Design and Modeling of a Magnetic-Coupling Monostable Piezoelectric Energy Harvester Under Vortex-Induced Vibration

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ABSTRACT Vortex-induced vibration (VIV) is used by piezoelectric energy harvesters to generate electricity from wind and water flow. In this study, we introduce the nonlinear magnetic force into piezoelectric energy harvesters and develop a nonlinear monostable piezoelectric VIV transducer. We build a distributed-parameter model based on the Euler-Bernoulli beam theory and Kirchhoff’s law to analyze the dynamic responses of the magnetic-coupling piezoelectric energy harvester (MCPEH). Model results show that the performance of the MCPEH varies greatly with the increase of the load resistance and the length of the used PZT. There are two optimal resistance values for the MCPEH. When $R < 31.6 \, \text{k}\Omega$, both the external load resistance and the PZT length affect the maximum power output. The little optimum resistance value will dwindle with the increase of the PZT length, whereas the large optimum resistance value still fixes at 1.78 M\Omega with the increase of the PZT length. Due to the resistive shunt damping effect and the kinetic energy of wind, the resonance domain becomes wider in these ranges of load resistance smaller than 31.6 k\Omega and larger than 316 k\Omega comparing that when the load resistance is larger than 31.6 k\Omega and smaller than 316 k\Omega. Besides, the performance is enhanced by the monostable nonlinear magnetic force, which can be improved by decreasing the value of the distance between moving magnets and fixed magnets. The energy harvester shows a maximum power output of 0.21 mW under excitation of wind velocity is 1.6 m/s when the cylindrical diameter is 20 mm, the PZT length is 30 mm and the load resistance is 0.5 M\Omega.

INDEX TERMS Vortex-induced vibration, monostable, magnetic force, distributed-parameter model.

I. INTRODUCTION

Electricity has been the necessary resource around the world. Whatever the raw energy is, the electricity is always the final transformation form to use directly in many respects, such as production, communication, transportation, and so on. With the increase of electricity consumption, it is vital to explore some methods for converting other forms of energy into electricity sustainably. Therefore, investigating the conversion methods fascinates many researchers. The photovoltaic solar technology [1], [2], the turbine energy harvesting technology [3]–[5], the nuclear power technology [6], [7] are all the great energy conversion technologies.

However, not only the large-scale power departments need electricity, but also the micro devices need. Wireless sensor devices, micro electro mechanical systems (MEMS), and unmanned probes or unmanned aerial vehicles have performed their irreplaceable position in the modern industrial community [8]–[10]. These low-energy consumption devices always install a severe environment and do not need frequent manual battery replacement, which should need a sustained and stable power supply mode, especially allowed various forms of energy to be obtained from the surrounding environment. Energy harvesting and converting technology have been developed in the recent 20 years, which is quite suitable for the above mentioned devices. Piezoelectric energy harvester (PEH) [11]–[14], electro-magnetic energy harvester [15], [16], electrostatic energy generator [17], [18],
and dielectric elastomers energy harvester [19], [20] are mainly the types of energy harvesting and converting technologies. Especially, the PEH has been studied extensively because of its high power density, no electro-magnetic interference, and stability. Based on the PEH, multiple mechanical oscillations have been used to harvest energy, such as base excitations [13], [21], [22], rotational vibrations [23], and flow-induced vibrations [24]–[27]. Flow-induced piezoelectric energy harvester is also divided into many types by the shape of bluff body, which includes vortex-induced vibration (VIV) [28]–[30], galloping [31]–[34], flutter [35], [36] and wake-induced vibration (WIV) [37], [38]. Vortex-induced vibration piezoelectric energy harvester (VIVPEH) can convert the low-velocity wind into electrical power. Song et al. [28] established the VIVPEH under the water and the maximum output power was 84.49 μW with 60.35 mW/m² energy density at the resonance vibration velocity of 0.35 m/s, and proposed a novel PEH with a bicylinder in water based on the VIV as well as WIV [29]. Shan et al. [11] compared a new underwater double piezoelectric energy harvesters system with the same parameters arranged in series to a single piezoelectric energy harvester system. Zhang et al. [17] designed and tested a broadband electrostatic energy harvester with a dual resonant structure, which consists of two cantilever-mass subsystems each with a mass attached at the free edge of a cantilever. And the results showed that the continuous power output up to 6.2–9.8 μW can be achieved under external vibration amplitude of 9.3 m/s² at a frequency range from 36.3 Hz to 48.3 Hz. As the above-mentioned research, they wanted to use multi-PEHs to enhance the output power. In order to enhance the performance, adding other force is also utilized by researchers, like nonlinear magnetic force. Zhou and Zuo [39] revealed the nonlinear dynamic mechanism of asymmetric bistable energy harvesters and enhance the energy harvesting performance, then proposed a stochastic energy harvester based on nonlinear vibration [40]. Zhao et al. [41] proposed an innovative approach for rotational piezoelectric wind energy harvesting using magnetic coupling and force amplification mechanisms and can generate an instantaneous power of 0.8 mW at a low wind speed (≈3 m/s). Zhang et al. [42] proposed an operable strategy to enhance the output power of piezoelectric energy harvesting from the VIV using nonlinear magnetic forces and then by changing the shape of bluff body, they designed and modeled a novel and efficient electromagnetic energy harvester for airflow power generation with the cross-section of Y-shape [15]. Zou et al. [43] proposed a two-degree-of-freedom flow-induced vibration (FIV) of a rotating circular cylinder system, which was induced in a range of 3.0 m/s < U* < 14 m/s. It was found from the simulation that the oscillation of cylinder was enhanced significantly by rotation in flow direction, but the opposite phenomenon was observed in cross flow direction. As well as based on the piezoelectric theory, researchers also explored the theory and application of ultrasonic motor [44]–[48]. However, there is few research on wind energy harvesting under nonlinear magnetic force.

