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Technical Paper

The micro-mechanical behaviour of sand–rubber mixtures under shear: An experimental study based on X-ray micro-tomography

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Abstract

The micromechanical behaviour of two sand–rubber mixture samples (i.e., a glass bead–rubber mixture sample (GB–R) and a Leighton Buzzard sand–rubber mixture sample (LBS–R)) under triaxial compression is investigated using X-ray micro-CT (µCT). Both samples contain sand and rubber grains with similar size and have a rubber content of 30% by mass. The results are compared to those of two pure sand samples (i.e., GB and LBS). Both sand–rubber mixture samples are found to exhibit a significantly lower strain localisation than the pure sand samples. The average coordinate number (CN) of sand–sand contacts of the sand–rubber mixtures experiences a significant decrease at the late shear stage, although volumetric compression of the samples occurs. Meanwhile, sand–sand contact fabrics of the sand–rubber mixtures are found to remain almost unchanged throughout the test, with an anisotropy degree of around 0.04 for GB-R and ~0.15 for LBS-R, respectively. In contrast, increasing contact areas and bias towards the major principal direction of sand–rubber contacts are observed in the sand–rubber mixtures as shear progresses, with the maximum anisotropy degree of about 0.27 for GB-R and 0.15 for LBS-R, respectively. Moreover, large-sized and small-sized sand–rubber contacts are found to concentrate towards the major and minor principal directions, respectively, throughout the test. The experimental observations highlight the important role of sand-rubber contacts in the stress-transmission behaviour of sand-rubber mixtures.

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Keywords: Sand–rubber mixtures; Sand–sand contacts; Sand–rubber contacts; Particle kinematics; X-ray CT

1. Introduction

Recycling and reuse of scrap tyres as construction materials has become a popular way to deal with the problem of waste tyre accumulation in both developed and developing countries. Sand–rubber mixtures, i.e., granular mixtures composed of shredded tyres and soil grains, have been widely used for construction purposes in various geotechnical applications, including slope engineering, retaining walls, highway embankments, drainage layers, protection for buried pipes, and seismic isolation of foundations (Poh and Broms, 1995; Bosscher et al., 1997; Reddy and Saichek, 1998; Lee et al., 1999; Garga and Shaughnessy, 2000; Uchimura et al., 2007; Tsang et al., 2012). The wide range of engineering applications of sand–rubber mixtures is attributed to their lightweight, desirable seismic response (i.e., high damping ratio) to dynamic loading, and their interesting acoustical and drainage properties, which are remarkably different from those of pure sands (Edil and Bosscher, 1994; Masad et al., 1996; Youwai and Bergado, 2003; Kaneda et al., 2007; Valdes and Evans, 2008).

A large number of laboratory studies have been carried out to investigate the mechanical behaviour of sand–rubber
mixtures. For instance, Youwai and Bergado (2003), among other colleagues (Foos et al., 1996; Zornberg et al., 2004; Attom, 2006), have investigated the effects of including rubber particles on the peak and critical state shear strength of sands. Lee et al. (2007), Kim and Santamarina (2008), Lee et al. (2014), Edincliler et al. (2012) and Li et al. (2020) have studied the effects of rubber size, rubber shape and rubber content on the mechanical behaviour of sand–rubber mixtures. Edincliler and others (Feng and Sutter, 2000; Pamukcu and Akbulut, 2006; Edincliler et al., 2013; Edincliler et al., 2018) have investigated the effects of rubber on the shear modulus and damping ratio of sand–rubber mixtures with different rubber contents. Fu et al. (2014, 2017) have tested two sands mixed with rubber grains, and found that the critical state framework could be applied to the sand–rubber mixtures. Cabalar and Karabash (Cabalar and Karabash, 2015; Karabash and Cabalar, 2015) have combined rubber grains and cement to improve the shear strength and stiffness of sandy soils. Meanwhile, various physical and constitutive models based on laboratory observations (Youwai and Bergado, 2004; Mashiri et al., 2016; Platzer et al., 2018) have been developed to simulate the mechanical behaviour of sand–rubber mixtures. As a result, valuable findings have been acquired regarding the effects of rubber type, rubber size, and rubber content on the mechanical response of sand–rubber mixtures to the applied stresses. However, the underlying particle-level mechanisms that govern this mechanical behaviour remain poorly understood. The main reason is that laboratory studies could not access particle-level information, such as rubber deformation and grain motion within the sand–rubber mixtures.

Recently, discrete element methods (DEM) have been used to model the grain-scale mechanical behaviour of sand–rubber mixtures (Valdes and Evans, 2008; Lee et al., 2014; Lopera Perez et al., 2017; Gong et al., 2019). This has enabled the acquisition of inter-particle contact forces and particle kinematics from sand–rubber mixture samples. As a result, the different contributions of sand–sand contacts, sand–rubber contacts, and rubber–rubber contacts to the shear strength of sand–rubber mixtures have been analysed. However, most of these studies adopt simplified particle shapes to model rubber grains and do not permit large rubber deformation in the DEM model, which does not represent the real scenario of irregularly shaped and highly deformable rubber grains. To take account of the deformation of rubber grains in DEM modelling, a bonded-particle method was adopted by Asadi et al. (2018) to investigate the compressive behaviour of sand–rubber mixtures under one-dimensional compression.

