Disturbance Observer-Based Adaptive Current Control With Self-Learning Ability to Improve the Grid-Injected Current for LCL-Filtered Grid-Connected Inverter

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ABSTRACT During the design of the conventional current controller for a grid-connected inverter with LCL filter, the parameter mismatches and disturbances are generally neglected, which may seriously affect the control performance, even result in instability. In order to improve the ability of disturbance rejection and ensure a desired control performance, this paper proposes an Adaptive PID (APID) controller with the self-learning ability based on the Disturbance Observer (DOB). First, the full state-feedback and state observer are utilized to achieve active damping and eliminate the effect of computational delay. Then, aiming to estimate and compensate the lumped disturbance, a DOB is designed. Beneficial from DOB, the steady-state performance is almost not affected by model uncertainties and unmodeled dynamics, however, the transient performance is still deteriorated inevitably due to the limitation of DOB. Thus, an online adaptive method using APID is finally proposed to further improve grid-injected current dynamics. The control parameters can be automatically adjusted in real time by adaptive learning rule, which significantly improves the system robustness and the control performance. Simulation and experiments are provided to demonstrate the effectiveness of the proposed strategy.

INDEX TERMS Adaptive PID controller, disturbance observer, full state-feedback, LCL filter, state observer.

I. INTRODUCTION

The LCL filter has been widely adopted in grid-connected inverters since they can provide improved attenuation on the pulse width modulation switching harmonics compared with the L filter [1], which allows the use of smaller inductance to satisfy the stringent harmonic requirements [2], [3]. In order to achieve excellent performances and address global stability, various control strategies have been proposed, such as PI-based control [4], [5], Repetitive Control (RC) [6], [7], Sliding Mode Control (SMC) [8], [9], Deadbeat Control (DBC) [10], [11], Model Predictive Control (MPC) [12], [13], etc. Most of the aforementioned methods are model-based control scheme, thus the grid-injected current is sensitive to the uncertainties, which means the unmodeled dynamics and disturbances of inverters, including nonlinear load, parameter uncertainties, grid inductance variation [14], dead time, on-state voltage drop of switching devices and diodes, would inevitably cause performance deterioration.

To eliminate the influence of disturbances, several Disturbance/Uncertainty Estimation and Attenuation (DUEA) schemes had been proposed [15]. Recently, DUEA schemes on the grid-connected inverters, such as Uncertainty
and Disturbance Estimator (UDE) [16], Extended State Observer (ESO) [17], Active Disturbance Rejection Control (ADRC) [18] and DOB [19], have become a research hotspot. In [16], a UDE-based grid-injected current control strategy was proposed. This method allows decoupling between disturbance attenuation and reference tracking, however, the introduced differential feedforward may be sensitive to noise. Wang et al. [17] separated observation dynamics and parameter mismatches error by utilizing ESO, which can guarantee robust adaption for parameter variation. Nevertheless, the method is only applicable for constant or slowly varying disturbance estimation. Benrabah et al. [18] presented a simplified robust control method based on ADRC [20] using Padé approximation. In this strategy, the first order linear ADRC controller is applied to deal with parameter variation of LCL filter, which greatly simplifies the controller design, yet brings adverse effects into the closed-loop control performances. In [19], DOB-based feedforward scheme was proposed by introducing the estimation of fluctuations into the reference of the inner loop, which can remarkably attenuate disturbance effects both in structure and external power.

Among these DUEA strategies above, owing to its relatively simple structure and prominent performance [21]–[23], DOB, combining with other controller, is increasingly adopted in the applications of inverters to improve the ability of disturbance rejection. In [24], PI controller based on feedback linearization technique and DOB, was proposed to provide good steady-state performance and improve robustness. However, its assumption that the lumped disturbance approaches to constant value may be not satisfied sometimes, especially when the background harmonic voltage exists. In [25], an internal model-based DOB was proposed to enhance the control performance. This controller fuses the merits of DOB and RC. Nevertheless, the tracking characteristic and stability are affected by the time delay element introduced by RC. Nguyen and Jung [26] investigated a DOB-based MPC, achieving fast dynamic response under the uncertain parameters. However, the unconstrained mode and constrained mode bring high computational effort.

It should be pointed out that although the steady-state performance is almost not be affected by model uncertainties or unmodeled dynamics under the contribution of DOB, the transient performance is still deteriorated inevitably. Because the model uncertainties and external disturbances cannot be completely eliminated due to limitation of DOB [25], and they will produce a transient component to the grid-injected current, which will degrade transient performance. However, few studies on DOB-based control of the grid-connected inverters have considered this issue so far. Aiming to overcome it and further improve grid-injected current dynamics, an effective way is to integrate online adaptive method into the designed controller [22], [27]. For example, in [28], DOB-based fuzzy SMC strategy was proposed. The switching gain can be adaptively adjusted, which achieves strong robustness and fast response. However, it only compensates constant or slowly varying disturbance.

Motivated by the aforementioned limitations, this paper proposes an Adaptive PID (APID) controller with the self-learning ability based on DOB. Firstly, the full state-feedback which plays a role of the active damping is utilized. It is well known that this feedback strategy can arbitrarily assign the position of the closed-loop poles, which can obtain the desired dynamic response [29]. While, since more sensors are required, a state observer is also constructed as a state predictor to reduce the number of sensors and eliminate the influence of computational delay. Secondly, aiming to compensate the lumped disturbance and relax the sensitivity issue, a DOB is designed. DOB possesses two-degrees-of-freedom structure, allowing to decouple the tracking performance and disturbance rejection by the independent design of nominal controller and DOB. Therefore, the excellent tracking performance and disturbance rejection can be achieved simultaneously. Finally, an adaptive PID regulator with the self-learning ability is proposed to prevent transient performance deterioration. The control parameters can be automatically adjusted in real time by adaptive learning rule, which further improves the system robustness and the control performance.

