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# Analysis of Point Daylight Factor (PDF) Average Daylight Factor (ADF) and Vertical Daylight Factor (VDF) under various unobstructed CIE Standard Skies

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**Abstract.** The usual method for computing natural illuminance for a given location in a building is the daylight factor (DF) approach, which is based on the traditional overcast sky without sunlight, as defined by the International Commission on Illumination (CIE). In 2003, the CIE adopted a range of 15 skies as international standard models that represent the actual skies of many places and cover the whole probable spectrum of skies found in nature. Previously, we developed a number of calculation tools to predict daylight illuminance for rooms facing different orientations under various CIE Standard skies. This paper presents the work on the calculation of the point DF (PDF), average DF (ADF) and vertical DF (VDF) under various overcast and non-overcast skies. The strong correlations between these three DF types can help to estimate the required DF values for architectural and daylighting designs and evaluations.

**Keywords:** Point Daylight Factor; Average Daylight Factor; Vertical Daylight Factor; CIE Standard Skies; Split-flux theory

## 1. Introduction

Estimation of the daylight level for any point within an interior space is essential to daylighting designs and assessments [1]. The usual method for computing natural illuminance for a given location in a building is the daylight factor (DF) approach, which is based on the traditional overcast sky without sunlight, as defined by the International Commission on Illumination (CIE). The DF approach can be analytically calculated using simple design aids [2]; annual daylight data are not required. The point DF (PDF) has shown the effectiveness of daylight-linked controls in saving energy on electric lighting [3]. The average DF (ADF), which is directly related to window area and less detailed input data than those provided by the PDF, could be a useful daylighting criterion [4]. The vertical DF (VDF) representing light coming directly from the sky and daylight reflected from surrounding buildings and the ground onto a vertical surface has been appropriately used for compact urban areas with large exterior obstructions, such as Hong Kong [5]. Such DF approaches have gained favour due to their simplicity. Practitioners have become accustomed to using the daylight criteria that are usually expressed in the DF approach, which has been adopted in many countries [6]. However, such DF approaches are not flexible enough to predict the dynamic daylight variations caused by the movement of the sun when the sky is not overcast [7]. Therefore, the daylight metric based on the DF lacks realism. In 2003, the CIE adopted a range of 15 skies as international standard models [8] that represent the skies of many places and cover the whole probable spectrum of skies found in nature [9]. Previously, we developed a number of calculation tools to predict daylight illuminance for rooms facing different orientations under various CIE Standard skies [10]. This paper presents the work on the calculation of the PDF, ADF and VDF



under various overcast and non-overcast skies. Calculation procedures are demonstrated. Characteristics of the findings and design implications are discussed.

## 2. Mathematical Expressions for calculating Daylight Factors

For flats at the top floors of high-rise buildings or rooms of houses located in low densely-built zones, the approaches for estimating DF should be computed under unobstructed skies. The DF for an internal space is often split into three components, viz. the sky component (SC), the externally reflected component (ERC) and internally reflected component (IRC). Under unobstructed skies, there is no outdoor illuminance reflected from opposing external vertical surfaces to the interior point (i.e. no ERC). Previously, a nomograph and a Waldram diagram for calculating the SC were established [11]. Fig. 1 presents the nomograph consisting of a chart with two scales A and B for calculating the SC under CIE standard clear sky (Sky 12). Similar nomographs can also be plotted for the other CIE Standard Skies. Based on a number of procedures, the required SC under various unobstructed CIE Standard Skies can be determined. The computation of the IRC can be based on the theory of the split-flux principle. Mathematically, average IRC can be given as:

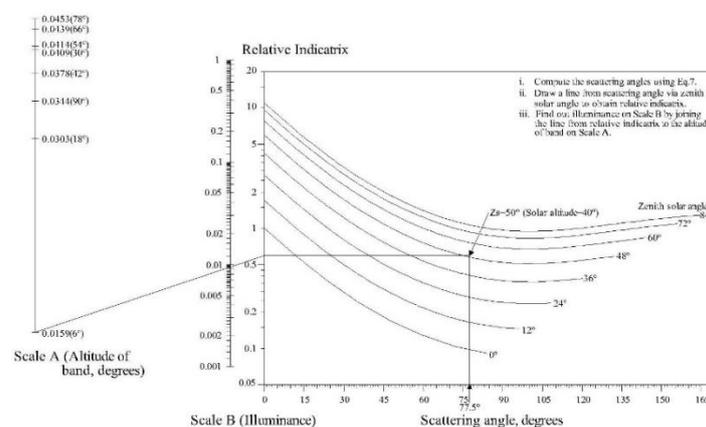
$$\bullet \quad IRC = tW \left( \frac{CR_{fw} + DR_{cw}}{A(1-R)} \right) \quad (1)$$