More recently, the rapid development of X-ray imaging equipment and image processing and analysis techniques has allowed non-destructive measurement of the internal structures of geomaterials with a high accuracy up to the micron scale. This provides new possibilities to explore the grain-scale mechanical behaviour of geomaterials under loading. As a result, X-ray tomography (CT) has been widely used to investigate the particle crushing, particle kinematics, particle shapes and fabric behaviour of pure sands (Hall et al., 2010; Andò et al., 2012; Fonseca et al., 2013; Higo et al., 2013; Zhao et al., 2015; Hurley et al., 2016; Vlahinic et al., 2017; Wiebicke et al., 2017; Alvarez-Borges et al., 2018; Cheng and Wang, 2018a; Karatza et al., 2018). However, the literature lacks of image-based investigations of the grain-scale mechanical behaviour of binary mixtures, especially for mixtures with two kinds of particles having very different stiffnesses, like sand–rubber mixtures.

This study aims to fill this gap by exploring the evolution of particle kinematics, strain localisation, and fabric behaviour of sand–rubber mixtures under shear using high-spatial resolution X-ray μCT and comparing the results with those of pure sands. For this purpose, in-situ triaxial compression tests of sand–rubber mixtures and pure sands are carried out with the use of X-ray μCT scanning. Here, in-situ implies that CT images of the samples are collected concurrent with triaxial testing. This is achieved by carrying out triaxial tests within a specially fabricated miniature loading apparatus, which is fitted into an X-ray μCT scanner. The scanner acquires the CT images of the samples at different loading stages of the tests. These images are used for image processing and analysis to extract the required grain-scale information.

The paper is organised as follows. Section 2.1 introduces the experimental setup (i.e., the X-ray μCT device and the loading apparatus used for triaxial testing), test materials, and the acquisition of CT images. Section 2.2 presents the detailed image processing and analysis techniques, including image segmentation, particle separation, inter-particle contact identification and extraction, particle tracking, and strain quantification, used to explore the grain-scale behaviour of the samples. Sections 3.1–3.3 then present the experimental results of particle kinematics, strain fields, inter-particle contact evolution, and fabric analysis of the samples. Concluding remarks are presented in Section 4.

2. Experiments and data analysis

2.1. Experimental setup, test materials, and data acquisition

The in-situ triaxial compression tests are carried out within a specially fabricated miniature loading apparatus (Cheng and Wang, 2019a), which is similar to a conventional triaxial apparatus from the structural point of view. A special advantage of the miniature apparatus is that it has no tie bars around the confining cell and can be fitted into an X-ray μCT scanner (Fig. 1a). This allows full-field scanning of the samples at different loading stages of the tests. For this study, the synchrotron-based X-ray μCT device at the 13 W beamline of the Shanghai Synchrotron Radiation Facility (SSRF) is used to acquire the scans. This device is composed of a parallel X-ray beam, a rotation stage, and a detector, as shown in Fig. 1a. The detector
has a high spatial resolution of 6.5 μm (i.e., the pixel size is 6.5 μm). An energy of 25 keV and 23 keV is used for scanning pure sands and sand–rubber mixtures, respectively. The test materials contain two pure sands, including glass beads (GB) and Leighton Buzzard sand (LBS), and two sand–rubber mixtures, including GB–rubber mixture (GB–R) and LBS–rubber mixture (LBS–R). The properties of GB, LBS and rubber are presented in Table 1. The rubber particles used in this study were bought from a commercial supplier. The normal production way is grinding the shredded waste tyres after removing the steel wires and fibres. GB is round while LBS is subangular. This allows to investigate the effect of particle shape on the testing results. As reported by Lopera Perez et al. (2017), rubber size and rubber contents have significant influences on the micromechanical behaviour of sand-rubber mixture. In their study, numerical simulations of sand-rubber mixtures with rubber-sand size ratios of 1–5 and rubber proportions of 0–30% by weight were carried out. They found that the dominant contribution to sand-rubber mixture shear strength transits from sand-sand contact forces to sand-rubber contact forces as the rubber content increases to 30% by weight, i.e., a rubber-dominant behaviour occurs. Here we focus only on the sand–rubber mixtures containing sand grains and rubber grains with similar grain sizes and volume percentages, with a particular attention paid to the quantification of their fabric anisotropy. This helps us to achieve qualitative insights into the micromechanical behaviour of sand-rubber mixtures with a high rubber content.

The GB and LBS samples have an initial grading (i.e., $d_{\text{min}}/d_{\text{max}}$) of 0.3–0.6 mm and 0.4–0.8 mm, respectively. Note that test data of these two samples was used to investigate the fabric evolution of pure sands in a previous study of the authors (Cheng and Wang, 2018b). The GB–R (or LBS–R) sample is produced by mixing 0.3–0.6 mm GB grains (or 0.3–0.8 mm LBS grains) with 0.3–0.6 mm rubber grains with a rubber content of 30% by weight. The GB, GB–R, LBS, and LBS–R samples have a pre-shearing porosity (i.e., the volume of voids over the total volume of the sample) of 0.331, 0.167, 0.343, and 0.182, respectively, which are determined based on CT images of the samples after the consolidation. It should be mentioned that it is difficult to produce samples with the same initial porosity because of the small sample size and large differences in the sample compressibility. Due to the limited access to the synchrotron source, only four representative tests were carried out. However, it is expected that the experimental observations on these samples are sufficient to reach conclusions regarding the general trends of the micromechanical behaviour of sand-rubber mixtures (with a high rubber content) under triaxial compression, especially on the qualitative level. For ease of visualisation, Table 2 summarises the pre-shearing conditions of the test samples.

