach the Heisenberg limit among the input coherent state, the input squeezed vacuum state, and the OPA process. In the presence of loss, the sensitivity can beat the SQL under a certain range of loss rates. Here, the phase shift studied is not the general phase but is sufficiently close to the optimal phase point [27, 28].

2. An SU(1,1) interferometer with coherent \otimes squeezed light input

2.1. Model

An SU(1,1) interferometer is shown in figure 1, where the OPAs replaced the 50-50 beam splitters in a traditional MZI. Here, we consider a coherent light combined with a squeezed vacuum light as input. After the first OPA, one output is retained as a reference, while the other experiences a phase shift process. After the beam recombines in the second OPA with the reference light, the output fields are dependent on the phase difference ϕ between the two beams. \hat{a} (\hat{a}^\dagger) and \hat{b} (\hat{b}^\dagger) are the annihilation (creation) operators for the two modes, respectively. The total transform of the operators by the SU(1,1) interferometer is given by [23, 29],

$$\hat{a}_2 = \mathcal{U}\hat{a}_0 - \mathcal{V}\hat{b}_0^\dagger, \quad (1)$$

$$\hat{b}_2 = e^{i\phi}(\mathcal{U}\hat{b}_0 - \mathcal{V}\hat{a}_0^\dagger), \quad (2)$$

where $\mathcal{U} = \cosh g_1 \cosh g_2 + e^{-i\phi} e^{i(\theta_2 - \theta_1)} \sinh g_1 \sinh g_2$ and $\mathcal{V} = e^{i\theta_1} \sinh g_1 \cosh g_2 + e^{-i\phi} e^{i\theta_2} \cosh g_1 \sinh g_2$, so $|\mathcal{U}|^2 - |\mathcal{V}|^2 = 1$. g_1 (g_2) and θ_1 (θ_2) describe the strength and phase shift in the process of OPA in the atomic cell 1 (2), respectively. The balanced situation is $\theta_2 - \theta_1 = \pi$ and $g = g_1 = g_2$ that the second OPA will ‘undo’ what the first did when the phase shift ϕ is 0.

In our scheme, the detected variable is amplitude quadrature \hat{X} instead of the photon number \hat{N} that has been studied in references [23, 29]. The output amplitude quadrature operator can be written as

$$\hat{X} \equiv \frac{1}{\sqrt{2}}(\hat{a}_2 + \hat{a}_2^\dagger). \quad (3)$$

Using the amplitude quadrature \hat{X} , the phase sensitivity of the SU(1,1) interferometer is given by

$$(\Delta\phi)^2 = \frac{\langle(\Delta\hat{X})^2\rangle}{|\partial\langle\hat{X}\rangle/\partial\phi|^2}. \quad (4)$$

Now, we first consider a balanced configuration, i.e., $g_1 = g_2 = g$ and $\theta_2 - \theta_1 = \pi$. For a coherent light $|\beta\rangle$ ($\beta = |\beta|e^{i\theta_\beta}$) together with a squeezed vacuum $|0, \xi\rangle$ ($\xi = re^{i\eta}$) as input, the noise of the output amplitude quadrature \hat{X} is given by

$$\langle(\Delta\hat{X})^2\rangle = \frac{|\mathcal{V}|^2 + |\mathcal{U}|^2[\cosh(2r) - \sinh(2r)\cos(\Theta)]}{2}, \quad (5)$$

where $\Theta = \eta + 2\theta_u$, θ_u is given by $\mathcal{U} = |\mathcal{U}| \exp(i\theta_u)$. With no squeezed vacuum input ($r = 0$), the noise of equation (5) can be written as $\langle(\Delta\hat{X})^2\rangle = 1/2 + |\mathcal{V}|^2$, which is larger than the vacuum noise $\langle(\Delta\hat{X})^2\rangle = 1/2$. With a squeezed-vacuum light input at one port, and letting $\phi = 0$ and $\eta = 0$, the noise of equation (5) can be written as $\langle(\Delta\hat{X})^2\rangle = e^{-2r}/2$, which is below the vacuum noise. And the slope of the output amplitude quadrature \hat{X} is

$$|\partial\langle\hat{X}\rangle/\partial\phi|^2 = \frac{|\beta|^2 \sinh^2(2g) \cos^2\Phi}{2}, \quad (6)$$