The rest of this paper is organized as follows. The modeling of three-phase grid-connected voltage-source inverter with LCL filter is provided in Section II. Then, in Section III, the theoretical analysis and design of the proposed strategy are addressed. The robust stability under model uncertainties is analyzed based on the small-gain theorem in Section IV. The effectiveness of the proposed method is demonstrated by a series of comparative simulations and experiments in Section V. Finally, a conclusion of the paper is drawn in Section VI.

FIGURE 1. The proposed control scheme.

II. MODELING OF GRID-CONNECTED VOLTAGE SOURCE INVERTER WITH LCL FILTER

A three-phase grid-connected voltage-source inverter with LCL filter is depicted in Fig. 1. Inverter-side inductor $L_1$, filter
capacitor $C$ and grid-side inductor $L_2$ give the LCL filter. $L_q$ is the equivalent grid inductor. $R_1$, $R_2$ are the equivalent series resistances of $L_1$ and $L_2$, respectively. $U_{dc}$, $u_i$ and $u_g$ represent the DC-link, inverter output and the Point of Common Coupling (PCC) voltages, respectively.

Thus, in the synchronous reference frame rotating at the grid angular frequency $\omega_g$, the state-space equation is expressed as:

$$
\begin{align*}
    \dot{x} &= \tilde{A}x + B_1u_i + B_2u_g \\
y &= C_c x
\end{align*}
$$

where

$$
\tilde{A} = \begin{bmatrix}
    -\frac{R_1}{L_1} - j\omega_g & -\frac{1}{L_1} & 0 \\
    \frac{1}{C} & -j\omega_g & -\frac{1}{C} \\
    0 & 1 & -\frac{R_2}{L_2} - j\omega_g
\end{bmatrix},
$$

$$
B_1 = \begin{bmatrix}
    \frac{1}{L_1} \\
    0 \\
    0
\end{bmatrix}^T,
$$

$$
B_2 = \begin{bmatrix}
    0 \\
    0 \\
    -\frac{1}{L_2}
\end{bmatrix}^T, \quad \text{and} \quad C_c = \begin{bmatrix}
    0 & 0 & 1
\end{bmatrix}.
$$

Due to the coupling terms between d- and q-axis, it is complicated to design a desired controller. In this paper, the coupling terms are treated as a part of internal disturbances, since they have a negligible influence on the stability of system, if the controller parameters are well tuned. And taken the parameter mismatches and unmodeled dynamics into consideration, the model is rewritten as:

$$
\begin{align*}
    \dot{x}_d &= Ax_d + B_1u_i + B_2u_g + \Phi \lambda_d \\
y_d &= C_c x_d \\
\dot{x}_q &= Ax_q + B_1u_i + B_2u_g + \Phi \lambda_q \\
y_q &= C_c x_q
\end{align*}
$$

where

$$
A = \begin{bmatrix}
    -\frac{R_1}{L_1} & -\frac{1}{L_1} & 0 \\
    \frac{1}{C} & 0 & -\frac{1}{C} \\
    0 & 1 & -\frac{R_2}{L_2}
\end{bmatrix}, \quad \text{and} \quad 
\Phi = \begin{bmatrix}
    -\frac{1}{L_1} & 0 & 0 \\
    0 & -\frac{1}{C} & 0 \\
    0 & 0 & -\frac{1}{L_2}
\end{bmatrix}.
$$

The subscript $d$ and $q$ are the component of d- and q-axis, respectively. $\lambda_d$ and $\lambda_q$ represent the lumped disturbance caused by the parameter variations, cross-coupling and other unstructured uncertainties, which can be deduced as:

$$
\begin{align*}
    \lambda_d &= \begin{bmatrix}
        \Delta L_1 i_{1d} - L_1 \omega_g i_{1q} - \Delta L_1 \omega_g i_{1q} + \Delta R_1 i_{1d} + \varepsilon_{1d} \\
        \Delta C \dot{u}_{cd} - \Delta \omega \dot{u}_{eq} - \Delta \omega \dot{u}_{eq} + \varepsilon_{2d} \\
        \Delta L_2 i_{2q} - L_2 \omega_g i_{2q} - \Delta L_2 \omega_g i_{2q} + \Delta R_2 i_{2q} + \varepsilon_{3d}
    \end{bmatrix} \\
    \lambda_q &= \begin{bmatrix}
        \Delta L_1 i_{1q} + L_1 \omega_g i_{1d} + \Delta L_1 \omega_g i_{1d} + \Delta R_1 i_{1q} + \varepsilon_{1q} \\
        \Delta C \dot{u}_{cd} + \Delta \omega \dot{u}_{eq} + \Delta \omega \dot{u}_{eq} + \varepsilon_{2q} \\
        \Delta L_2 i_{2q} + L_2 \omega_g i_{2q} + \Delta L_2 \omega_g i_{2q} + \Delta R_2 i_{2q} + \varepsilon_{3q}
    \end{bmatrix}
\end{align*}
$$

where the symbol “$\Delta$” denotes the deviation from the nominal values, $\varepsilon_{d}$ and $\varepsilon_{q}$ represent the unmodeled uncertainties of d- and q-axis, respectively.

Obviously, the expressions in the d- and q-axis are identical, in essence. Hereafter, for the brevity of notation, the subscript $d$ and $q$ will be omitted.