In this work, a novel magnetic-coupling monostable piezoelectric energy harvester under vortex-induced vibration was proposed and analyzed. The performance of the MMPEH was investigated by the numerical simulation analysis. The physical model is constructed in section 2. The distributed-parameter model is established in section 3. The simulation results are discussed in section 4. And some valuable conclusions are drawn in section 5.

II. DESIGN OF THE MCPEH SYSTEM

In the present work, a nonlinear magnetic-coupling piezoelectric energy harvester (MCPEH) based on the vortex-induced vibration is proposed, designed, and analyzed. As shown in Fig. 1, the MCPEH is composed of a cantilever beam attached with two piezoelectric sheets and connected to a cylindrical rigid body. The materials of substrate layer and PZT layer are aluminum and PZT-5H respectively. The PZT layer is stuck on the substrate layer by using the epoxy adhesive (LOCTITE E-120HP). The material of cylindrical rigid body is foamed plastic. In order to introduce the nonlinear magnetic force, two pairs of magnets (including two fixed magnets and two moving-magnets) are constructed. The fixed magnets are equipped near the two ends of cylinder respectively. The moving-magnets are embedded in the cylindrical rigid body along the axis, which is against the fixed magnets when the cylinder is in an equilibrium position. When the wind comes with a specific velocity, the Karman Vortex Street appears and the PZT will convert the vibration energy into electric energy due to the piezoelectric effect. Besides, the nonlinear magnetic force should have great effect on the vibration responses during vibrating, which the effect will be shown in next discussion.

The length of substrate layer and PZT layer are L_s and L_b respectively. w_p and w_s are the widths of substrate layer and PZT layer respectively. h_p and h_s are the thickness of PZT layer and substrate layer respectively. D and 2R_m are individually the diameters of cylinder and magnets including fixed magnets and moving-magnets. L_c is the length of the
The piezoelectric beam.

\( \frac{d}{dt} \) means substrate layer and piezoelectric layer, respectively.

The partial derivative of time \( t \) and the partial derivative of arc length \( x \) can be drafted first: (1) ' \( t \) is the partial derivative of time \( t \) and \( \rho_s \) and \( \rho_p \) are the volume.

(2) ' \( x \) is the partial derivative of arc length \( x \) and the equivalent capacitance of the piezoelectric sheets.

\( R \) is the external load resistance. By consulting Fig. 2 and Fig. 3, it can be got that \( h_a = h_d = h_p = h_s/2 \), \( h_c = h_b = h_s/2 \).

III. MODELING

A. ESTABLISHING THE DISTRIBUTED PARAMETER MODEL

The distributed-parameter model can be established according to the Lagrangian Equation and Hamilton Principle. Firstly, the kinetic energy of the system \( (T_b) \) is the sum of the kinetic energy of piezoelectric beam \( (T_{pb}) \) and cylinder \( (T_c) \), the potential energy of the system \( (U_e) \) is the sum of the elastic potential energy of piezoelectric beam \( (U_{eb}) \), electric potential energy \( (U_e) \) and magnet-induced nonlinear potential energy \( (U_m) \). In order to simplify expressions, several specifications can be drafted first: (1) ' \( t \) is expressed as a partial derivative of arc length \( x \). (2) ' \( t \) is expressed as a partial derivative of time \( t \). (3) Subscript ' \( x \) ' and ' \( p \) ' are used to means substrate layer and piezoelectric layer, respectively. (4) \( w(x, t) \) is expressed as the displacement of the free end of the piezoelectric beam.

In order to establish the mathematical model of the MCPEH, some assumptions should be made as following:

1.-PZT layer is adhered to substrate layer and the adhesive is neglected.

2.-Incoming wind flow is a stable laminar flow field.

3.-Electric field intensity of the piezoelectric crystal is uniformly distributed along the thickness direction.

4.-Moving magnet and fixed magnet are distributed on the same axis in the initial position.

\( T_b \) is the sum of the kinetic energy of piezoelectric layer and substrate layer, which can be expressed as

\[
T_b = \frac{1}{2} \int_{V_s} \rho_s w^2 (x, t) \, dV_s + \frac{1}{2} \int_{V_p} \rho_p w^2 (x, t) \, dV_p \quad (1)
\]

where, \( V_s \) and \( V_p \) are the volume, \( \rho_s \) and \( \rho_p \) are the volume density.

\( T_c \) is the kinetic energy of cylinder, which can be expressed as

\[
T_c = \frac{1}{2} M_c \left[ \dot{w} (L_s, t) + \frac{D}{2} \dot{w}' (L_s, t) \right]^2 + \frac{1}{2} J \dot{w}' (L_s, t)^2 \quad (2)
\]

where, \( M_c \) is the total mass of cylinder \( (m_c) \) and moving magnets \( (m_{mag}) \), \( J \) is the rotary inertia of cylinder with respect to the axis \( y \).

\( U_b \) is the sum of the elastic potential energy piezoelectric layer and substrate layer, which can be expressed as

\[
U_b = \frac{1}{2} \left[ \int_{V_s} \varepsilon_s \sigma_s dV_s + \int_{V_p} \varepsilon_p \sigma_p dV_p \right] \quad (3)
\]

where, \( \varepsilon_s \) and \( \varepsilon_p \) are respectively the strain of substrate layer and piezoelectric layer, \( \sigma_s \) and \( \sigma_p \) are respectively the stress of substrate layer and piezoelectric layer, which can be given by

\[
\varepsilon_s = \varepsilon_p = -yw'' (x, t) = \varepsilon
\]

\[
\sigma_s = E_s \varepsilon
\]

\[
\sigma_p = E_p (\varepsilon - d_{31} E_3) = E_p \varepsilon - e_{31} E_3, \quad d_{31} = E_p d_{31} \quad (4)
\]

where \( d_{31} \) is the piezoelectric strain constant, \( E_3 \) is the electric field intensity given by

\[
E_3 = -\frac{V(t)}{2h_p} \quad (5)
\]

and \( V(t) \) is the output voltage of piezoelectric sheets.