where $\Phi = \theta_2 - \theta_\beta - \phi - \pi/2$. When $\Phi = 0$, the slope of the SU(1,1) interferometer is $|\beta|^2 \sinh^2(2g)/2$. However, for a traditional MZI using the same input state (coherent \otimes squeezed-vacuum light) the slope is $|\beta|^2$ [3]. For $\sinh^2(2g)/2 > 1$, the slope of the SU(1,1) interferometer is enhanced compared with a traditional MZI. Therefore, under certain conditions the SU(1,1) interferometer can decrease the ΔX and increase the slope $d\langle X\rangle/d\phi$ at the same time to improve phase sensitivity enough to beat the SQL.

When $\Phi = 0$ and at the optimal phase point $\phi = 0$, from equations (5) and (6) the best phase sensitivity of our scheme is

$$(\Delta\phi')^2 = \frac{1}{e^{2r}} \frac{1}{N_\beta} \frac{1}{N_{\text{OPA}}(N_{\text{OPA}} + 2)}, \quad (7)$$

where the factor e^{2r} results from the input squeezed-vacuum light, $N_\beta = |\beta|^2$ is the amount of input coherent light, and $N_{\text{OPA}} = 2 \sinh^2 g$ is the amount of light emitted from the OPA with vacuum input. If vacuum input is present, the sensitivity by intensity detection is $(\Delta\phi)^2 = 1/N_{\text{OPA}}(N_{\text{OPA}} + 2)$, which is the result of Yurke's scheme [17].

2.2. Heisenberg Limit

In this section, we compare the optimal phase sensitivity of our scheme with the HL. In our scheme the HL is

$$\Delta\phi_{\text{HL}} = \frac{1}{N_{\text{Tot}}}, \quad (8)$$

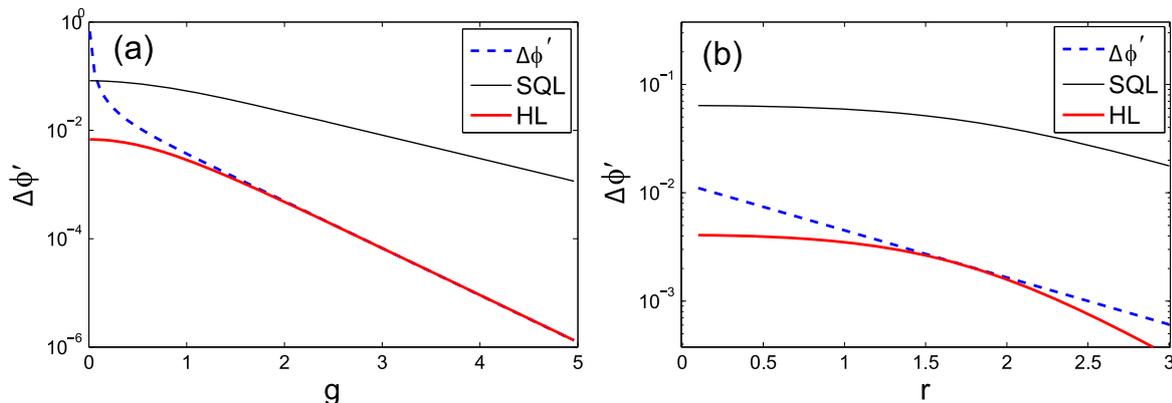


Figure 2. The phase sensitivity $\Delta\phi'$ of the SU(1,1) interferometer (a) versus g with $r = 2.5$, and $|\beta| = \tanh(2g)e^r/2$; (b) versus r with $|\beta| = 3$, and $g = 2$.

where $N_{\text{Tot}} (\equiv \langle \hat{a}_1^\dagger \hat{a}_1 + \hat{b}_1^\dagger \hat{b}_1 \rangle)$ is the total photon number inside the SU(1,1) interferometer, not the input photon number as in the traditional MZI. According to transformation, the total photon number is

$$N_{\text{Tot}} = (N_{\text{OPA}} + 1)(|\beta|^2 + \sinh^2 r) + N_{\text{OPA}}. \quad (9)$$

The first term on the right-hand side corresponds to the amplification process of the input photon number $|\beta|^2 + \sinh^2 r$ and the second term results from the spontaneous process. In the SU(1,1) interferometer the HL is dependent not only on the input photon number but also on the strength g of the parametric process.