For digital implementation of the control algorithm, the state space equation is represented in discrete-time domain as follows:

$$
\begin{align*}
    x(k+1) &= Gx(k) + H_1 u_i(k) + H_2 u_g(k) + \Gamma \lambda(k) \\
y(k) &= C_c x(k)
\end{align*}
$$

where

$$
G = e^{A \tau_s}, \quad H_1 = \int_0^{T_s} e^{A \tau} d \tau B_1, \\
H_2 = \int_0^{T_s} e^{A \tau} d \tau B_2, \quad \Gamma = \int_0^{T_s} e^{A \tau} d \tau \Phi,
$$

$T_s$ is sample period and $k$ is the discrete sampling instant.

### III. THEORETICAL ANALYSIS AND DESIGN OF THE PROPOSED CONTROL STRATEGY

The structure of the proposed strategy is depicted in Fig. 1, from which it can be observed that the control objectives can be met by the three cascaded control loops. The inner loop is to achieve active damping via full state-feedback using predicted state variables; the middle loop is utilized to combine the disturbance compensation and tracking performance together; whereas the outer loop is designed to improve the quality of grid-injected current by minimizing grid-injected current error online both in ideal and perturbed conditions.

#### A. STATE-FEEDBACK AND STATE OBSERVER

The full state-feedback is utilized in inner loop for the active damping, as described in Fig. 1. Accordingly, the control law is produced as:

$$
u_i(k) = u_i^o(k) - K_v x(k)
$$

where $u_i^o(k)$ denotes the output of the designed controller with disturbance compensation, and $K_v = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$ is the state-feedback gain vector.
Substituting (5) into (4) yields
\[
\begin{cases}
  x(k + 1) = (G - H_1 K_e)x(k) + H_1 u^*_g(k) + H_2 u_g(k) \\
  + \Gamma \lambda(k) \\
  y(k) = C_e x(k) \cdot 
\end{cases}
\]
(6)

Here, the analytical relationship from \( u^*_g(k) \) to \( i_g(k) \) is treated as a generalized controlled plant \( P_G(z) \). Thus, from (6), the transfer function of the generalized plant is derived as
\[
P_G(z) = C_e(zI_3 - G + H_1 K_e)^{-1} H_1 
\]
(7)
where \( I_3 \) is a three-dimensional identity matrix.

According to the guidelines for selecting the pole locations [30], [31], let the characteristic polynomial of (7) be
\[
\det(zI_3 - G + H_1 K_e) = (z - \alpha_1)(z - \alpha_2)(z - \alpha_3) \quad (8)
\]
where \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are the desired poles of the state-feedback, then the state-feedback gain vector \( K_e \) can be derived.

It should be noted that the full state-feedback requires more sensors compared with capacitor current feedback, which increases the system cost and vulnerability. Moreover, the computational delay has not been considered. In practice, which increases the system cost and vulnerability. Moreover, the computational delay has not been considered. In practice, however, one-step delay associated with the digital implementation exists between the control voltage and inverter output voltage. To account for the computational delay, the system dynamics is rewritten as follows
\[
\begin{cases}
  x(k + 1) = Gx(k) + H_1 u_1(k - 1) + H_2 u_2(k) + \Gamma \lambda(k) \\
  y(k) = C_e x(k) \cdot 
\end{cases}
\]
(9)

Referring to [16], [32], an effective way to compensate for this time delay is to make one-step-ahead prediction of the state variables and utilize predicted states instead of the current states in determining the control law.

In order to reduce the number of sensors and predict state variables one-step ahead, a full-order state observer is utilized in this paper. Since the grid-injected current \( i_g(k) \), PCC voltage \( u_2(k) \) and the inverter output voltage \( u_1(k) \) is internally known, the remaining state variables can be estimated precisely, i.e.
\[
\begin{cases}
  \hat{x}(k + 1) = \hat{G}\hat{x}(k) + H_1 u_1(k - 1) + H_2 u_2(k) \\
  + L[y(k) - \hat{y}(k)] \\
  \hat{y}(k) = C_e \hat{x}(k) \cdot 
\end{cases}
\]
(10)
where the subscript „^“ denotes the estimated value, \( L = [l_1 \ l_2 \ l_3]^T \) is the observer gain vector. With (9) and (10), the dynamics of the estimation error \( e_x(k) = x(k) - \hat{x}(k) \) is
\[
e_x(k + 1) = (G - LC_e)e_x(k) + \Gamma \lambda(k). \quad (11)
\]

According to the pole placement, the observer gain vector \( L \) can be determined, if the characteristic polynomial of the observer dynamics is selected as:
\[
\det(zI_3 - G + LC_e) = (z - p_1)(z - p_2)(z - p_3) \quad (12)
\]
where \( p_1, p_2, \) and \( p_3 \) are the desired poles of the state observer.

From (11), the estimation error will asymptotically converge under the elaborately selected poles, as long as the sufficient and necessary condition below is satisfied
\[
\rho(G - LC_e) = \max_{1 \leq \lambda \leq 3} |\lambda_i(G - LC_e)| < 1 \quad (13)
\]
where \( \lambda_i(G - LC_e), \rho(G - LC_e) \) denote the eigenvalues and spectral radius of \( G-LC_e \), respectively.

Consequently, when the steady-state is reached, \( e_x(k+1) \) and \( e_x(k) \) will converge to the same, i.e.
\[
\lim_{k \to \infty} |e_x(k + 1)| \approx \lim_{k \to \infty} |e_x(k)| \approx \lim_{k \to \infty} |(I_3 - G + LC_e)^{-1} \Gamma \lambda(k)|. \quad (14)
\]

It is obvious that there exists estimation error in the presence of lumped disturbance, which will inevitably degrade the control performance. Thus, aiming to improve the ability of disturbance rejection, a DOB is constructed.