\( U_e \) is the electric potential energy of piezoelectric layer, which can be expressed as

\[
U_e = \frac{1}{2} \int_{V_p} E_3 D_3 dV_p \quad (6)
\]

where, \( D_3 \) is the electric displacement given by

\[
D_3 = e_{33} \varepsilon + e_{31} E_3 \quad (7)
\]

and \( e_{33} \) is the piezoelectric permittivity under constant stress.

\( U_m \) is the magnet-induced nonlinear potential energy \( [49] \), which can be expressed as

\[
U_m = \frac{\mu_0 M^2}{2\pi} \left[ w^2 (L_s, t) + d_0^2 \right]^{-\frac{1}{2}} \quad (8)
\]

where, \( \mu_0 \) is the vacuum permeability, \( M \) is the magnetization intensity which is the ratio of residual magnetic induction intensity \( (B_r) \) to vacuum permeability \( (\mu_0) \).
Secondly, the virtual work of the system \( (\delta W_t) \) includes the virtual work of external resistance \( (\delta W_c) \), mechanical damping \( (\delta W_F) \), and lift force \( (\delta W_{R}) \), which can be given by

\[
\delta W_c = -c \int_0^L \dot{w}(x, t) \delta w(x, t) \, dx
\]
\[
\delta W_F = \left[ \frac{C_{L0}}{\rho D_2} \right] \dot{w}(L_s, t) + \frac{D_2}{2} \dot{w}'(L_s, t)
\]
\[
\delta W_{R} = -V(t) Q_R
\]

where, \( Q_R \) is the amount of charge produced by the piezoelectric vibrator, \( g(t) \) is introduced to describe the wake behavior of near cylinder. \( C_{L0} = 0.3 \) is the reference lift coefficient on an unfixed cylinder undergoing vortex shedding, \( C_D = 1.2 \) is the drag coefficient.

\[
\ddot{q} + \lambda \omega \left( q^2 - 1 \right) \dot{q} + \omega^2 q = \left( \frac{A}{D} \right) \left[ \dot{w}(L_s, t) + \frac{D_2}{2} \dot{w}'(L_s, t) \right]
\]

where, \( \lambda = 0.3 \), \( A = 12 \), \( \omega \) is the vortex shedding frequency.

Then, the kinetic energy of the system \( (T_t) \), the potential energy of the system \( (U_t) \), and the virtual work of the system \( (\delta W_t) \) can be obtained by

\[
T_t = T_e + T_c
\]
\[
\frac{1}{2} \int_{V_s} \rho \dot{w}^2 (x, t) \, dV_s + \frac{1}{2} \int_{V_p} \rho \rho_s \dot{w}^2 (x, t) \, dV_p
\]
\[
+ \frac{1}{2} \int_{V_s} \int_{V_p} \rho_s \sigma_y \dot{w} \, dV_s \, dV_p
\]

\[
U_t = U_p + U_e + U_m
\]
\[
= \frac{1}{2} \int_{V_s} \int_{V_p} E_s \sigma_y \, dV_s \, dV_p + \frac{1}{2} \int_{V_s} \int_{V_p} E_p \sigma_y \, dV_s \, dV_p
\]

\[
\delta W_t = -V(t) \delta Q_R - c \int_0^L \dot{w}(x, t) \delta w(x, t) \, dx
\]
\[
\delta W_c = \left[ \frac{C_{L0}}{\rho D_2} \right] \dot{w}(L_s, t) + \frac{D_2}{2} \dot{w}'(L_s, t)
\]

B. SOLUTION OF THE DISTRIBUTED PARAMETER MODEL

In order to solve the mathematical model, the above equations are discretized by using the Galerkin procedure to obtain the reduced-order model. The displacement of the end of the piezoelectric beam \( w(x, t) \) can be obtained by

\[
w(x, t) = \sum_{i=0}^{n} \varphi_i(x) r_i(t)
\]

In the low-frequency excitation environment, the first-order vibration mode has the largest contribution to the vibration response of the vibration energy capture device, and the power generation in the second-order and higher-order modes is very weak. Therefore, \( n \) is equal to one. Equation (14) can be rewritten as

\[
w(x, t) = \varphi_1(x) r_1(t)
\]

The mode function \( \varphi_1(x) \) can be defined as

\[
\varphi_1(x) = A_1 \left[ \cos \left( \frac{\lambda_1 L_s}{x} \right) - \cos \left( \frac{\lambda_1 L_s}{x} \right) \right] + \beta_1 \left( \sin \left( \frac{\lambda_1 L_s}{x} \right) - \sin \left( \frac{\lambda_1 L_s}{x} \right) \right)
\]

where, \( \beta_1 \) can be obtained by

\[
\beta_1 = \frac{\beta_0}{\beta_2}
\]

\[
\beta_0 = \sin \lambda_1 - \sinh \lambda_1 + k_1 \lambda_1 (\cos \lambda_1 - \cosh \lambda_1) - k_2 \lambda_2^2 (\sin \lambda_1 + \sinh \lambda_1)
\]

\[
\beta_2 = \cos \lambda_1 + \cosh \lambda_1 - k_1 \lambda_1 (\sin \lambda_1 - \sinh \lambda_1) - k_2 \lambda_2^2 (\cos \lambda_1 - \cosh \lambda_1)
\]

\[
k_1 \text{ is the mass ratio of the cylinder to piezoelectric beam, } k_2 \text{ is the ratio of cylinder diameter to width of piezoelectric beam. } \lambda_1 \text{ can be obtained by}
\]

\[
1 + \cos \lambda_1 \cosh \lambda_1 + k_1 (\cos \lambda_1 \sin \lambda_1 - \cosh \lambda_1 \sinh \lambda_1)
\]

\[
-2k_1 k_2 \lambda_1^2 \sin \lambda_1 \sinh \lambda_1
\]

\[
-\frac{1}{3} k_1 k_2 \lambda_1^4 (\cos \lambda_1 \sin \lambda_1 + \cosh \lambda_1 \sinh \lambda_1)
\]

\[
+ \frac{1}{3} k_1 k_2 \lambda_1^4 (1 - \cos \lambda_1 \cosh \lambda_1) = 0
\]