where  $A$  is the total area of all the interior surfaces,  $m^2$ ;  $C$  and  $D$  are the configuration factors of the daylight flux incident on the mid-height of the window pane from above and below the horizon, respectively, dimensionless;  $R$  is the average reflectance of all the interior surfaces, dimensionless;  $R_{cw}$  is the average reflectance of the ceiling and upper walls above the mid-height of the window, excluding the window wall, dimensionless;  $R_{fw}$  is the average reflectance of the floor and lower parts of the walls below the mid-height of the window, excluding the window wall, dimensionless;  $t$  is the visual transmittance of the window, dimensionless;  $W$  is window area,  $m^2$

The important issue is to determine the configuration factors  $C$  and  $D$  under various sky conditions. For an unobstructed sky,  $C$  is the vertical sky component (VSC) which is the ratio of the vertical ( $E_{VD}$ ) to the horizontal ( $E_{HD}$ ) sky-diffuse illuminance available at the same point. Constant VSC values of 40, 46 and 50% representing, respectively, the Standard Skies 1, 3 and 5 were obtained. For other 12 CIE Standard Skies, the VSC substantially relies on the solar location and the VSC could be identified by the scattering angle ( $\chi$ ). To provide precise VSC values for subsequent study, the template of Gaussian functions [12] that depict the symmetric bell curve shape were used to correlate VSC with the  $\chi$  for the 12 CIE Standard Skies as shown in Eq. 2.

$$\bullet \quad VSC = A_1 \exp \left\{ - \left[ \frac{\chi_{ref} - B_1}{C_1} \right]^2 \right\} + A_2 \exp \left\{ - \left[ \frac{\chi_{ref} - B_2}{C_2} \right]^2 \right\} \quad (2)$$

where  $A_1$ ,  $B_1$ ,  $C_1$ ,  $A_2$ ,  $B_2$  and  $C_2$  are the regression coefficients



**Figure 1.** Nomograph calculating daylight illuminance for the CIE standard clear sky.

Table 1 summarizes the regression coefficients for Eq. 2 under the 12 CIE Standard Skies other than Skies 1, 3 and 5 [12]. Accordingly, the VSC can be computed. For non-overcast skies (i.e. Skies 7 to 15) the ground reflected component due to direct sunlight ( $E_{HB}$ ) should be included.

**Table 1.** Coefficients  $A_1$ ,  $B_1$ ,  $C_1$ ,  $A_2$ ,  $B_2$  and  $C_2$  (for the Gaussian Function with 2 terms).

Sky No.	$A_1$	$B_1$	$C_1$	$A_2$	$B_2$	$C_2$
2	0.528	10.9	106	0.306	198	70.7
4	0.626	5.93	95.7	0.424	222	107
6	0.688	2.41	91.9	0.549	257	146
7	0.82	6.76	75.1	0.958	336	178
8	1	7.3	66.7	0.745	294	150
9	0.891	-2.5	95.1	0.624	243	142
10	1.04	2.33	75.9	2.01	461	246
11	1.3	-2.48	73.3	2	427	211
12	1.45	-1.5	68.1	2.60	510	267
13	1.86	-13.1	71.8	7	801	372
14	2.39	-24.9	78.3	1.8	499	278
15	3.91	-56.3	93.6	1.31	445	222

Mathematically, D can be written as:

- $D = \frac{(E_{HD} + E_{HB})\rho_g}{2E_{HD}}$  (with  $E_{HB}$ ) (3)

- $D = 0.5\rho_g$  (without  $E_{HB}$ ) (4)