![FIGURE 2. (a) Block diagram of DOB. (b) The standard M-Δ configuration of DOB for the small-gain theorem.](image)

**B. DISTURBANCE REJECTION BY DISTURBANCE OBSERVER**

As mentioned above, the disturbance and model uncertainties may result in performance deterioration. For estimating and compensating this lumped disturbance, a DOB is designed, as depicted in Fig. 2. \( u_g(z), d(z), \hat{d}(z) \) and \( n(z) \) represent the controller output, external disturbance, estimated disturbance and measurement noise, respectively. \( Q(z) \) is a low-pass filter which needs to be elaborately designed. \( P_G(z) \) is the actual generalized plant defined by (7) and \( \bar{P}_G^{-1}(z) \) is the inverse of nominal generalized plant. To account for the mismatches between \( P_G(z) \) and \( \bar{P}_Gn(z) \), \( P_G(z) \) is formed into the output multiplicative perturbation description as follows
\[
\left\{ \begin{array}{l}
P_G(z) = [1 + \Delta(z)]P_Gn(z) \\
\Delta(z) = W(z)\delta(z) \\
\|\delta(z)\|_\infty \leq 1 \end{array} \right. \quad (15)
\]
where \( \Delta(z), W(z), \) and \( \delta(z) \) represent the modeling error, weighting function and variation, respectively.
Based on the block diagram in Fig. 2, the estimated disturbance can be derived as
\[
\hat{d}(z) = \frac{Q(z)\Delta(u) + Q(z)[1 + \Delta(z)]d(z) + Q(z)n(z)}{1 + Q(z)\Delta(z)}
\]
and the grid-injected current \(i_g(z)\) is written as follows:
\[
i_g(z) = G_{rl}(z)u_r(z) + G_{dl}(z)d(z) + G_{nl}(z)n(z)
\]
where \(G_{rl}(z), G_{dl}(z),\) and \(G_{nl}(z)\) denote the transfer functions from \(u_r(z), d(z),\) and \(n(z)\) to \(i_g(z),\) respectively. They are given by
\[
\begin{align*}
G_{rl}(z) &= \frac{P_G(z)P_G(z)}{P_G(z) + Q(z)[P_G(z) - P_G(z)]} \\
G_{dl}(z) &= \frac{[1 - Q(z)]P_G(z)P_G(z)}{P_G(z) + Q(z)[P_G(z) - P_G(z)]} \\
G_{nl}(z) &= \frac{Q(z)[1 + \Delta(z)]P_G(z)}{1 + Q(z)\Delta(z)}
\end{align*}
\]

It is noted that the disturbance \(d(z)\) is dominant in the low frequency segment, while the noise \(n(z)\) is dominant in the high frequency segment. Thus, from (18), in the low frequency segment, \(G_{dl}(z) \approx 0\) provided that \(Q(z) \approx 1\), which implies the disturbance can be effectively attenuated. Therefore, within the bandwidth of \(Q(z)\), the grid-injected current \(i_g(z)\) is deduced as
\[
i_g(z) \approx P_{Gn}(z)u_r(z).
\]

It can be seen from (19) that even if the actual plant is perturbed, the DOB forces it to maintain the characteristic of the nominal plant, which significantly improves the ability of disturbance rejection and relaxes the lumped disturbance sensitivity issue.

Then, we transfer the perturbed plant and DOB into the standard M–\(\Delta\) configuration, as shown in Fig. 2(b). Based on the small-gain theorem [22], [25], the sufficient condition for the system stability can be easily obtained
\[
\|\Delta(z)Q(z)\|_{\infty} < 1.
\]

From (20), an improper \(Q(z)\) would deteriorate the stability. Hence, when selecting the parameters of \(Q(z)\), including the relative order and bandwidth, attention should be paid. In fact, the higher order of \(Q(z)\) is, the faster response of the DOB becomes, but with the increasing order, phase lag may make the system unstable. On the other hand, a high bandwidth \(Q(z)\) can enhance the anti-disturbance ability, however, it may easily result in system instability and increase the sensitivity to measurement noise \(n(z)\). Thus, a tradeoff among the stability, disturbance suppression ability and noise sensitivity, must be made when designing DOB.

Considering that \(P_G(z)\) is a three-order plant, to make \(Q(z)P_G^{-1}(z)\) realizable, this paper defines \(Q(z)\) as
\[
Q(z) = Z\left(\frac{1}{(\tau s + 1)^3}\right)
\]
where \(\tau, Z[\cdot]\) denote the time constant and ZOH-transformation, respectively.

It can be obtained from (21) that \(Q(z)\) is slightly less than unity within its bandwidth, which results in a fact that \(G_{dl}(z)\) is not strictly equal to zero. Therefore, the lumped disturbance cannot be completely eliminated and it will produce a transient component to the grid-injected current, which will degrade the dynamic performance. To further improve the quality of grid-injected current, an online adaptive control strategy with the self-learning ability is finally proposed.

C. ADAPTIVE PID CONTROLLER WITH THE SELF-LEARNING ABILITY

Fig. 3 shows the schematic diagram of the APID controller, where \(e(k)\) is tracking error of grid-injected current at the \(k\)th sampling instant, which is transformed into three internal variables, denoted \(\chi_1, \chi_2,\) and \(\chi_3,\) respectively; \(w_1, w_2,\) and \(w_3\) are the weight coefficients of the each internal variable; \textit{rule} means the adaptive learning rule, which is utilized to adjust the weight coefficients online; \(K\) is the gain coefficient and \(u_{r}(k)\) is the output of the APID controller.