A_1 is the amplitude of modal function, which can be obtained by the orthogonality conditions of mass and stiffness expressed as

\[
\int_{V_s} \rho_0 \varphi_1^2(x) \, dV_s + \int_{V_p} \rho_0 \varphi_1^2(x) \, dV_p + \frac{1}{2} \int \varphi_1 \left( \frac{\rho_0 \varphi_1^2(x)}{D_2} \right) \, dV_p = 1
\]

\[
\int_{V_s} \lambda^2 E_s \varphi_1'^2(x) \, dV_s + \int_{V_p} \lambda^2 E_p \varphi_1'^2(x) \, dV_p = \omega^2
\]

where, \( \omega \) is the natural frequency of the MCPEH system.

According to (15), the (10) to (13) can be rewritten as

\[
\ddot{q} + \lambda \omega \left( q^2 - 1 \right) \dot{q} + \omega^2 q = \left( \frac{A}{D} \right) \left[ \varphi_1(L_s) + \frac{D_2}{2} \varphi_1'(L_s) \right]
\]

\[
T_i = \frac{1}{2} \int_{V_s} \rho_0 \varphi_1^2(x) \, r_1^2(t) \, dV_s + \frac{1}{2} \int_{V_p} \rho_0 \varphi_1^2(x) \, r_1^2(t) \, dV_p
\]

\[
+ \frac{1}{2} M_c \left[ \varphi_1(L_s) \, \dot{r}_1(t) + \frac{D_2}{2} \varphi_1'(L_s) \, \dot{r}_1(t) \right]^2
\]

\[
+ \frac{1}{2} J \varphi_1'^2(x) \, r_1^2(t)
\]
\[ U_t = \frac{1}{2} \int_{V_s} E_0 x^2 \varphi_1^2 (x) r_1^2 (t) \, dV_s + \frac{1}{2} \int_{V_p} E_x y^2 \varphi_1'' (x) r_1^2 (t) \, dV_p - \int_{V_p} e31 V (t) \frac{V^2 (t)}{2h_p} \varphi_1'' (x) r_1 (t) dV_p - \frac{1}{2} \int_{V_p} \frac{\epsilon 33 V^2 (t)}{4h_p^2} dV_p + \frac{\mu_0 M^2}{2\pi} \left[ \varphi_1^2 (L) r_1^2 (t) + d_0^2 \right]^{\frac{3}{2}} \]  

(22)

\[ \delta W_t = -V (t) \delta Q_R - c \int_0^{L_s} \varphi_1 (x) \dot{r}_1 (t) \, dx + \left[ \frac{c L_0 q (t) \rho_0 D U^2 L_c}{2} - \frac{C_D \rho_0 D U L_c}{2} \left[ \varphi_1 (L) \dot{r}_1 (t) + \frac{D}{2} \varphi_1' (L) \dot{r}_1 (t) \right] \right] \cdot \delta \varphi_1 (L) \dot{r}_1 (t) + \frac{D}{2} \delta \varphi_1' (L) \dot{r}_1 (t) \]  

(23)

Submitting the (19) and (20) into Lagrange term \( L \), which is expressed as

\[ L = T_t - U_t \]  

(24)

And then submitting the (20), (23) and (24) into Lagrange equation, as shown following

\[ \begin{cases} \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{r}_1} \right) - \frac{\partial L}{\partial r_1} = \frac{\delta W_t}{\delta r_1} \\ \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{\gamma}} \right) - \frac{\partial L}{\partial \gamma} = \frac{\delta W_t}{\delta \gamma} = -\frac{V}{\dot{r}} \end{cases} \]  

(25)

Finally, the reduced-order coupling model of vortex-induced vibration energy harvesting considering the magnetic force effecting, can be given by

\[ \begin{cases} \ddot{\gamma} + C \dot{\gamma} + \omega^2 r - \xi r \left[ r^2 + \eta^2 \right]^{-5/2} - \theta V = aq \\ \theta \ddot{r} + C_p \dot{V} + \frac{V}{R} = 0 \end{cases} \]  

(26)

where,

\[ C = c \int_0^{L_s} \varphi_1^2 (x) \, dx + \frac{C_D \rho_0 D U L_c}{2} \left[ \varphi_1 (L) + \frac{D}{2} \varphi_1' (L) \right] \]  

(27)

\[ \xi = \frac{3 \mu_0 M^2}{2\pi \varphi_1^3 (L)} \]  

(28)

\[ \eta = \frac{d_0}{\varphi_1 (L)} \]  

(29)

\[ \theta = \frac{e31 w_p (h_s + h_p)}{2} \left[ \varphi'(L_0 + L_p) - \varphi'(L_0) \right] \]  

(30)

\[ \alpha = \frac{c L_0 \rho_0 D U^2 L_0}{4} \left[ \varphi_1 (L_0) + \frac{D}{2} \varphi_1' (L_0) \right] \]  

(31)

\[ C_p = \frac{w_p L_f^3 \varphi_1}{2h_p} \]  

(32)