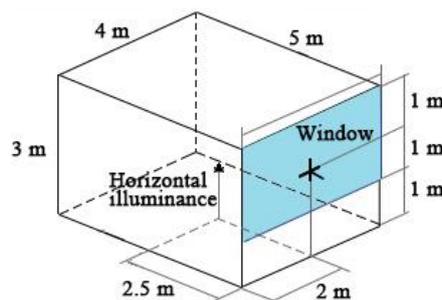
- $E_{HB} = E_{NB} \sin \alpha_s$  (5)

where  $E_{NB}$  is the direct normal sunlight (lux);  $\alpha_s$  is the solar altitude (degrees);  $\rho_g$  is the ground reflectance (dimensionless)

The approximate  $E_{HD}$  and  $E_{NB}$  may be either measured or obtained from a number of articles [13]. Under non-overcast skies, the sun can cast a large and well defined shadow in front of the window façade. To cater such effect, it can be assumed that only diffuse component was considered for those shaded areas (using Eq. 4) [14]. The study can be extended to include the determination of the VDF and ADF for unobstructed skies [15]. VDF is the sum of C (VSC under unobstructed skies) and D. Referring to the Longmore equation [16], the additional term is the ratio of C to the area of floor and lower parts of the walls below the mid-height of the window ( $A_{fw}$ ). Knowing the C and D values, the IRC, VDF and ADF for various unobstructed CIE Standard Skies can then be estimated accordingly.

### 3. Model for estimating DF

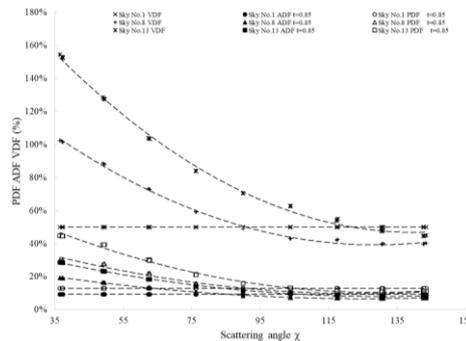
The PDF, ADF and VDF under the 15 unobstructed CIE Standard Skies were estimated based on the following case studies. The vertical window area of 4m (W) x 2m (H) with the sill height of 1m off the floor was set. Figure 2 presents the layout of the room and the reference points for estimating the DF. The centre of the vertical window faces south (i.e. sun-shading vertical surface) on 21 December (winter solstice) solar noon ( $\alpha_s=45^\circ$  and  $\phi_s=180^\circ$ ) with façade reflectance 0.6 and ground reflectance 0.2 under 15 CIE unobstructed skies.



**Figure 2.** The room layout and the reference points.

#### 4. Results Analysis and Design Implications

Accordingly, the C, D, SC and IRC for ADF and PDF were computed and analysed. Figure 3 presents the ADF, PDF and VDF at various scattering angle ( $\chi$ ) under Skies 1, 8 and 13 representing overcast, partly cloudy and clear skies, respectively. The VDF is higher than the corresponding PDF and ADF. Sky 13 has the maximum ADF of 29%, PDF (centre point) of 45% and VDF of 154%. Under dark overcast condition, all the three DF types in Sky 1 are independent of  $\chi$ . For non-overcast skies (Skies 8 and 13), the DFs change with  $\chi$ . The peak DFs appear at low  $\chi$  of 36.5° and the DFs drop with the increase of  $\chi$ . When  $\chi$  is between 120° and 130°, the DFs are at their minimum values and then increase slightly as the  $\chi$  goes further.



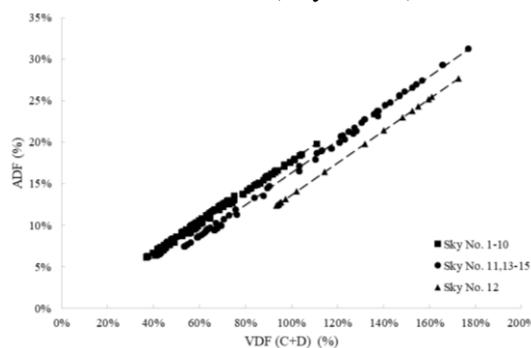
**Figure 3.** The  $\chi$  against the ADF, PDF and VDF for 15 CIE unobstructed skies.

Figure 4 displays the correlation between the VDF and ADF for all cases. Three linear trends can be observed and Eqs. 6 to 8 present the regression equations for Skies 1-10, Skies 11, 13-15 and Sky 12, respectively. The  $R^2$  values close to 1 indicating quite perfect relation. It shows that once the C and D values are obtained, the required ADF can be computed using appropriate room parameters.