The tracking error and internal variables are defined as follows:
\[
\begin{align*}
\chi_1(k) &= \Delta e(k) = e(k) - e(k - 1) \\
\chi_2(k) &= e(k) \\
\chi_3(k) &= e(k) - 2e(k - 1) + e(k - 2).
\end{align*}
\]
Correspondingly, \(u_r(k)\) can be expressed as
\[
u_r(k) = u_r(k - 1) + K \sum_{i=1}^{3} w_i(k) \chi_i(k).
\]

Referring to optimal control theory, for pursuing excellent tracking performance, the quadratic performance index about \(e(k)\) can be utilized. Thus, we insert a performance index function into the self-learning algorithm of the APID controller. The discrete-type quadratic error function is defined as
\[
E(k) = \frac{1}{2} \left[ i_g, ref(k) - i_g(k) - n(k) \right]^2 = \frac{1}{2} e(k)^2.
\]
In order to minimize the error function $E(k)$, one can evaluate the following equation:

$$\frac{\partial E(k)}{\partial w_1(k)} = -e(k) \frac{\partial \hat{g}_1(k)}{\partial u_1(k)} \frac{\partial u_1(k)}{\partial w_1(k)} \quad i = 1, 2, 3.$$ (25)

For the steepest descent algorithm, the changes in weight coefficients can be deduced as

$$\begin{align*}
\Delta w_1(k) &= w_1(k) - w_1(k - 1) = -\eta P \frac{\partial E(k)}{\partial w_1(k)} \\
&= \eta P e(k) \frac{\partial \hat{g}_1(k)}{\partial u_1(k)} \frac{\partial u_1(k)}{\partial w_1(k)} \\
\Delta w_2(k) &= w_2(k) - w_2(k - 1) = -\eta I \frac{\partial E(k)}{\partial w_2(k)} \\
&= \eta I e(k) \frac{\partial \hat{g}_2(k)}{\partial u_2(k)} \frac{\partial u_2(k)}{\partial w_2(k)} \\
\Delta w_3(k) &= w_3(k) - w_3(k - 1) = -\eta D \frac{\partial E(k)}{\partial w_3(k)} \\
&= \eta D e(k) \frac{\partial \hat{g}_3(k)}{\partial u_3(k)} \frac{\partial u_3(k)}{\partial w_3(k)}
\end{align*}$$ (26)

where $\eta P$, $\eta I$, $\eta D$ are the proportional, the integral and the differential learning rates, respectively.

It can be observed from (23) that $\frac{\partial u_1(k)}{\partial w_1(k)}(i = 1, 2, 3)$ is equal to $K_\chi_1(k)(i = 1, 2, 3)$. However, $\frac{\partial \hat{g}_1(k)}{\partial u_1(k)}$ in (26) is uncertain. According to the engineering experience, when tuning the controller parameters, the $e(k)$ and $\Delta e(k)$ are expected to quickly converge to zero, which indicates the controller parameters tuning is associated with $e(k)$ and $\Delta e(k)$, so this paper utilizes $e(k) + \Delta e(k)$ to replace $\frac{\partial \hat{g}_1(k)}{\partial u_1(k)}$. The calculation error can be compensated by the adjustable learning rates.

Therefore, the adaptive learning rule is rewritten as

$$\begin{align*}
w_1(k) &= w_1(k - 1) + \eta P K_\chi_1(k)e(k) [e(k) + \Delta e(k)] \\
w_2(k) &= w_2(k - 1) + \eta I K_\chi_2(k)e(k) [e(k) + \Delta e(k)] \\
w_3(k) &= w_3(k - 1) + \eta D K_\chi_3(k)e(k) [e(k) + \Delta e(k)].
\end{align*}$$ (27)

Then, in order to ensure the convergence and robustness of (23) and (27), $w_i(k)(i = 1, 2, 3)$ is normalized as:

$$\bar{w}_i(k) = \frac{w_i(k)}{\sum_{j=1}^{3} |w_j(k)|}, \quad i = 1, 2, 3.$$ (28)

Accordingly, $u_i(k)$ in (23) can be modified as follows:

$$u_i(k) = u_i(k - 1) + K \sum_{j=1}^{3} \bar{w}_j(k) \chi_j(k).$$ (29)

Expanding (29) and comparing with the incremental digital PID regulator, one can easily obtain the equivalent proportional gain $K_p$, the integral gain $K_I$ and the differential gain $K_D$ of the APID controller, i.e.

$$\begin{align*}
K_P &= K \bar{w}_1(k) \\
K_I &= K \bar{w}_2(k) \\
K_D &= K \bar{w}_3(k).
\end{align*}$$ (30)

From the theoretical analysis above, the equivalent gains are able to be adjusted online to minimize grid-injected current error $e(k)$ and ensure robustness by the adaptive learning rule at each sampling instant. Hence, when the grid-injected current error occurs, caused by no matter the step-changed reference, parameters variation or disturbances, the performance index function $E(k)$ will be optimized, which will significantly improve the control performance.

In addition, the proposed adaptive strategy is designed with low computational burden. This strategy minimizes the requirements on the complexity and computational capacity. The relevant control parameters can be obtained by the trial-and-error method. And a rule of thumb to determine the values of $K$, $\eta P$, $\eta I$, and $\eta D$ is briefly summarized as follows:

1) Firstly, the values of initial weight coefficient $w_i(0) (i = 1, 2, 3)$ can be set arbitrarily.
2) Then, the gain coefficient $K$ needs to be elaborately selected. $K$ can be adjusted by the experimental results. The response of system becomes fast as the value of $K$ increases, but with the increasing value, the system may oscillate even become unstable.
3) Finally, determine the learning rates $\eta P$, $\eta I$, and $\eta D$. They are tuned to balance the overshoot and settling time.