In order to solve the (26), state variable \((X_1, X_2, X_3, X_4, X_5)\) is induced, which can be written as

\[ X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} w \\ \dot{w} \\ V \\ q \\ \dot{q} \end{bmatrix}, \quad \dot{X} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ \dot{X}_5 \end{bmatrix} \]  

(33)

The space state equation of the MCPEH is established by (20) and (33), expressed as

\[ \dot{X} = \begin{bmatrix} -\omega^2 X_1 - CX_2 + \theta X_3 + \xi X_1 \left[ X_1^2 + \eta^2 \right]^{-5/2} + aX_4 \\ -\frac{\theta_1}{C_p} X_2 - \frac{1}{C_p R} X_3 \\ X_5 \end{bmatrix} \]  

(34)

where,

\[ \dot{X}_5 = \left( \frac{A}{D} \right) \left[ \varphi_1 (L) + \frac{D}{2} \varphi_1' (L) \right] \cdot \left[ -\omega^2 X_1 - CX_2 + \theta X_3 + \xi X_1 \left[ X_1^2 + \eta^2 \right]^{-5/2} + aX_4 \right] - \lambda \omega_x \left( X_1^2 - 1 \right)^2 X_5 - \omega_x^2 X_4 \]  

(35)

At last, the vibration response and energy harvesting performance of the MCPEH can be achieved by solving (34) using ode45 in MATLAB®. The output power can be respectively expressed as

\[ P = \frac{1}{T} \int_0^T \frac{V^2 (t)}{R} \, dt \]  

(36)

The output power is one of significant indexes to present the performance of the MCPEH in the following simulating and discussing.

**IV. RESULTS AND DISCUSSION**

**A. VALIDATION WITH THE EXPERIMENT RESULTS**

In order to determine the accuracy of the mathematical model for the piezoelectric energy harvesting form VIVs based on wind field, we validate the numerical predictions approach with the experimental measurements of Akaydin et al. [50]. The geometric and physical parameters of the harvester in Akaydin et al. experiment are shown in Table 1. In Fig. 4 (a), the plot shows that the tendency of numerical simulation is similar to the trend of experiments of Akaydin et al. In fact, in the experiments of Akaydin et al. the maximal output power is 0.1 mW when the wind velocity is 1.192 m/s, whereas the maximal output power obtained by the present mathematical model is 0.0993 mW when \( U = 1.22 \) m/s. Besides, the resonance region is well predicted by the present mathematical model and particularly its starting and its peak. However, it should be mentioned that there is an underestimate in simulating the output power when the wind velocity is
FIGURE 4. (a) Comparisons of the variations in the output power with wind velocity when using the mathematical model and the experimental measurements of Akaydin et al. [50]; (b) Variation in the maximum generated voltage with wind velocity. Both figures are plotted while load resistance is equal to 2.46 MΩ.

TABLE 1. Geometric and physical parameters of the harvester [50].

<table>
<thead>
<tr>
<th>physical parameters</th>
<th>PZT layer</th>
<th>Substrate layer</th>
<th>Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length $L_p$, $L_c$, $L_e$ (mm)</td>
<td>31.8</td>
<td>267</td>
<td>203</td>
</tr>
<tr>
<td>Width $w_p$, $w_e$ (mm)</td>
<td>25.4</td>
<td>32.5</td>
<td>—</td>
</tr>
<tr>
<td>Thickness $h_p$, $h_c$ (mm)</td>
<td>0.267</td>
<td>0.635</td>
<td>—</td>
</tr>
<tr>
<td>Diameter $D$ (mm)</td>
<td>—</td>
<td>2*19.8</td>
<td>—</td>
</tr>
<tr>
<td>Mass density $\rho_p$, $\rho_r$ (kg/m³)</td>
<td>7800</td>
<td>2730</td>
<td>—</td>
</tr>
<tr>
<td>Young’s modulus $E_p$, $E_s$ (GPa)</td>
<td>66</td>
<td>73</td>
<td>—</td>
</tr>
<tr>
<td>Permittivity at constant strain $\varepsilon_{33}$ (nF/m²)</td>
<td>13.28</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Mass $m_p$, $m_s$, $m_c$ (g)</td>
<td>1.68</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

TABLE 2. Geometric and physical parameters of the MCPEH.

<table>
<thead>
<tr>
<th>physical parameters</th>
<th>PZT layer</th>
<th>Substrate layer</th>
<th>Cylinder</th>
<th>Moving magnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length $L_p$, $L_c$, $L_e$ (mm)</td>
<td>30</td>
<td>100</td>
<td>100</td>
<td>—</td>
</tr>
<tr>
<td>Width $w_p$, $w_e$ (mm)</td>
<td>20</td>
<td>25</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Thickness $h_p$, $h_c$ (mm)</td>
<td>0.2</td>
<td>0.6</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Diameter $D$ (mm)</td>
<td>—</td>
<td>20</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Mass density $\rho_p$, $\rho_r$ (kg/m³)</td>
<td>7386</td>
<td>2700</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Young’s modulus $E_p$, $E_s$ (GPa)</td>
<td>59.77</td>
<td>70</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Permittivity at constant strain $\varepsilon_{33}$ (nF/m²)</td>
<td>41.78</td>
<td>—</td>
<td>—</td>
<td>8.2</td>
</tr>
<tr>
<td>Mass $m_c$, $m_{mag}$ (g)</td>
<td>—</td>
<td>—</td>
<td>8.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

larger than 1.324 m/s (post-resonance region). This condition of a sharp drop in the model results may be due to the accuracy of the wake oscillator model to the starting of the resonance region and its peak [51]. As shown in Fig. 4 (b), the maximal output voltage obtained is 22.07 V while the external load resistance is 2.46 MΩ. It is excellently similar to the experimentally measured value of 22.181 V at the same external load resistance.