Let $\Delta\phi' \simeq \Delta\phi_{\text{HL}}$, according to equations (7) and (8) the optimal condition is

$$|\beta| = \frac{\tanh(2g)e^r}{2}. \quad (10)$$

The condition gives the requirement for the input coherent state $|\beta|$, the input squeezed vacuum state r , and the OPA process g . The difference between $\Delta\phi'$ and $\Delta\phi_{\text{HL}}$ can be seen in figure 2(a), where we plot the phase sensitivity $\Delta\phi'$ as a function of the strength g for this optimal condition. Given $r = 2.5$, the phase sensitivity $\Delta\phi'$ can approach the HL when $g > 2$ ($\tanh(2g) \simeq 1$). Under this condition, equation (10) is reduced to $|\beta| \simeq e^r/2 \simeq \sinh r$. The total photon number inside the interferometer is $N_{\text{Tot}} \simeq 2N_{\text{OPA}}N_\beta = 2N_{\text{OPA}}\sinh^2 r$. The phase sensitivity $\Delta\phi'$ can approach the HL:

$$\left(\Delta\phi'\right)^2 = \frac{1}{2N_{\text{OPA}}N_\beta} \frac{1}{2N_{\text{OPA}}\sinh^2 r + e^{2r}} \simeq \frac{1}{N_{\text{Tot}}^2}. \quad (11)$$

When g is very small, increasing the input squeezed strength r and increasing the input coherent light intensity enables the phase sensitivity to beat the SQL, but it cannot approach the HL.

To approach the HL, the optimal condition is $|\beta| \simeq e^r/2 \simeq \sinh r$ and $\tanh(2g) \simeq 1$. The mean photon numbers in the coherent state and the squeezed vacuum state must balance, which means that for a MZI fed by coherent and squeezed-vacuum light the phase sensitivity of a single measurement is $1/\sqrt{|\beta|^2 e^{2r} + \sinh^2 r}$ [12]. When $|\beta|^2 \simeq \sinh^2 r \simeq \tilde{n}/2$ and $\tilde{n} \gg 1$, the phase sensitivity reaches the HL $1/\tilde{n}$, where the mean photon numbers in two input ports are balanced under this optimal condition [12]. For an SU(1,1) interferometer, the parametric processes only

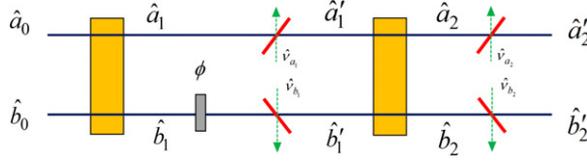


Figure 3. A lossy interferometer model; the losses in the interferometer are modeled by adding fictitious beam splitters.

amplify the input lights and generate correlations between them. Similar to MZI, the photon numbers in two input ports of the SU(1,1) interferometer also need to balance to approach the HL when $N_{\text{OPA}} \gg 1$, which is also obviously seen in figure 2(b). It is shown that there is an optimal degree of squeezing r at the other input for given values of $|\beta|$ and g , which makes the $\Delta\phi'$ approach the HL. The numerical optimal degree meets the condition $|\beta| \simeq e^r/2$ when $\tanh(2g) \simeq 1$. That is, the mean photon numbers in two input ports also need to balance to approach the HL in the SU(1,1) interferometer.

2.3. Losses

As has been previously pointed out, the loss is the limiting factor in precision measurement [21, 30–32]. Now, we investigate the effects of photon losses on the phase sensitivity for coherent \otimes squeezed-vacuum light as input. Here, we concentrate on the losses generated by light field propagation in both arms of the interferometer and by the imperfect detectors. As shown in figure 3, losses can be modeled by adding the fictitious beam splitters. Considering both arms of the interferometer to have the same internal and outside transmission rates T_i , passing through the internal beam splitters, the mode transform of the fields \hat{a}_1 and \hat{b}_1 is $\hat{a}'_1 = \sqrt{T_1} \hat{a}_1 + \sqrt{1-T_1} \hat{v}_{a_1}$, and $\hat{b}'_1 = \sqrt{T_1} \hat{b}_1 e^{i\phi} + \sqrt{1-T_1} \hat{v}_{b_1}$. And passing through the outside beam splitters the mode transform of the fields \hat{a}_2 and \hat{b}_2 is $\hat{a}'_2 = \sqrt{T_2} \hat{a}_2 + \sqrt{1-T_2} \hat{v}_{a_2}$, and $\hat{b}'_2 = \sqrt{T_2} \hat{b}_2 + \sqrt{1-T_2} \hat{v}_{b_2}$.