For each of the test materials, a dry cylindrical sample of size 8 mm × 16 mm (diameter × height) is produced and installed within the loading apparatus, which is fixed onto the rotation stage (see Fig. 1a). To prepare a sand-rubber mixture (LBS–R or GB–R) sample, we firstly weigh sand grains and rubber grains, respectively based on the required rubber content (i.e., 30% by weight). The sand grains and rubber grains are then carefully filled into the sample from a certain height in a batchwise and alternating manner, i.e., tens of sand grains followed by tens of rubber grains are repeatedly filled into the sample until it is fully filled. The sample preparation process, although time-consuming, has effectively avoided segregation to occur.

A confining stress of 500 kPa and a strain-controlled axial load with a loading rate of 0.2%/min are applied to the sample. This confining pressure ensures that sand grains
have little crushing while rubber grains have sufficient deformation during the test. The confining stress is maintained while the axial loading is paused at different axial strains to acquire the scans. For pure sands (i.e., LBS and GB) where grain deformation is negligible, CT scans are only performed at characteristic loading states (i.e., the pre-shearing state, pre-peak state, around peak state, and post-peak state). For sand-rubber mixtures (i.e., LBS-R and GB-R), the frequency of scans is increased with a strain interval of around 2% so that the deforming process of rubber grains can be well captured. For each scan, the sample together with the entire loading apparatus is rotated across 180° at a constant rotation rate. Around 1,080 projections of the sample are acquired at different angles, which are used to reconstruct CT slices of the sample. As seen in Fig. 1b, the current parameters used for CT scanning provide CT images of all test materials with easily distinguishable phases.

Fig. 2a and b present the stress ratio \( q/p \) and volumetric strain vs. axial strain of the four samples, respectively. In the figures, the locations of CT scanning are marked with red circles. Note that the volumetric strains are calculated according to the CT image-based measurement of sample volumes at different scans (Cheng et al., 2020). The GB and LBS samples exhibit a clear peak stress ratio, which is located around the axial strain of 3.96% and 4.94%, respectively. Both samples experience a volumetric dilation (negative values denote dilation) during the post-peak stage of shear. In contrast, GB–R and LBS–R exhibit an increasing stress ratio and volumetric compression as the shear progresses. The higher deviator stress of LBS-R than GB-R is mainly resulted from the higher shear resistance mobilized from particle interlocking of irregularly shaped LBS grains than spherical GB grains (Li et al., 2019). The change from a mainly dilative response of the pure sands to a compressive response of the sand-rubber mixtures is attributed to the addition of rubber grains rather than the difference in sample porosities, which is obvious considering that the sand-rubber mixture samples have much lower initial porosities than the pure sand samples. Meanwhile, GB–R shows a lower initial stiffness, but a higher stress ratio at large axial strains (e.g., when axial strain is larger than 12%) than GB, even if GB–R has a lower initial porosity than GB (see Table 1). Similarly, LBS–R exhibits a lower initial stiffness than LBS. The stress–strain behaviour of these samples indicates that adding rubber grains into a host sand tends to decrease the initial stiffness of

<table>
<thead>
<tr>
<th>Sample</th>
<th>Pre-shearing porosity</th>
<th>Confining stress</th>
<th>Grain size</th>
<th>Rubber content by weight/%</th>
<th>Scanned acquired at axial strain/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>GB</td>
<td>0.331</td>
<td>500 kPa</td>
<td>0.3–0.6 mm</td>
<td>0</td>
<td>0, 2.02, 3.96, 8.06, 12.14</td>
</tr>
<tr>
<td>GB-R</td>
<td>0.167</td>
<td>500 kPa</td>
<td>0.3–0.6 mm glass beads mixed with 0.3–0.6 mm rubber grains</td>
<td>30</td>
<td>0, 2.67, 3.85, 5.77, 7.68, 9.66, 11.81, 13.5, 15.53, 17.49, 18.92</td>
</tr>
<tr>
<td>LBS</td>
<td>0.343</td>
<td>500 kPa</td>
<td>0.4–0.8 mm</td>
<td>0</td>
<td>0, 0.98, 4.94, 104, 15.34</td>
</tr>
<tr>
<td>LBS-R</td>
<td>0.182</td>
<td>500 kPa</td>
<td>0.3–0.8 mm LBS grains mixed with 0.3–0.6 mm rubber grains</td>
<td>30</td>
<td>0, 1.26, 2.27, 4.19, 5.98, 7.84, 9.8, 11.52, 13.4, 15.39</td>
</tr>
</tbody>
</table>

Table 1
Properties of materials used for triaxial test.