B. THE EFFECTS OF THE PHYSICAL PARAMETERS OF CYLINDER

There are some physical parameters that the analysis will take in next simulation, which is shown in Table 2.

It is noted that the control variate method is taken in simulation, thus when one parameter is under studying, the others will keep unchanged and employ the values in Table 2.

As shown in the Fig. 5 (a), the curves of output power with the velocity of wind were obtained in the simulation when the diameter of cylinder was 20 mm, the mass of cylinder (including the mass of moving magnets) was 10 g, the length of PZT was 30 mm and the external load resistance was 0.5 MΩ. It can be got from the Fig. 5 (a) that the utmost output power can be obtained at $U = 1.6$ m/s nearly. And the energy harvester is able to obtain 212.5 μW of power while $U = 1.6$ m/s. When the resonance happens, the vibration frequency is locked at the natural frequency in a specific range of wind speed, which is called the “lock-in” region. The Fig. 5 (b) shows the relationship between the output voltage and time when the optimum wind speed is at almost 1.6 m/s. The Fig. 5 (c) is the enlarged view of the Fig. 5 (b) among 20 and 25 seconds. It can be found that stable voltage output can be achieved in the simulation due to the vortex-induced vibration, and the frequency of voltage output is 18.34 Hz with a peak output voltage of 11.97 V being obtained under the FFT analysis, as shown in Fig. 5 (d). In Fig.5 (e), it is noted that the modeling result of output power with magnetic force is more excellent than that without magnetic force by using the values of parameters in Table 1. The curve of simulation without magnetic force is the same as
FIGURE 5. (a) Curves of output power with velocity of wind; (b) Output voltage changing with time; (c) Output voltage enlarged drawing in 5 s; (d) FFT of the simulating results of output voltage at $U = 1.6 \text{ m/s}$; (e) Effect of magnetic force on output power with velocity of wind.

the curve of simulation in Fig. 4 (a). The peak of output power and the resonance domain with nonlinear magnetic force is higher and wider than those without nonlinear magnetic force respectively. However, it should be noted that there is a short sharp rise in the front-resonance domain, which may be also due to the accuracy of the wake oscillator model.

In order to explore the performance and the rule for energy harvesting of the MCPEH based on the vortex-induced vibration, the parametric studies were performed to further analyze the output power and the energy harvesting efficiency, such as the load resistance, the physical parameters of cylinder, the length of PZT and substrate layer as well as velocity of wind. And converting flow energy (water and wind energy) to electricity, the flow-induced PEH was proposed, mainly including flutter harvesters, galloping harvesters, vortex-induced harvesters, and so on. According to some researches, the free end of the cantilever beam connecting bluff bodies with different shapes will produce different types of vibration [52], [53]. And while the bluff body is cylinder, the flow-induced vibration will be vortex-induced vibration. Therefore, investigating the effects of the physical parameters of cylinder is necessary. When the load resistance value is 0.5 MΩ, the performances of the MCPEH are studied with the cylindrical diameter $D$ of 12 mm, 15 mm, 20 mm, and 22 mm, as shown in Fig. 6. It can be found that the output power and vibration displacement increase firstly and then decrease with the increase of wind velocity. There is a maximum of output power and vibration displacement for
the MCPEH when the vortex-induced resonance occurs. With increasing the cylindrical diameter, the output power and vibration displacement increase distinctly. This is because that the vortex-induced force will increase with the increase of cylindrical diameter. At the same time, the velocity of resonance domain is gradually increasing as well as the resonance domain. For example, the width of resonance domain is $\Delta_1 = 1.375 - 1.075 = 0.3 \text{ m/s}$ when $D = 15 \text{ mm}$, but when $D = 20 \text{ mm}$, the $\Delta_2 = 1.8 - 1.3 = 0.5 \text{ m/s}$. It is obvious that $\Delta_2 > \Delta_1$. As shown in Fig. 6 (a) and (b), the vibration frequency at optimal wind velocity will decrease with the increase of cylindrical diameter, therefore more large velocity of resonance domain is needed to fit the natural frequency. It can be figure out that the performance of the MCPEH is influenced greatly by the cylindrical diameter. And it also affects the velocity of vortex-induced resonance domain.

After studying the cylindrical diameter, the length of cylinder was studied as well. As shown in Fig. 7, it can be found that the output power and vibration displacement will increase with the increase of cylindrical length. The optimal wind velocities of the different cylindrical lengths are near $U = 1.6 \text{ m/s}$. The resonance domain increases with the
increase of cylindrical length, but not obviously as Fig. 6. It should be mentioned that the starting velocity of wind always keep the same value of velocity when \( L_c = 90 \text{ mm} \), \( L_c = 100 \text{ mm} \), and \( L_c = 120 \text{ mm} \), whereas the starting velocity of wind becomes lower when \( L_c = 130 \text{ mm} \). The reason of this condition may be that the vortex-induced force increase with the increase of cylindrical length, but the increment is not enough to produce an obvious effect on the response of the MCPEH system especially the resonance domain. Besides, the starting velocities of resonance domain are almost at the same point and the ending velocities increase gradually with the increase of cylindrical length but the increment is not obvious. It can be concluded that the cylindrical length mainly affects the output power and vibration displacement against Fig. 6.

The mass of cylinder is also an important element for influencing the performance of the energy harvester. According to the results of simulating in Fig. 8, the output power and the vibration displacement will increase with the decrease of cylindrical mass. The velocity of vortex-induced resonance will decrease with the increase of cylindrical mass. The reason of this condition is mainly due to the natural frequency. It can be got in Fig. 9 (a) that the natural frequency will increase with the decrease of cylindrical mass. In addition, the vortex shedding frequency in vortex-induced resonance domain is always approximate to the natural frequency. And no matter what the value of cylindrical mass takes, the output power and the vibration displacement will increase firstly and then decrease with the increase of wind velocity among the vortex-induced resonance domain. But contrasting with Fig. 6, the width of resonance domain is greater and the velocity of vortex-induced resonance is higher too. Because that the cylindrical mass has greater effect on the natural frequency than that cylindrical diameter has. Therefore, an energy harvester can possess a great performance under a desired wind velocity by adjusting the parameters of cylinder.