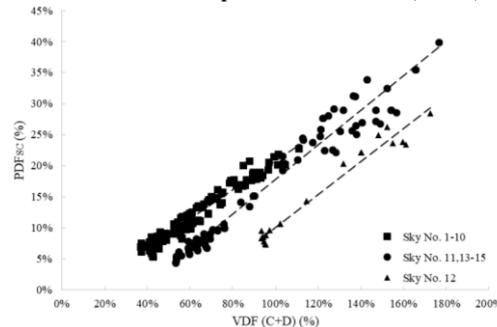
- $ADF = 0.186VDF - 0.008 \quad R^2 = 0.993$  (Sky No.1-10) (6)

- $ADF = 0.194VDF - 0.03 \quad R^2 = 0.998$  (Sky No.11,13-15) (7)

- $ADF = 0.193VDF - 0.057 \quad R^2 = 1$  (Sky No.12) (8)



**Figure 4.** The relationship between VDF(C+D) and ADF



**Figure 5.** The relationship between VDF and PDF<sub>sc</sub>

Similarly, the simulated data of VDF and the SC of PDF ( $PDF_{SC}$ ) are plotted in Fig. 5 and Eqs.9-11 give the mathematical functions for these two components. The  $R^2$  ranges from 0.96 to 0.97 indicating that 96-97% of the  $PDF_{SC}$  can be explained by the VDF. It means that the  $PDF_{SC}$  and PDF can be computed by the corresponding VDF [10]. Such issue can help using open loop daylight-linked lighting controls.

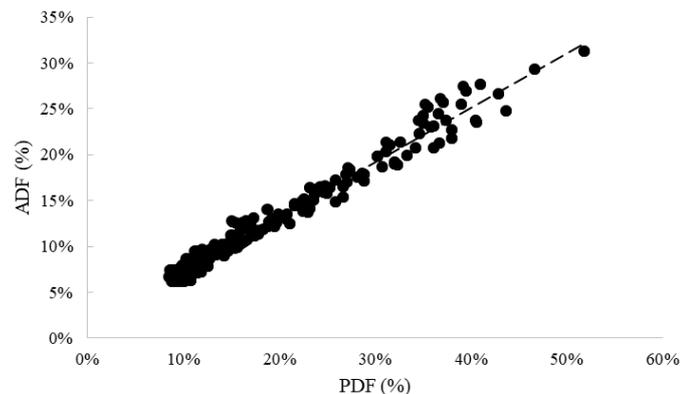
$$\bullet \quad PDF_{SC} = 0.23VDF + 0.03 \quad R^2 = 0.955 \quad (\text{Skies.1-10}) \quad (9)$$

$$\bullet \quad PDF_{SC} = 0.29VDF + 0.1 \quad R^2 = 0.968 \quad (\text{Skies.11,13-15}) \quad (10)$$

$$\bullet \quad PDF_{SC} = 0.26VDF + 0.16 \quad R^2 = 0.972 \quad (\text{Sky 12}) \quad (11)$$

Likewise, the PDF and ADF are depicted in Fig. 6 and the linear regression is shown in Eq. 12. The strength of correlation is considered very good as  $R^2$  is 0.97. The mathematical model can estimate the point that PDF is equal to ADF.

$$\bullet \quad ADF = 0.59 PDF + 0.014 \quad R^2 = 0.97 \quad (\text{All 15 CIE Skies}) \quad (12)$$



**Figure 6.** The relationship between PDF and ADF

The daylight glare index (DGI) and the corresponding subjective perception levels are : '<16 imperceptible', '16-20 perceptible', '20-24 acceptable', '24-28 uncomfortable' and '>28 intolerable', respectively. The DGIs were calculated when the vertical window faces south on 21 December solar noon ( $\chi=44.3^\circ$ ) and Table 2 shows the results. For overcast skies, the DGIs are almost under 22. For non-overcast skies, the indoor daylight environment is only acceptable under Sky 11. The DGIs of other clear skies are in the range of 24-28 which cause the uncomfortable daylight environment. The results show that discomfort glare is caused by high luminance between the glare source (window) and its surroundings. Glare from window usually arises when large amount of direct sunlight enters the room. A high availability of daylight in an indoor environment could be a disadvantage for achieving optimal visual conditions, not only for psychological reactions, but also for the variability of the characteristics of daylight during the time.