<table>
<thead>
<tr>
<th>Material property</th>
<th>LBS</th>
<th>GB</th>
<th>Rubber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity</td>
<td>2.65</td>
<td>2.60</td>
<td>1.15*</td>
</tr>
<tr>
<td>Grain diameter/mm</td>
<td>0.4–0.8</td>
<td>0.3–0.6</td>
<td>0.3–0.6</td>
</tr>
<tr>
<td>Maximum void ratio</td>
<td>0.812</td>
<td>0.731</td>
<td></td>
</tr>
<tr>
<td>Minimum void ratio</td>
<td>0.585</td>
<td>0.543</td>
<td></td>
</tr>
<tr>
<td>Young's modulus/kPa</td>
<td>5.2×10⁷ $</td>
<td>$</td>
<td>5.8×10⁷ $</td>
</tr>
<tr>
<td>Bulk modulus/kPa</td>
<td>3.47×10⁷ $</td>
<td>$</td>
<td>4.83×10⁷ $</td>
</tr>
<tr>
<td>Shear modulus/kPa</td>
<td>2.08×10⁷ $</td>
<td>$</td>
<td>2.23×10⁷ $</td>
</tr>
<tr>
<td>Poisson’s ratio/kPa</td>
<td>0.25 $</td>
<td>$</td>
<td>0.30 $</td>
</tr>
</tbody>
</table>

$|$ Li et al. (2020).
$|$ Sandeep and Senetakis (2018).
$|$ Kim and Santamarina (2008).
the mixture. This observation is consistent with those reported in the literature (Li et al., 2019).

2.2. Image processing and analysis

2.2.1. Image segmentation and particle separation

To investigate the grain-scale mechanical behaviour of a sample, the raw 3D CT image (i.e., a stack of 2D greyscale images) of the sample acquired at each scan is subject to a series of image processing and analysis steps. First, each phase is identified from the 3D CT image. For the GB and LBS samples, the CT image contains only two phases, i.e., the sand phase and the void phase. Note that as the porous stones at the sample ends have similar greyscale intensities as sand grains, for the purpose of simplicity they are not distinguished from the sand grains in the image segmentation process. A low-pass filter (Perona and Malik, 1990) is implemented on the raw CT image, and a global thresholding is implemented on the filtered image to convert it to a binary image using Avizo (Visualization Science Group). The binary image contains pixels with an intensity of either 1 or 0, representing the sand phase and the void phase, respectively. It should be noted that using a global threshold for image binarisation may lead to an over-detection of inter-particle contacts in subsequent image processing stages (Wiebicke et al., 2017). However, a parametric investigation in a previous study (Cheng, 2018) indicates that if a high-resolution X-ray scanner is used, the over-detection of inter-particle contacts that results from global thresholding of a CT image does not have a significant influence on the fabric behaviour. For the GB–R and LBS–R samples, the CT image contains three phases, i.e., the sand phase, the rubber phase, and the void phase. The identification of these three phases requires a ‘trinarisation’ process which converts the CT image to an image containing pixels with only three intensity values (e.g., 0, 1, and 2). It should be mentioned that there is a membrane included in the CT image, which is regarded as the void phase in the image trinarisation because of its small thickness (around 0.3 mm) and its close CT values to the void phase. This does not have significant influence on the image segmentation of the materials encompassed by the membrane. To implement the image trinarisation, an improved region growing method (Cheng and Wang, 2020) is applied to the CT image. Note that alternative image segmentation techniques are available to deal with the trinarisation of CT images with three phases (Higo et al., 2013; Wang et al., 2019). The improved region growing method can avoid over-estimation of the middle phase (i.e., the rubber phase in the current study) that results from using general region growing methods. To correct the image segmentation results within the surrounding membrane region, the segmented image is filtered using a mask, which is a binary image containing only one solid region representing the sample without membrane. The mask is acquired through binarising the sample CT image using a global threshold and then implementing morphological operations including dilation, filling holes, and erosion to remove void regions within the sample. The process of determining the mask is similar to that used for calculating the sample volume (Cheng et al., 2020). For ease of visualisation, the 2D image trinarisation results for the GB–R (Fig. 3a) and LBS–R (Fig. 3c) samples, and their intensity histograms are presented in Fig. 3. There are many pixels with a grey level of 255 in Fig. 3f, which is associated with the inclusion of highly X-ray absorptive minerals of some LBS grains. Note that in the current study, image trinarisation is implemented on 3D images. It can be seen from Fig. 3b and d that the image segmentation method provides satisfactory results for both samples.

Then, for each trinarised image (or binarised image for GB and LBS), the sand phase (or glass bead phase) is extracted and stored in a binary image. A marker-based watershed algorithm (Wa¨hlby et al., 2004) is applied to the binary image to separate the individual grains. The results are stored in a labelled image, in which each grain has a unique intensity value. Fig. 4a and b present the particle separation results of GB–R on the vertical slice shown.
in Fig. 3b, and LBS–R on the vertical slice shown in Fig. 3d, respectively. Note that contact voxels between the grains are removed from these images and that different particles are indicated by different colours. From these figures it can be seen that for both samples most of the individual particles are effectively separated.

2.2.2. Extraction of sand–sand contacts and sand–rubber contacts

Sand–sand contacts (i.e., GB–GB contacts of GB and GB–R, and LBS–LBS contacts of LBS and LBS–R) of the samples are extracted based on the results of the watershed segmentation. Here, GB-GB and LBS-LBS represent contacts among GB grains and contacts among LBS grains, respectively. Specifically, the sand–sand contact voxels of a sample are extracted by implementation of a logical ‘and’ between the binary image of the sand phase and a binary image of watershed lines acquired from the marker-based watershed segmentation. These contact voxels are stored in a labelled image where each contact region has a unique intensity value. Note that the contact regions with less than 10 voxels are not considered as real contacts and are removed from the labelled image. The threshold of 10 voxels is used to consider the over-estimation of inter-particle contacts caused by partial volume effects of the CT image. It should be noted that the results would not be significantly different if a different threshold, for example, 5 voxels is used. A 3D dilation is implemented on the labelled image with a $3 \times 3 \times 3$ cubic structural element mask, which is intrinsically a $3 \times 3 \times 3$ matrix with all elements equaling 1. For each contact region of the dilated image, we check each particle in the labelled image of particles: the particle who has an overlap with this region are determined as the particles sharing this contact. A MATLAB (Mathworks, Natick, MA, USA) script is used to extract the sand–sand contacts and assign them to the corresponding particles. A full description of this process can be found in Cheng and Wang (2018b).