For calculation and analysis convenience, we use loss rates $L_i \equiv 1 - T_i$ ($i = 1, 2$). After taking into account these losses, we introduce the idea that \hat{X}_L , compared with equation (3), can be written as

$$\hat{X}_L \equiv \frac{1}{\sqrt{2}} \left[\hat{a}'_2 + \left(\hat{a}'_2 \right)^\dagger \right], \quad (12)$$

where the field $\hat{a}'_2 = \sqrt{(1-L_1)(1-L_2)} \hat{a}_2 + \sqrt{L_1(1-L_2)} (\cosh g_2 \hat{v}_{a_1} - e^{-i\theta_2} \sinh g_2 \hat{v}_{b_1}^\dagger) + \sqrt{L_2} \hat{v}_{a_2}$. Considering the losses, the slope is $|\partial \langle \hat{X}_L \rangle / \partial \phi|^2 = (1-L_1)(1-L_2) |\beta|^2 \sinh^2(2g) \cos^2 \Phi / 2$, and the fluctuation is $\langle (\Delta \hat{X}_L)^2 \rangle = (1-L_1)(1-L_2) \langle (\Delta \hat{X})^2 \rangle + L_1(1-L_2) \cosh(2g) + L_2$. Then, in the presence of loss the sensitivity for the balanced case is given by

$$\Delta\phi_L = \left[(\Delta\phi)^2 + \frac{1}{|\beta|^2 \sinh^2(2g)} \frac{\cosh(2g)L_1(1-L_2) + L_2}{(1-L_1)(1-L_2) \cos^2 \Phi} \right]^{1/2}, \quad (13)$$

where $\Delta\phi = \Delta \hat{X} / |\partial \langle \hat{X} \rangle / \partial \phi|$ is from equations (5) and (6). The second term on the right-hand side of equation (13) is the extra term due to the internal and outside losses. The extra term is generated from the vacuum noise, which can be seen from the $\langle (\Delta \hat{X}_L)^2 \rangle$. When considering only the internal loss ($L_1 \neq 0$ and $L_2 = 0$), the extra term is

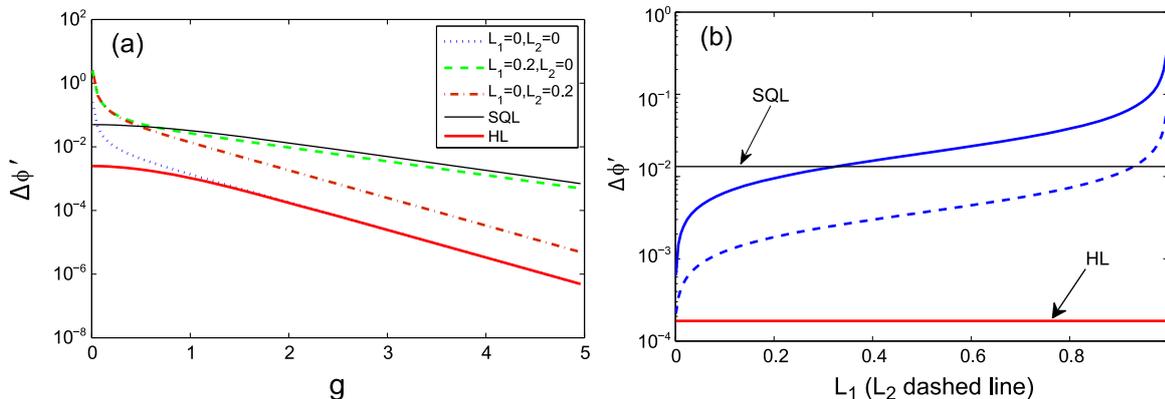


Figure 4. (a) The phase sensitivity as a function of g in the presence of loss with $r = 3$ and $|\beta| = \tanh(2g)e^r/2$. (b) The phase sensitivity of the solid line (dashed line) versus the loss rate L_1 (L_2) with $g = 2$, $r = 3$ and $|\beta| = \tanh(2g)e^r/2$.