**Table 2.** Vertical daylight factor (VDF) and daylight glare index (DGI) under 15 CIE skies

Sky No.	VDF	DGI	Sky No.	VDF	DGI	Sky No.	VDF	DGI
1	0.5	18.08	6	0.72	23.61	11	1.28	23.98
2	0.58	21.06	7	0.85	24.63	12	1.61	22.73
3	0.56	19.31	8	0.94	25.87	13	1.38	25.71
4	0.66	21.92	9	0.93	24.81	14	1.49	25.45
5	0.6	20.62	10	1.05	24.57	15	1.57	27.29

## 5. Conclusions

Calculating procedures for indoor daylight determination under the 15 CIE Standard Skies were elaborated. A number of equations, figures and charts were presented for estimating the VDF, ADF

and PDF. The methods for determining VSC and ground reflected components were adopted to estimate the configuration factors C and D such that the IRC can be computed. With the required SC, VSC and IRC, the PDF, VDF and ADF can then be obtained for subsequent analysis. The calculation procedures explicitly illustrate the interdependency of various daylight parameters. The DGI has also been calculated to show the visual discomfort resulting from high window luminance. The analysis and calculation of DFs and DGIs are necessary in order to achieve a good indoor daylight environment and avoid visual discomfort. As the 15 CIE skies represent the actual skies for many places and cover the whole probable spectrum of skies found in nature, the findings could be globally adopted and would be useful to practitioners engaged in architectural and daylighting designs and evaluations. Further research analyses for validating more complex cases using parametric study will be conducted in near future.

### References

- [1] Li DHW. A review of daylight illuminance determinations and energy implications. *Appl. Energy* 2010;87(7):2109-2118
- [2] Chartered Institution of Building Services Engineers. Code for interior lighting. London, UK: CIBSE, 1997.
- [3] Yu X, Su Y. Daylight availability assessment and its potential energy saving estimation – A literature review. *Renew. Sustain. Energy Rev.* 2015;52:494-503.
- [4] Lynes J, Littlefair P. Lighting energy savings from daylight: Estimation at the sketch design stage. *Light Res Technol* 1990;22(3):129-137.
- [5] Cheung HD, Chung TM. Calculation of the vertical daylight factor on window facades in a dense urban environment. *Archit Sci Rev* 2005; 48(1):81-92.
- [6] Hraska J. Criteria of daylighting and sunlight access in sustainable construction evaluation systems. In: *Proceedings of the 5th International Conference on Solar Radiation and Daylighting*, 10-11 Aug. 2011, Brno University of Technology, Czech Republic, pp.98-103.
- [7] Littlefair PJ. Predicting hourly internal daylight illuminances for dynamic building energy modeling. Garston, UK: Building Research Establishment: 1989.
- [8] International Standardisation Organisation. CIE S 011/E:2003. Spatial Distribution Daylight-CIE Standard General Sky, ISO Standard 15469: 2004, Geneva: 2004.
- [9] Li DHW, Lau CCS, Lam JC. A study of 15 sky luminance patterns against Hong Kong data. *Archit Sci Rev* 2003;46(1):61-68.
- [10] Li DHW, Lou S, Ghaffarianhoseini A, Alshaibani KA, Lam JC. A review of calculating procedures on daylight factor based metrics under various CIE Standard Skies and obstructed environments. *Building and Environment* 2017; 112:29-44.
- [11] Li DHW, Cheung GHW, Lau CCS. A simplified procedure for determining indoor daylight illuminance using daylight coefficient concept. *Build Environ* 2006;41(5):578-589.
- [12] Li DHW, Li C, Lou SW, Tsang EKW, Lam JC. Analysis of vertical sky components under various CIE Standard General Skies. *Indoor Built Environ* 2016;25(4):703-711.
- [13] Kittler R, Darula S, Perez R. A set of standard skies, Polygrafia, Bratislava. 1998.
- [14] Li DHW. Daylight and energy implication for the CIE standard skies. *Energy Convers Manage* 2007;48(3):745-755.
- [15] Li DHW, Cheung GHW. Average daylight factor for the 15 CIE standard skies. *Light Res Technol* 2006;38(2):137-152.
- [16] Longmore J. Daylighting: a current view. *Light and Lighting* 1975;68:113-119.

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