To determine the sand–rubber contacts (i.e., GB–rubber contacts of GB–R, and LBS–rubber contacts of LBS–R), the rubber phase of the trinarised image is extracted and stored in a binary image similar to the sand phase. A 3D image dilation is implemented on the binary image using a $3 \times 3 \times 3$ cubic structural element mask. The sand–rubber contacts are determined by applying a logical ‘and’ between the dilated image and the labelled image of sand grains. Each contact region is assigned to a sand particle if the extracted contact region belongs to that sand particle. The procedure of detecting sand–rubber contacts from the trinarised image is illustrated in Fig. 5.

In this study, sample fabric is quantified using the branch vectors of contacts. Note that fabric can also be quantified using contact normals, inter-particle voids and particle orientations. Here, inter-particle voids refer to voids encompassed by several neighboring particles. Fig. 6 presents the definition of branch vectors for sand–sand contacts and sand–rubber contacts. The branch vector of a sand–sand contact is determined as the centroid coordinate difference between the two particles sharing this contact. Note that because of the irregular surface of sand particles, especially the LBS particles, there may be multiple contact regions between two particles. In this situation, these contact regions are taken as the same contact if they are shared by the same two particles. It should be pointed out that rubber grains deform so dramatically during the test that it is impossible to separate individual rubber grains based on the CT image. In this condition, the centroid coordinates of individual rubber grains cannot be
determined. Therefore, the traditional definition of the branch vector for grain–grain contacts is not adopted for sand–rubber contacts. The branch vector of a sand–rubber contact is defined as the difference in centroid coordinates between the contact region and the sand particle.

2.2.3. Particle tracking and strain quantification

Particle kinematics of the samples during each shear increment are quantified based on a grain tracking approach (Cheng and Wang, 2018a). The method uses either particle volume or particle surface area as the particle tracking criterion, and requires the input of centroid coordinates of particles at both the reference and the deformed configurations (i.e., the loading states at the start and end of the shear increment) to track the sand particles (i.e., GB or LBS grains) during the shear increment. The centroid coordinate, volume, and surface area of each particle are acquired from the labelled images of sands using a MATLAB intrinsic function ‘regionprops’ (Cheng, 2018; Cheng and Wang, 2018a). The displacement of a particle during the shear increment is calculated as the difference in centroid coordinates of the particle at the start and the end of the shear increment. Particle rotation is quantified according to the change in orientation of particle principal axes during the shear increment. The particle principal axes are determined using the ‘pca’ function of MATLAB and are considered to have sufficient precision for sub-angular particles such as LBS grains (Fonseca, 2011; Cheng and Wang, 2018a). Note that as spherical grains such as GB do not have unique principal axes, their grain rotations are not quantified in the current study.

![Fig. 4. Separation of GB particles (a) and LBS particles (b) from the GB-R and LBS-R samples, respectively.](image)

![Fig. 5. Illustration of the determination of sand-rubber contacts: (a) trinarised image with sand phase denoted in green, void phase in white and rubber phase in yellow; (b) binary image of rubber phase after image dilation; (c) extracted sand-rubber contacts of two individual sand particles.](image)
The strain field of a sample during each shear increment is calculated according to a tetrahedral mesh-based method (Wang et al., 1999; Cheng and Wang, 2019b). Specifically, a Delaunay triangulation is performed on the tracked particle centroids to generate a set of tetrahedral meshes. The strain tensor $\varepsilon_{ij}$ of each tetrahedral element that joints the centroids of four neighbouring sand particles is calculated based on the particle displacement. The octahedral shear strain $\varepsilon_s$ and volumetric strain $\varepsilon_v$ are calculated by Eqs. (1) and (2), respectively:

$$\varepsilon_s$$

$$\varepsilon_v$$

---

Fig. 6. Illustration of the definition of sand-rubber branch vectors and sand-sand branch vectors.

Fig. 7. Displacement and rotation of sand particles in: (a) LBS-R; (b) LBS.
\[ e_{ij} = \frac{2}{3} \times \sqrt{(e_{11} - e_{33})^2 + (e_{11} - e_{33})^2 + (e_{22} - e_{33})^2 + 6e_{12}^2 + 6e_{13}^2 + 6e_{23}^2}, \]

where \( e_{ij} \) is an element of \( \varepsilon_{ij} \).

3. Experimental results

3.1. Particle kinematics and strain fields

Fig. 7 presents 3D maps of particle displacements and particle rotations of LBS–R and LBS at different shear increments. Note that only the results of several shear increments are presented due to the limited space of the paper. As shown in Fig. 7a, LBS–R exhibits a gradual increase of particle displacement from the sample base upwards at the early shear stages (e.g., at the axial strain of 0–1.26% and 4.19–5.98%). This is anticipated, as the sample is loaded from the top downwards. A localised band of particle displacement is initiated at the shear increment of 9.8–11.52% and is more obviously shaped at the shear increment of 13.4–15.39%. In contrast, the sample shows a similar disorganised distribution of low particle rotations (usually smaller than 20°) at all shear increments. These observations differ significantly from those of LBS. As shown in Fig. 7b, there is no clear localisation of particle displacements or particle rotations at the early shear stages (i.e., 0–0.98% and 0.98–4.94%) of LBS prior to the peak shear–stress ratio. However, LBS reveals a clear localised band in both the particle displacement maps and the particle rotation maps at the post-peak shear stages (i.e., 4.94–10.4% and 10.4–15.34%). Compared to LBS–R, LBS reveals a clearly higher degree of particle rotation within the shear band.