IV. ROBUSTNESS ANALYSIS BASED-ON THE SMALL-GAIN THEOREM UNDER MODEL UNCERTAINTIES

It should be pointed out that, in essence, the APID controller is a nonlinear incremental digital PID regulator with the self-learning ability. To evaluate the robustness, we simplify it as a linear one, denoted $C_r(z)$, whose equivalent gains are obtained under the nominal plant. This simplification for the purpose of analysis is feasible, since once the steady-state is reached, the APID controller will be converted as incremental digital PID regulator. While, during the transient state, the APID controller has stronger adaptability due to the adjustable learning rule, which will achieve better performance than PID regulator. Hence, conclusions about the robustness drawn based-on PID regulator is also suitable for the APID controller.

In this section, the potential influence of parameter perturbation which may result in unexpected dynamics will be investigated. For simplification, the output multiplicative perturbation form in (15) is modified as follows:

$$P_G(z) = [1 + W(z)\delta(z)] P_Gn(z) \|\delta(z)\|_\infty \leq 1.$$ (31)

Then, $W(z)$ can be derived for all frequency range as

$$|W(z)| \geq \left| \frac{P_G(z)}{P_Gn(z)} - 1 \right|.$$ (32)

The inequality (32) implies that the frequency response curve of $W(z)$ is supposed to lie above the clustering frequency response curves of relative perturbation $[P_G(z)/P_Gn(z) - 1]$. Assuming that the filter parameters $L_1$, $L_2$ can drift $\pm25\%$, $C$ can drift $\pm5\%$ around the nominal values [33] listed in Table 1, the clustering frequency response curves of $[P_G(z)/P_Gn(z) - 1]$ can be easily obtained, which are
TABLE 1. Parameters of the system.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_x$</td>
<td>PCC voltage</td>
<td>110 V (RMS)</td>
</tr>
<tr>
<td>$U_d$</td>
<td>DC link voltage</td>
<td>350 V</td>
</tr>
<tr>
<td>$L_i$</td>
<td>Inverter-side inductor</td>
<td>1.2 mH</td>
</tr>
<tr>
<td>$L_g$</td>
<td>Grid-side inductor</td>
<td>1.2 mH</td>
</tr>
<tr>
<td>$C$</td>
<td>Filter capacitor</td>
<td>6 μF</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Equivalent series resistance of $L_e$</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>$R_g$</td>
<td>Equivalent series resistance of $L_g$</td>
<td>0.1 Ω</td>
</tr>
<tr>
<td>$K_s$</td>
<td>State-feedback gain vector</td>
<td>[13.1253, -0.3149, -3.1668]</td>
</tr>
<tr>
<td>$L$</td>
<td>State observer gain vector</td>
<td>[0.1463, -13.7063, 0.5122]</td>
</tr>
<tr>
<td>$K$</td>
<td>Gain coefficient of APID</td>
<td>1.8</td>
</tr>
<tr>
<td>$\eta_w, \eta_{i_x}, \eta_{i_y}$</td>
<td>Learning rate</td>
<td>0.8, 0.1, 0.1</td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>Switching frequency</td>
<td>10 kHz</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Sampling frequency</td>
<td>10 kHz</td>
</tr>
</tbody>
</table>

FIGURE 4. The frequency responses of the weighting function and relative perturbation.

FIGURE 5. The robustness analysis based on the small-gain theorem.

FIGURE 6. The transient responses of the grid-injected current in d- and q-axis under the step change in the active power and performance comparison between the APID + DOB and PI + DOB.

For simplicity, (36) can be transformed to

$$|W(z)| \leq \left| 1 + \frac{1}{1 + P_{Gn}(z)K_{eq}(z)} \right|.$$  (37)

From (37), the frequency response curve of $W(z)$ is expected to lie below the curve of the right-hand part. Both of them are depicted in Fig. 5. It is easily observed that the robustness under model uncertainties can be guaranteed.

V. SIMULATION AND EXPERIMENTAL VERIFICATION

A. SIMULATION VERIFICATION

In order to verify the effectiveness of the proposed strategy (APID + DOB), simulation tests on a 110 V/50 Hz/3 kW grid-connected inverter with LCL filter are carried out in MATLAB/Simulink environment based-on the system and current controller presented in Fig. 1, and the parameters utilized in simulations are listed in Table 1. In addition, to prove...
more advantageous performances of the proposed strategy, the comparative studies with the conventional PI controller based on DOB (PI + DOB) are also conducted. And for a fair comparison, the optimal control parameters have been designed for PI + DOB.

1) TRANSIENT PERFORMANCE UNDER THE STEP CHANGE IN THE ACTIVE POWER

The objective of this simulation is to verify the transient performance and tracking ability of the grid-injected current under the proposed strategy. To this end, the q-axis current reference is set to zero, while the d-axis current reference is stepped up from 0 A to 6.43 A at \( t = 0 \) s, and then stepped up to 12.86 A at \( t = 0.06 \) s. The dynamic performances of the grid-injected current controlled by the APID + DOB and PI + DOB are shown in Fig. 6. It can be observed that the transient response without any overshoot under the APID + DOB is faster than that under PI + DOB. This can be explained as follows: when the reference is suddenly changed, the tracking error \( e(k) \) occurs in the grid-injected current, and the discrete-type quadratic error function \( E(k) \) defined in (24) will be optimized online by the steepest descent algorithm and self-learning rule. Therefore, the actual grid-injected current quickly converges to the reference and the error \( e(k) \) asymptotically converges to zero. This confirms that the proposed strategy can achieve better transient performance and tracking ability compared with the traditional method.