C. THE EFFECTS OF THE LENGTH OF PIEZOELECTRIC SHEETS

The flow-induced PEH can obtain vibrational energy from the external environment on the resonance domain. For the vortex-induced harvester, while the vortex shedding frequency is closed to the natural frequency, it can convert the external environment energy to electric energy. Therefore, the length of piezoelectric sheets was investigated to explore the energy harvesting law. As shown in Fig. 9 (a) and (b), the natural frequency increases with the increase of the length of PZT. But increasing the cylindrical mass, the natural frequency will decrease. In addition, the cylindrical mass also has greater effect on the natural frequency than PZT length has. The lower natural frequency the energy harvester is, the lower the wind velocity of vortex-induced resonance domain is, as shown in Fig. 8 (a), (b) and Fig. 9 (a), (b). Within the range of 20mm to 35mm in the length of PZT as shown in Fig. 9 (c) and (d), the output power and the velocity of vortex-induced resonance increase with the increase of PZT length, but the peaks of the vibration displacement are very close to each other. In Fig. 9 (c), the values of output power with different PZT length are same as each other when the wind velocity is near 1.45 m/s. When \( U < 1.45 \text{ m/s} \), the output power is high under low values of PZT length. But while \( U > 1.45 \text{ m/s} \), the output power is always high under high values of PZT length with increase of wind velocity. What’s more, in Fig. 9 (c) and (d), the velocity of resonance domain and output power increase with the increase of PZT length, whereas the width of resonance domain remains unchanged. This is because that increasing the PZT length will increase the area of power generation and the natural frequency, but have little effect on the cantilever beam. It can be concluded that the cylindrical mass has a greater influence than the length of PZT on the natural frequency, which can be also embodied by the vortex-induced resonance domain.
D. THE EFFECTS OF THE LOAD RESISTANCE

As shown in Fig. 10 (a) and (b), there are two optimum resistance values for the energy harvester to obtain the most output power for distinct wind velocity or PZT length. Inspecting the plotted curves in Fig. 10 (a) and (c), it is noted that the output power is almost 0 for $U = 1.8$ m/s when the load resistance between 40 kΩ and 600 kΩ, which is similar to the variations of vibration displacement. Besides, the load resistance region in which the output power values are high matches the load resistance region which the vibration displacement values are low. This is because of the resistive shunt damping effect [54], which takes place over a specific region of load resistance between 40 kΩ and 600 kΩ. The variation rule is also shown in Fig. 10 (b) and (d). It should be mentioned in Fig. 10 (d) that the values of vibration displacement increase with the increase of PZT length when $R < 3.16$ kΩ and $L_p < 30$ mm, whereas the values of vibration displacement decrease with the increase of PZT length and the vibration displacement values are low when $5.62$ kΩ $< R < 316$ kΩ, which matches the load resistance region in which the output power values are high. This realization is useful for employing low vibration displacement to obtain high output power.

In Fig. 10 (b), when the load resistance $R$ is less than 178 kΩ, the output power increases firstly and then decreases with the increase of the load resistance. When the load resistance $R$ is greater than 178 kΩ, the output power increases firstly and then decreases with the increase of the load resistance too. The output power is approximately the value of 178 kΩ symmetry. It can be noted that the output power increases with the increase of the length of PZT when $R$ is less than 31.6 kΩ and 20 mm $< L_p < 30$ mm. However, when 30 mm $< L_p < 50$ mm, the output power increases firstly and then decrease with the increase of load resistance until $R = 100$ kΩ, next the output power will increase until $R = 1.78$ MΩ and then decrease, whereas when 20 mm $< L_p < 30$ mm, the output power will increase while $178$ kΩ $< R < 1.78$ MΩ and then decrease. The peaks of output power are affected not only by the load resistance but also by the length of PZT. When $R > 316$ kΩ, the peaks of output power under the different length of PZT are nearly at 1.78 MΩ, the growth amplitude is very close and steady with the increase of the length of PZT. Inspecting the plotted curves in Fig. 10 (b) and (d), the performance of the MCPEH is affected greatly by the PZT length when $R < 316$ kΩ. This is because that while the external load resistance value is low, the PZT length is the main one of effect elements on the response for the MCPEH system. In order to explore the relationship among the output power, the load resistance, and the length of PZT, the further study was investigated between them and the mechanical power of wind. The mechanical
The power of wind can be given by

$$P_{\text{wind}} = \frac{S_{\text{wind}}}{2} \rho_0 U^3$$  (37)

where, $S_{\text{wind}}$ is the windward area, which can be expressed as

$$S_{\text{wind}} = 2 \left( \frac{D}{2} + w_{\text{amp,c}} \right) L_c$$  (38)

where, $w_{\text{amp,c}}$ is the maximum of the cylindrical displacement. Therefore, the energy conversion efficiency can be expressed as $\eta = P_{\text{out}} / P_{\text{wind}}$.