$1/|\beta|^2 \sinh^2(2g) \times \cosh(2g)L_1/(1-L_1) \cos^2\Phi$, and when considering only the extra outside loss ($L_1 = 0$ and $L_2 \neq 0$), the extra term is $1/|\beta|^2 \sinh^2(2g) \times L_2/(1-L_2) \cos^2\Phi$. When $\cosh(2g) > 1$, the influence of the internal loss on the phase sensitivity is greater than the outside loss. Because the vacuum noise induced by the internal beam splitters is amplified.

Since $N_{\text{Tot}} \gg \sinh^2 g$, at the optimal point the preceding equation (13) can be rewritten as

$$\Delta\phi_L = \Delta\phi' \left[1 + \frac{L_1}{(1-L_1)} \frac{e^{2r} N_{\text{Tot}}}{|\beta|^2 + \sinh^2 r} + \frac{e^{2r} L_2}{(1-L_1)(1-L_2)} \right]^{1/2}. \quad (14)$$

Comparing the internal loss, this equation is similar to equation (24) in [29] but with an external term e^{2r} , because without considering the losses our sensitivity is almost e^r times better than that of [29] (see equation (18)). In figure 4(a), we plot the sensitivities with the same internal (dashed line) and outside loss (dot-dashed line), respectively. It is obviously seen that the sensitivity reduction by the internal loss is larger when $\tanh(2g) \simeq 1$. The sensitivities as a function of loss rate L_1 (solid line) and L_2 (dashed line) are plotted in figure 4(b). In the presence of photon losses, as can be seen in figure 4, the sensitivity cannot reach the HL, but it can beat the SQL for a certain range of L_1 and L_2 .

3. Discussion and conclusions

Now we consider the unbalanced situation $g_1 \neq g_2$ and still assume that $\theta_2 - \theta_1 = \pi$ as above. In figure 5 the behavior of the phase sensitivity $\Delta\phi$ as a function of g_2/g_1 is shown. Obviously, a large unbalance will degrade the phase sensitivity. The optimal form is $g_2 \simeq g_1$ and g_2 slightly larger than g_1 . In terms of phase sensitivity and convenience in conducting experiments, the balance case is optimal.

If the squeezed vacuum is replaced by another coherent light $|\alpha\rangle$, that is, at the input ports are coherent states $|\alpha\rangle$ and $|\beta\rangle$, as in [23] and [29]. From equation (14) of [29] by Marino *et al* the sensitivity at the optimal phase point according to intensity detection is $(\Delta\phi_M^I)^2 = (|\alpha|^2 + |\beta|^2)/[4|\alpha|^2|\beta|^2 N_{\text{OPA}}(N_{\text{OPA}} + 2)]$,³ where the subscript M denotes the

³ For convenience, we use N_{OPA} to replace n_s in the expression.

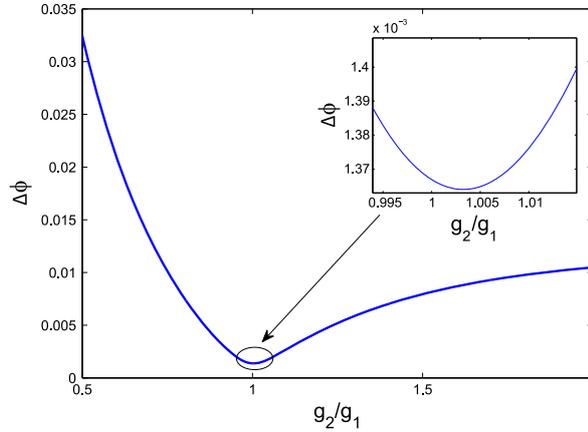


Figure 5. The phase sensitivity as a function of g_2/g_1 . The parameters are as follows: $r = 3$, $g_1 = 1$, $|\beta| = 10$, and $\Theta = 0$. The inset shows a zoom of the graph around $g_2 = g_1$.