2) ROBUSTNESS TO THE PARAMETER VARIATION

To evaluate the robustness to parametric variation of the proposed control strategy, a set of simulations under parameter mismatches have been carried out. Fig. 7(a)-(c) present the transient responses under the proposed strategy when \( L_1 \) varies \( \pm 25\% \), \( C \) varies \( \pm 5\% \) and \( L_2 \) varies \( \pm 25\% \) from the nominal values, respectively. It can be seen that,
compared with the transient performance under the nominal values, the dynamics almost keep unchanged regardless of the parameter variation. Thus the parameter variation in LCL filter brings little adverse effect on the grid-injected current under the proposed strategy, which sufficiently verifies the theoretical analysis in Section IV. In order to further highlight this advantage, the performance comparison between the APID + DOB and PI + DOB under parameter variation of $-25\%$ in $L_1$, $-5\%$ in $C$ and $-25\%$ in $L_2$ is presented in Fig. 7(d). Clearly, during the transient state, the ripple is observed in the grid-injected current controlled by PI + DOB. Because the effect of the parameter uncertainties cannot be completely eliminated due to limitation of DOB, and it produces a transient component to grid-injected current, which degrades transient performance. This fully matches the theoretical analysis in Section III. While, it should be noted that the proposed strategy has overcome this issue, and it can remain the prominent performance under the severe parameter variation.

3) ADAPTABILITY FOR THE GRID INDUCTANCE VARIATION
This test is conducted to verify the ability of the proposed strategy to adapt the grid inductance variation. Fig. 8(a) and Fig. 8(b) depict the transient performances when the grid inductance varies from 0 mH to 4 mH under the PI + DOB and proposed strategy, respectively. In Fig. 8(a), the overshoot and oscillation are easily observed in the grid-injected current when the grid inductance increases. However, in Fig. 8(b), the proposed strategy can ensure fast convergence characteristics and precise tracking ability regardless of the grid inductance variation. As well known, the control bandwidth decreases as the grid inductance increases. The proposed strategy attenuates the influence of the grid inductance on the performance. This is due to the inherent self-tuning capability, which enables the controller to be redesigned in real time to achieve optimal performances. Thus the proposed strategy can achieve fast convergence properties with enough robustness and high control bandwidth.

4) ABILITY TO REJECT THE GRID DISTURBANCES
In this scenario, the effectiveness of the proposed strategy will be verified under the grid disturbances, such as the unbalanced grid voltage and the background harmonic voltage.

For a weak grid, the unbalanced grid condition commonly occurs due to the grid fault, highly unbalanced loads, etc. Fig. 9 depicts the simulated result of the grid-injected current under the unbalanced grid voltage which is emulated by 10% grid voltage sag in Phase B and 20% grid voltage sag in Phase C. From Fig. 9, it can be found that the grid-injected current has a perfect sinusoidal waveform with zero tracking error and low total harmonic distortion (THD), which demonstrates the ability of the disturbance rejection under the unbalanced grid voltage.

In addition, there may exist large amounts of the background harmonic voltage caused by the nonlinear loads in the grid. In order to further enhance the harmonic rejection ability, a harmonic compensator (HC regulator) [34] can be added in parallel with the APID controller to attenuate the grid-injected current distortions. Considering the
high bandwidth characteristics of the proposed strategy, an 11\textsuperscript{th} HC regulator is utilized:

$$G_{hc}(z) = Z \left[ \sum_{h=2,4,6,8,10} \frac{K_{h}s}{s^2 + (\omega_g h)^2} \right]$$

$$= \sum_{h=2,4,6,8,10} K_{h} \frac{\sin(\omega_g h T_s)}{2 \omega_g h} \frac{z^2 - 1}{z^2 - 2z \cos(\omega_g h T_s) + 1}$$

where $K_{h}$ denote the resonant gains and all of them are selected as 800 in this paper. Fig. 10 shows the simulated result of the grid-injected current under the grid voltage distorted by the 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 9\textsuperscript{th}, and 11\textsuperscript{th} harmonics, whose magnitudes with respect to the grid fundamental voltage are 1.2\%, 2.8\%, 1.3\%, 2.4\%, and 1.5\%, respectively. It can be seen that the grid-injected current remains sinusoidal waveform with zero steady-state error and low THD, which indicates that the proposed strategy can effectively reject up to 11\textsuperscript{th} harmonic under the condition of $L_g = 4$ mH.

5) ACCURACY OF THE STATE VARIABLE ESTIMATED BY THE FULL-ORDER STATE OBSERVER

As analyzed in Section III, the accuracy of the state variables estimated by the full-order state observer affects the control performance. Fig. 11 presents the simulated waveform of the measured grid-injected current and the estimated
grid-injected current under a sudden change in the active power. In Fig. 11, $i_g$, $i_{g_{\text{est}}}$ and $i_{g_{\text{err}}}$ denote the measured grid-injected current, the estimated grid-injected current and the estimation error, respectively. It can be observed that the estimation is consistent with the measurement, and the estimation error is significantly small, which can be neglected. Thus, the high accuracy can be guaranteed. Similarly, the remaining state variables $i_1$ and $u_c$ can be precisely estimated, as well, which are omitted due to the length of this paper.

### B. EXPERIMENTAL VERIFICATION

In order to further verify the theoretical analysis, a 110 V/50 Hz/3 kW prototype is constructed based on dSPACE DS1202, Danfoss-FC320, Chroma 61830 and Chroma 62150H-600S. The experimental setup is shown in Fig. 12 and the experimental parameters coincide with those utilized in simulations. To emulate the grid inductance, the external inductors are utilized. In addition, the comparative experiments based on PI + DOB are also carried out to highlight the superiority of the proposed strategy. And all the experimental waveforms are captured from a Yokogawa DL 1640 digital oscilloscope.

#### 1) STEADY-STATE PERFORMANCE UNDER NOMINAL POWER OPERATION

In this scenario, the d-axis current reference is set to 12.86 A, while the q-axis current reference is set to zero. The corresponding steady-state performances under the PI + DOB and proposed strategy are shown in Fig. 13(a) and Fig. 13(b), respectively. It can be seen that both of them can achieve the zero steady-state error and unity power factor.