Note that for the same granular materials sheared under a low confining pressure, more particle rotations are generally associated with samples with lower initial porosities (Alshibli et al., 2017). In this study, the higher degree of particle rotation of LBS than LBS-R is attributed to the different interparticle contact types between the two samples rather than their initial porosity difference (LBS-R has a much lower initial porosity than LBS). In the LBS sample the external loads are carried by LBS–LBS contacts, which generally have rather small contact areas. In contrast, the LBS grains of LBS–R sample are surrounded by a large number of rubber grains. These grains participate not only in LBS–LBS contacts, but also LBS–rubber
contacts, which also carry the external loads but usually have large contact areas. Due to the larger contact areas of LBS–rubber contacts than LBS–LBS contacts, the LBS–R sample has a higher anti-rotation resistance for the LBS grains than the LBS sample. As a result, the LBS grains have a higher degree of particle rotation within the shear band of the LBS sample than the LBS–R sample.

Fig. 8 presents the strain results on a 2D slice of the four samples. Note that only several shear increments are selected for illustration. The vertical slice is chosen such that it contains the axis of the cylindrical sample and is parallel to the normal vector of the shear band plane. The shear strain maps of LBS and LBS–R exhibit a pattern similar to the particle displacement maps shown in Fig. 7. However, a much higher degree of shear strain magnitudes is observed in LBS than in LBS–R. A similar trend appears in the GB and GB–R samples. Meanwhile, GB and LBS show significantly localised volumetric dilation in the last two shear increments. In contrast, even with significantly lower sample porosities, GB–R and LBS–R reveal volumetric compressions with notably lower localisation. This phenomenon implies that rubber grains prevent the formation of shear bands, which is consistent with observations from previous studies (Gong et al., 2019). These results also indicate that sand grains in GB–R and LBS–R samples move closer to each other as shear progresses, mainly due to the high deformability of rubber particles.
3.2. Inter-particle contact evolution

Fig. 9 depicts the evolution of the average coordination number (CN) of sand–sand contacts within the samples. As seen in Fig. 9a, for both GB–R and LBS–R the average CN of sand–sand contacts experiences a gradual increase at the early shear stage (e.g., 0–5.77% for GB–R and 0–4.19% for LBS–R). This is followed by a stage where the average CN is relatively stable, before it gradually decreases at the last shear stage (e.g., 9.66–18.92% for GB–R and 11.52–15.39% for LBS–R). Given that both samples experience volumetric compression throughout the entire test (see Figs. 2b and 8), these phenomena imply that the average CN does not have a negative correlation with sample porosity. These are dramatically different to those observed in pure sands. As shown in Fig. 9b, the average CN decreases throughout the shear progress where volumetric dilation occurs (i.e., 0–12.14% for GB and 0.98–15.34% for LBS). This is attributed to the different interparticle contact interaction modes between pure sands and sand–rubber mixtures, which will be discussed in Section 3.4.

Fig. 10 presents the evolution of the normalised sand–rubber contact area of sand (LBS or GB) grains. Here, the normalised sand–rubber contact area of a sand grain is defined as the total area of rubber–sand contacts (i.e., multiple isolated sand–rubber contacts may exist for a single sand grain) on the particle divided by its surface area. As seen in Fig. 10a and b, the probability density function (PDF) curves of normalised rubber–sand contact area shifts right for both GB–R and LBS–R, indicating that the probability of larger normalised rubber–sand contact areas increases as the shear progresses for both samples. This is also reflected in the increasing mean values of normalised rubber–sand contact area, as shown in Fig. 10c.

Fig. 9. Sand-sand contact coordination number evolution: (a) in GB-R and LBS-R samples; (b) in GB and LBS samples.

Fig. 10. Evolution of normalized contact area between sand and rubber: (a) PDF of normalized contact area between GB and rubber; (b) PDF of normalized contact area between LBS and rubber; (c) mean and standard deviation.
However, it is noted that the peak of the PDF curve of GB-R gradually lowers and the shape of the curve becomes more symmetrical during the shearing process; while such a phenomenon is largely absent in LBS-R. This observation suggests that the particle shape plays an important role in the evolution of rubber-sand contact area: the spherical...
shape of GB offers a higher chance to reduce those rubber-sand contacts with lower contact areas and form a more homogeneous distribution than the sub-angular shapes of LBS as shear progresses.

3.3. Fabric analysis

The fabric of sand–sand contact branch vectors is quantified using a second-order fabric tensor (Satake, 1982) according to Eq. (3):

$$ F_{ij} = \frac{1}{N} \sum_{k=1}^{N} n_k^i n_k^j, $$

where $N$ is the total number of sand–sand contact branch vectors, and $n_k^i$ is the component of the $k^{th}$ unit orientation vector along direction $i$.