sensitivity from Marino *et al* and the superscript I denotes intensity detection. If homodyne detection is used instead of intensity detection, the sensitivity at the optimal phase point is $(\Delta\phi_M^H)^2 = 1/[\alpha^2 N_{\text{OPA}}^2 + |\beta|^2 N_{\text{OPA}} (N_{\text{OPA}} + 2) + 2 |\alpha| |\beta| N_{\text{OPA}} \sqrt{N_{\text{OPA}} (N_{\text{OPA}} + 2)}]$, where the superscript H denotes homodyne detection. For conveniently comparing our obtained phase sensitivity with the one for coherent states at both inputs, we consider both of them to be at optimal condition and take $|\alpha| = |\beta|$, $|\beta| = \sinh r$. Thus,

$$(\Delta\phi_M^I)^2 = \frac{1}{2 |\beta|^2 N_{\text{OPA}} (N_{\text{OPA}} + 2)}, \quad (15)$$

$$(\Delta\phi_M^H)^2 = \frac{1}{2 |\beta|^2 N_{\text{OPA}} [N_{\text{OPA}} + 1 + \sqrt{N_{\text{OPA}} (N_{\text{OPA}} + 2)}]}, \quad (16)$$

and

$$(\Delta\phi')^2 = \frac{1}{4 |\beta|^4 N_{\text{OPA}} (N_{\text{OPA}} + 2)}. \quad (17)$$

Comparing the two results $\Delta\phi_M^I$ and $\Delta\phi_M^H$ with $\Delta\phi'$, we then obtain

$$\frac{(\Delta\phi_M^I)^2}{(\Delta\phi')^2} = 2 |\beta|^2 \simeq e^{2r}/2, \quad (18)$$

$$\frac{(\Delta\phi_M^H)^2}{(\Delta\phi')^2} = 2 |\beta|^2 \frac{N_{\text{OPA}} + 2}{N_{\text{OPA}} + 1 + \sqrt{N_{\text{OPA}} (N_{\text{OPA}} + 2)}}. \quad (19)$$

When $\sqrt{N_{\text{OPA}} (N_{\text{OPA}} + 2)} > 1$, the sensitivity $\Delta\phi_M^H$ is slightly better than $\Delta\phi_M^I$. Using the same intensity input for the SU(1,1) interferometer, and a coherent light combined with a squeezed vacuum light as input, the sensitivity is almost $2 |\beta|^2$ times higher than that of one for coherent states at both inputs. To obtain the same sensitivity with two coherent states as input, the

Table 1. The quantum limits for different interferometers with different input states.

Input states	MZI	SU(1,1) interferometer
vacuum states		$\Delta\phi = 1/\sqrt{N_{OPA}(N_{OPA} + 2)}$ [17]
one coherent state	$\Delta\phi = 1/\sqrt{N_\beta}$ [3]	$\Delta\phi = 1/\sqrt{N_\beta N_{OPA}(N_{OPA} + 2)}$ [23]
coherent \otimes squeezed vacuum state	$\Delta\phi = e^{-r}/\sqrt{N_\beta}$ [3]	$\Delta\phi = e^{-r}/\sqrt{N_\beta N_{OPA}(N_{OPA} + 2)}$

coherent \otimes squeezed-vacuum light as input can reduce the intensity inside the interferometer and eliminate the disadvantages, such as radiation-pressure noise.

In table 1, we summarize the quantum limits for MZI and an SU(1,1) interferometer with different input states. For the same input states, the sensitivities of the SU(1,1) interferometer are higher than those of the MZI by a factor of $\sqrt{N_{OPA}(N_{OPA} + 2)}$ due to the amplification process. For the SU(1,1) interferometer with coherent \otimes squeezed-vacuum light as input, the phase sensitivity is improved the most due to the noise reduction and phase-sensing field amplification. In an optical parameter process, $g = 2.25$ has been reported [33]. According to equation (8), the photon number is improved substantially, as is the absolute accuracy.

In conclusion, we investigated the phase sensitivity of an SU(1,1) interferometer for coherent \otimes squeezed-vacuum light as input. The phase sensitivity was improved compared with that of the traditional MZI as a result of the noise reduction and phase-sensing field amplification. The optimal condition to approach the HL for the SU(1,1) interferometer is given, which will help others realize it experimentally.

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