#### 2) TRANSIENT RESPONSE UNDER THE STEP CHANGE IN THE ACTIVE POWER

The following experiments are performed to investigate the disturbance rejection performance and the dynamic response under a sudden change in the active power. Fig. 14 shows the dynamic waveforms of the grid-injected current when the q-axis current reference is kept equal to zero, while the d-axis current reference is suddenly stepped up from 6.43 A to 12.86 A. It can be easily observed from Fig. 14(a) that the transient response with settling time about 8 ms under PI + DOB is slower than the proposed strategy in Fig. 14(b). As shown in Fig. 14(b), the grid-injected current references are well tracked and the proposed scheme provides excellent...
dynamic performance with settling time about 3 ms and without overshoot. This also demonstrates the proposed strategy has stronger disturbance rejection capability under the sudden change in active power.

3) ROBUSTNESS TO THE PARAMETER VARIATION

These tests evaluate the robustness of the proposed method by investigating the sensitivity of the grid-injected current against the parameter uncertainties under a reference step-changed condition. To emulate the parameter uncertainties, the values of $L_1$, $L_2$, $C$ are set with $-25\%$, $-25\%$ and $-5\%$ variation from the nominal values. The dynamic performances with parameter uncertainties are shown in Fig. 15, revealing that the perturbed plant behaves as the same as the nominal plant during the steady state, regardless of the filtering deterioration due to the physical change of an $LCL$ filter. That is because the DOB eliminates the influence of the parameter variation. However, the transient response under PI + DOB (with settling time about 7 ms in Fig. 15(a)) is still slower than that under the proposed strategy (with settling time about 3 ms in Fig. 15(b)). Especially, there are some current ripples in Fig. 15(a) during the transient state, which completely agrees with the simulation result, because the parameter variation as the internal disturbance defined in (3) results in the transient component superimposed on the grid-injected current. Compared with PI + DOB, the APID fully removes the ripple by optimizing the grid-injected current error, which is consistent with the theoretical analysis. This simultaneously verifies the effectiveness and robust stability of the proposed strategy.

4) ADAPTABILITY FOR THE GRID INDUCTANCE VARIATION

In actual applications, the grid inductance may change in a large range. Thus, these experiments are performed to highlight the benefits of the proposed method to maintain the stability and fast dynamics in the presence of the grid inductance. To emulate the grid inductance, the external inductors ($L_g = 4$ mH) are adopted. Fig. 16 shows the dynamic performances when the d-axis current reference is stepped down from 12.86 A to 6.43 A. In Fig. 16(a), the oscillation can be easily observed under PI + DOB. In addition, the settling time and overshoot are up to 10 ms and 40\%, respectively. The performance evidently deteriorates under the traditional method because the control bandwidth significantly reduces due to the great value of grid inductance. While, it can be
obtained from Fig. 16(b) that the fast transient performance against grid inductance variation is achieved with no oscillation and settling time about 4 ms. Different from the PI controller with fixed parameters, the APID can be automatically tuned online. Thus, the proposed strategy can achieve stronger adaptability and more prominent transient performance with high control bandwidth characteristics when the grid impedance varies widely.

5) ABILITY TO REJECT THE GRID DISTURBANCES
In applications, the grid-connected inverters are expected to normally operate no matter under the balanced grid condition or the unbalanced one. Fig. 17 depicts the experimental waveforms of the grid-injected current controlled by the proposed strategy under the unbalanced grid voltage. In Fig. 17(a), it is effectively demonstrated the ability to maintain the sinusoidal grid-injected current with zero tracking error even under the severely unbalanced grid condition. Simultaneously, Fig. 17(b) further reveals the prominent property of the fast performance recovery of the proposed strategy under the step change in the active power.

In order to further prove the ability of the proposed strategy to reject the grid disturbances, a programmable AC source (Chroma 61830) is utilized to emulate the grid voltage which is distorted by the 3rd, 5th, 7th, 9th and 11th harmonics. The magnitudes of harmonics with respect to the grid fundamental voltage are 1.2%, 2.8%, 1.3%, 2.4% and 1.5%, respectively. Fig. 18(a) depicts the experimental waveform of the grid-injected current under \( L_g = 4 \, \text{mH} \) when an 11th HC regulator is in parallel with the PI controller. The THD of the grid-injected current is 4.7%. As a contrast, Fig. 18(b) depicts the experimental waveform of the grid-injected current under \( L_g = 4 \, \text{mH} \) when an 11th HC regulator is in parallel with the APID controller. The THD is 1.8%, which is evidently lower than that under the traditional method. It can be seen that the grid-injected current maintains perfect sinusoidal waveform with low THD and the 11th harmonic has been successfully attenuated. These tests further verify that the proposed strategy can maintain the high-bandwidth characteristics under the great value of grid inductance and distorted grid voltage.

VI. CONCLUSION
In this paper, based on disturbance observer, an adaptive current control strategy with the self-learning ability is proposed to overcome the lumped disturbance sensitivity issue and improve the control performance. The principle of the proposed strategy is deduced in detail and the robust stability is analyzed based on the small-gain theorem. By theoretical analysis, the following conclusions can be drawn.

1) Embedding a DOB into control loop can effectively suppress the influence of disturbances and maintain the characteristic of the nominal plant, which significantly enhances the robust stability and ability of the disturbance rejection.

2) Optimizing the grid-injected current error through the steepest descent algorithm and self-learning rule, APID can achieve high control performance and tracking accuracy, even when the filter parameters and grid inductance vary widely.

Simulations and experiments on a 110 V/50 Hz/3 kW LCL-filter-based three-phase grid-connected inverter prototype verify the effectiveness of the proposed strategy.

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