For the sand–rubber contact branch vectors, a fabric tensor accounting for the different contributions of contacts with various contact sizes is defined according to Eq. (4):

$$ F_{ij} = \frac{1}{\sum \omega_k} \sum_{k=1}^{N} \omega_k n_k^i n_k^j, $$

where $N$ is the total number of sand–rubber contacts, $n_k^i$ is the component of the $k^{th}$ unit orientation vector along direction $i$, and $\omega_k$ is the area weight of the $k^{th}$ unit orientation vector.

Referring to Yimsiri and Soga (2010), the degree of anisotropy of a fabric tensor can be quantified by a single parameter $a$ as given by Eq. (5):

$$ a = \frac{1}{\sum \omega_k} \sum_{k=1}^{N} \omega_k n_k^i n_k^j. $$

Fig. 12. Sand-rubber contact fabric evolution: (a) variation of anisotropy degree with axial strain; (b) normalized orientation frequency distribution of the branch vectors.
\[ a = \frac{1}{3} \left( \frac{15F_{11} - 5}{5F_{11} - 3} + \frac{15F_{22} - 5}{5F_{22} - 3} + \frac{15F_{33} - 5}{5F_{33} + 1} \right), \]

where \( F_{ii} (i = 1, 2, \text{or } 3) \) is a diagonal element of the fabric tensor given by Eqs. (3) or (4). While \( a = 0 \) denotes that the branch vectors are isotropic, \( a > 0 \) and \( a < 0 \) indicate that the branch vectors tend to concentrate in the vertical direction (direction 3) and the horizontal direction (directions 1 and 2), respectively.

Fig. 11 presents the variation of anisotropy degree \( a \) of sand–sand contacts with the axial strain for the four samples. As seen in Fig. 11a, the GB–GB contacts of GB and GB–R reveal a similar initial degree of anisotropy around 0, which indicates a nearly isotropic distribution of GB–GB branch vectors for both samples at the isotropic compression state. While the anisotropy degree of GB–GB contacts of GB–R remain nearly unchanged, that of GB increases dramatically as the shear–stress ratio increases to the peak and undergoes little change thereafter. This is attributed to the difference of force-transmission mechanisms between GB and GB–R. In GB, the directions of GB–GB contacts change to the vertical direction to resist the deviatoric loads as the shear progresses. While in GB–R, the deviatoric loads are also carried by GB-R contacts, which will be discussed in the following context.

Similar trends have been found for the LBS and LBS–R samples, for which a negative anisotropy degree is observed at the initial state. This denotes that the initial LBS–LBS contact branch vectors exhibit a bias towards the horizontal direction at the isotropic compression state. This is anticipated, as LBS particles have low sphericity and tend to lie in the most stable position when they are deposited. For ease of visualisation, Fig. 11b presents the normalised orientation frequency of the sand–sand contact branch vectors of the four samples at selected axial strains. The normalised frequency of branch vectors along a specific orientation is calculated as the frequency of branch vectors along this orientation divided by the maximum frequency for all orientations. The sand–rubber mixture samples reveal a nearly unchanged orientation distribution of branch vectors at all axial strains, while the pure sand samples exhibit an orientation preference of sand–sand branch vectors towards the vertical direction at the latter two axial strains. These observations are in agreement with those shown in Fig. 11a.

The sand–rubber contacts reveal a significantly different fabric behaviour from the sand–sand contacts within the sand–rubber mixture samples, as shown in Fig. 12a. The sand–rubber contact branch vectors of both samples experience a significant increase of anisotropy degree throughout the test. This indicates a change of orientation distribution of the branch vectors from a nearly isotropic state (e.g., the LBS–R) or a state having a slight preference towards the horizontal direction (e.g., the GB–R), to a significant bias towards the vertical direction during shear, as depicted in Fig. 12b. These phenomena and those revealed by Fig. 11 suggest an increasingly important role of sand-rubber contacts in the transmission of deviatoric load as shear proceeds.

To investigate the roles of sand–rubber contacts with different sizes in sample fabric evolution, the sand–rubber contacts are categorised into 10 groups according to their contact area ranges. The fabric of each group of contacts (i.e., from the 1st group to the 10th group of contacts with increasing contact area) is quantified according to Eqs. (3) and (5). The ten contact area ranges are determined according to the cumulative probability distribution of sand–rubber contact area such that each range has the
same number of sand–rubber contacts, as shown in Fig. 13a. Interestingly, for both samples the sand–rubber contacts with small sizes (e.g., contacts in the 1st to 8th size ranges) have an anisotropy degree lower than zero at all axial strains, as seen in Fig. 13b and c. There is a fluctuation of anisotropy degree in these small-sized sand-rubber contacts for GB-R. This might be attributed to the initial inhomogeneity of sand-rubber contact distribution induced in sample formation. In contrast, the sand–rubber contacts with large sizes (i.e., contacts in the 9th and 10th size ranges) have a positive and increasing anisotropy degree with the increase of axial strain. These results suggest that large-sized sand–rubber contacts which concentrate towards the major principal stress direction carry higher deviatoric load during shearing, while small-sized sand–rubber contacts that concentrate towards the minor principal stress direction serve as a lateral support for the load-bearing contacts.