Fig. 11 (a) shows that the conversion rate increases gradually firstly and then decreases with the load resistance when $R > 31.6$ k$\Omega$, but the conversion rate almost keeps stable with the increase of the length of PZT. Then, it can be found from Fig. 11 (b), (c), (d) that the output power is strongly affected by the load resistance. Besides, the relationship between output power and load resistance also depended on the choice of the length of PZT. For instance, at $R = 16$ k$\Omega$, when $L_p = 20$ mm, the output power value is the least one as shown in Fig. 11 (b), but when $L_p = 50$ mm in Fig. 11 (d), the output power value is the utmost one. But under $R = 300$ k$\Omega$, the peaks of output power values with different length of PZT are very close to each other. Much interesting is that the peak values of output power are very close to each other when $R = 8$ M$\Omega$, whereas the velocity of resonance domain increase with the increase of PZT length. Next, it can be also found that the vortex-induced resonance domain is greater not only in the range of $R < 31.6$ k$\Omega$ but also $R > 316$ k$\Omega$ than that in the range of 31.6 k$\Omega < R < 316$ k$\Omega$. And in Fig. 11 (e), curves show that the output power changes with the PZT length under different load resistances. When $R = 16$ k$\Omega$, 50 k$\Omega$, and 3 M$\Omega$, the output power values are close to each other, especially within the range of 30 mm $< L_p < 35$ mm. this realization is very useful for PEH to adapt to different load resistance conditions. As for when $R = 3$ M$\Omega$, the output power almost remains unchanged with PZT length, which matches what shows in Fig. 10 (b) and (d) with value of load resistance, R, being near 3.16 M$\Omega$. This value of load resistance implies that the value of PZT length is not the main effect element for the response of the MCPEH system. Besides, when $R = 1.78$ M$\Omega$, the output power value is always the utmost, no matter what the value of the length of PZT takes. The reason of this condition is that the velocity of vortex-induced resonance will increase with the increase of the length of PZT as shown in Fig. 9 due to the natural frequency and the mechanical power of wind will increase with the increase of the velocity of vortex-induced.
resonance. Especially, when the load resistance is large, the energy harvester will present the resistive shunt damping effect, which will further enhance the velocity of vortex-induced resonance. And the incremental velocity of vortex-induced resonance may keep a balance with the incremental length of PZT.

E. THE EFFECTS OF $d_0$ ON THE ENERGY HARVESTER SYSTEM

It is no doubt that the parameter, $d_0$, is one of the vital elements affecting the performance of the vortex-induced PEH system under nonlinear magnetic force. As shown in Fig. 12 (a), the performance of the energy harvester system is greatly affected by the distance between the moving magnet and the fixed magnet ($d_0$). The output power increase with the decrease of $d_0$. The velocity of vortex-induced resonance as well as the width of resonance domain increase with the decrease of the value of $d_0$. This is because that the nonlinear magnetic force can improve the performance of PEH as shown in Fig. 5 (e). When $d_0 > 5$ mm, the lower value of $d_0$ is, the greater enhancement of output power is. Besides, it can be also found that the resonance frequency may be
adjusted by changing the value of $d_0$. The lower value of $d_0$ is, the higher value of resonance frequency is. But if a good value of output power is expected under a low resonance frequency, adjusting the length of PZT, the load resistance and the parameters of cylinder is necessary. In Fig. 12 (c), the curves show the tendency of magnetic force in $y$ axis with vibration displacement under distinct $d_0$, and are about point $(0, 0)$ centrosymmetric. Besides, the potential energy [39] of MCPEH system can be expressed as

$$U_{sys} = \frac{1}{2} \omega^2 y^2 + U_{mag}(y)$$  \hspace{1cm} (39)$$

where, $y$ is the vibration displacement. As shown in Fig. 12 (b), the number of extreme points of the potential energy increases with the decrease of $d_0$. And the number of extreme points is always used to judge the steady state of a system by researchers. The number of minimal value points is defined as the number of stable state points of a system [52]. For example, one minimal value point indicates a monostable state, two minimal value points indicate the bistable state. It can be found in Fig. 12 (b) that the energy harvester system is a monostable state system when $d_0 > 5$ mm, whereas the system will be a bistable state system when $d_0 < 5$ mm. With regard to a bistable or a multistable system, their vibrations are complex and the interwell oscillations need more exciting force or acceleration. Besides, a standard lever for the flexibility of piezoelectric materials is high and severe due to the great vibration amplitude and exciting force. Therefore, this study forced on the monostable system.

V. CONCLUSION

In this study, a magnet-induced monostable nonlinear VIV piezoelectric energy harvester is proposed, modeled, and analyzed. The effects of wind velocity, cylindrical structure, piezoelectric beam, and load resistance on the vibration response and energy harvesting performance are investigated respectively. It can be found that the resonance wind velocity increases with the increase of cylindrical diameter and the length of PZT, with the decrease of cylindrical mass. As for the nonlinear magnetic force, it not only influences the output power of MCPEH greatly but also decides the steady state of the system. The output power will increase with the decrease of $d_0$, whereas the response of the MCPEH is not investigated when $d_0 < 5$ mm. Because the system will become a bistable state system while $d_0 < 5$ mm. It should be mentioned that the output power and vibration displacement can increase with the decrease of cylindrical mass and the increase of the cylindrical diameter. But the output power except the vibration displacement will increase with the increase of the length of PZT. As far as the load resistance is concerned, there are two optimal resistances for the energy harvesting.
system. And the load resistance region with the high output power values matches the load resistance region with the low vibration displacement values. When the load resistance is smaller than 31.6 kΩ and larger than 316 kΩ, the energy harvesting system will obtain a wider resonance domain. In addition, 316 kΩ of load resistance can be defined as the critical point, which means the PZT length is not the main effect element on the performances of the MCPEH system. Therefore, the energy harvesting efficiency will keep a stable state between the increase of PZT length and kinetic energy of wind due to the resistive shunt damping effect and kinetic energy of wind particularly while \( R > 316 \) kΩ. Whereas whatever the length of PZT takes, the energy harvester always obtains the utmost output power value while \( R = 1.78 \) MΩ. Finally, 212.5 μW of power can be obtained by the energy harvester when \( R = 0.5 \) MΩ, \( D = 20 \) mm, \( L_p = 30 \) mm, and \( U = 1.6 \) m/s.

REFERENCES


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