3.4. The micromechanical processes of sand–rubber mixture deformation

During the deformation of sand-rubber mixtures, sand-sand contacts are continuously formed and separated as a result of inter-sand voids’ shrinkage. To illustrate the formation and separation of sand-sand contacts, Fig. 14 presents typical CT images acquired at different loading stages of GB-R. At the axial strain of 0%, GB particle A is in contact with particle B, and not in contact with particle C (see the top left plot of Fig. 14). Rubber grains deform and reach the corner of the inter-sand void between A and B at the axial strain of 5.77%, as shown by the arrows on the top middle plot of Fig. 14. As the rubber grains continuously deform, a separation of the contact between A and B occur at the axial strain of 9.66%, and an obvious distance between the boundaries of the two particles appears at the end of the test (i.e., the axial strain of 18.92%). Meanwhile, particle A becomes closer to particle C as the sample deforms. As a result, a contact between the two particles creates at the end of the test.

In light of the above findings regarding sand–sand contact and sand–rubber contact fabric behaviour, the deformation of a sand–rubber mixture sample can be divided into three stages, as shown in Fig. 15. Note that the blue region represents a deforming rubber grain, and that the grains denoted by grey circles can be either spherical or irregularly shaped sand grains. At the first shear stage, most rubber grains are located at the ‘centre’ of inter-sand voids (i.e., the voids between sand particles) and the sand grains become closer to each other as external loads increase. As a result, both the average CN of sand–sand contacts and sand–rubber contact areas increase. At the second shear stage, more and more rubber grains start to reach the ‘corner’ of these inter-sand voids, as a result of the shrinkage of these voids and the continuous deformation of the rubber grains. This leads to a relatively stable average CN of sand–sand contacts. While the number of sand–sand contacts increases due to the shrinkage of inter-sand voids, the deformation of rubber grains towards the ‘corner’ of inter-sand voids also tends to separate the sand–sand contacts, which occurs when the ‘driving forces’ from the sand–rubber contacts are larger than the corresponding sand–sand contact forces. These two factors work as two competing roles in determining the overall average CN of sand–sand contacts. As shear progresses to the third stage, the inter-sand void shrinks and more
rubber grains deform to the void corner. The increase of sand–rubber contact forces, as indicated by the increase of sand–rubber contact areas at the void corner, leads to the separation of more sand–sand contacts. This role dominates over the generation of new sand–sand contacts which results from inter-sand void shrinkage. As a result, the average CN of sand–sand contacts decreases at this stage.

During the test, the deviatoric loads are increasingly carried by the sand–rubber contacts, as implied by the increase of sand–rubber interface area and sand–rubber contact anisotropy degree.

4. Conclusions

In this paper, X-ray μCT is used to investigate the micromechanical behaviour of sand–rubber mixtures under triaxial compression. Typical spherical and sub-angular sand grains (i.e., GB and LBS, respectively) are used to produce the sand–rubber mixtures (i.e., GB–R and LBS–R) with a rubber content of 30% by weight. The results are compared to those of pure sands. This has allowed conclusive remarks to be made regarding the general trends of micromechanical behaviour of sand-rubber mixtures with high rubber contents. The main findings are summarised below.

Both sand–rubber mixture samples exhibit a lower initial shear stiffness than the corresponding sands without rubber grains. A significant localisation of particle rotations of sand grains arises in the LBS sample, but does not occur in the LBS–R sample. This is due to the stronger anti-rotation effects of sand–rubber contacts on the sand grains than sand–sand contacts, which results from the larger contact areas of the former in the sand–rubber mixture than the latter in the pure sand sample. Both sand–rubber mixture samples show significantly lower magnitudes of shear strain than those of pure sands, which indicates that rubber grains can prevent the formation of shear bands.

Evolution of the average CN of sand–sand contacts can be divided into three stages during shearing. At the first stage, most rubber grains are located at the ‘centre’ of inter-grain voids of the sand particles. Shrinkage of these voids as the sample deforms leads to a gradually increasing average CN of sand–sand contacts. At the second stage, increasingly deforming rubber grains start to reach the ‘corner’ of these voids, which results in the separation of some sand–sand contacts. A balance is reached at this stage between the generation of new sand–sand contacts caused by void shrinkage and the separation of old sand–sand contacts resulting from rubber deformation. Consequently, the sand–rubber mixtures reveal a relatively average CN of sand–sand contacts. At the final stage, a decreasing average CN of sand–sand contacts arises, which results from the dominant role of rubber deformation over void shrinkage.

Increasing contact area and anisotropy degree of sand–rubber contacts are observed as shear progresses. In contrast, sand–sand contacts reveal an almost unchanged and nearly isotropic orientation frequency distribution throughout the test, suggesting a dramatically different stress-transmission mechanism of sand-rubber mixtures from pure sands. Moreover, the large-sized sand–rubber contacts reveal an increasing directional preference to the major principal direction, while the small-sized sand–rubber contacts exhibit a bias towards the minor principal direction throughout the test. This phenomenon suggests that higher deviatoric loads are carried by the large-sized sand–rubber contacts, while the small-sized sand–rubber contacts provide a lateral support to the load-bearing sand–rubber contacts.

This study highlights the important role of sand-rubber contacts in the stress-transmission behaviour of sand–rubber mixtures containing high proportions of rubber grains. It provides important experimental evidences to
previous numerical studies on the micromechanics of sand-rubber mixtures. For sand-rubber mixtures with significantly lower rubber contents or sand-rubber size ratios, the mechanical behaviour may be very different, as reported in previous numerical studies. Future research work will focus on such sand-rubber mixtures to unravel the effects of rubber content and rubber size on the micromechanical behaviour of sand-rubber mixtures.

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