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Case study

Application of response surface methodology: Predicting and optimizing the properties of concrete containing steel fibre extracted from waste tires with limestone powder as filler

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**Abstract**

This study showcases the predictive and optimization capabilities of response surface methodology with respect to the fresh and hardened properties of waste tyre steel fibre reinforced concrete containing limestone powder. Response surface methodology has the advantage of simultaneously varying chosen independent variables to provide a useful model for overall response variation. The study identifies aspect ratio (50–140), water cement ratio (0.2–0.4) and cement content (25%–40%) as independent variables while limestone powder was kept constant at 5% by weight of concrete. Predictive equations for the water intake/absorption, compressive strength, flexural strength, split tensile strength and slump of fibre reinforced concrete were obtained using the independent variables. The analysis of variance (ANOVA) for all properties indicates that the modified quadratic model was able to effectively predict the fresh and hardened properties of fibre reinforced concrete with coefficient of determination ranging between 0.86 and 0.98. In addition, RSM model predictive efficiency was classified as very good for compressive strength, splitting tensile strength, slump and water absorption and acceptable for FS in terms of Nash & Sutcliffe coefficient of model efficiency. An optimum condition of 140 for the aspect ratio, 0.26 for water cement ratio and 40% for cement content corresponding to 0.94%, 42.69 N/mm\(^2\), 7.97 N/mm\(^2\), 5.23 N/mm\(^2\), 7.65 cm for water intake/absorption, compressive strength, flexural strength, split tensile strength and slump respectively was achieved. These predictions were validated and a good correlation was observed between the experimental and predicted values judging by the absolute relative percent error of 0.842, 11.35, 3.6, 18.22 and 2.04 for water intake/absorption, compressive strength, flexural strength, split tensile strength and slump respectively. The proposed mathematical models are capable of predicting the required fresh and hardened properties of fibre-reinforced concrete as to inform early decision making when utilized in construction.

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1. Introduction

The ever increasing population growth, urbanization and rising standards of living resulting from technological innovations have contributed to the increasing quantity of solid wastes generated. The management of these disposed wastes becomes a major environmental problem in the long term. One of such solid wastes is tires. Owing to the rise in the number of vehicles being purchased, many tires end up as waste and if not properly disposed it results to environmental pollution in major towns and cities globally [1].

It is estimated that about 1000 million tires are being discarded at the end of their useful life annually across the globe, and there is need for proper handling in order to prevent severe ecological hazard. It is also anticipated that this number will increase by 20% by the year 2030, thereby increasing the estimated quantity to 5000 million (including stock piled). Therefore, the large volume of this waste has made it a material of research interest [2,1,5,2,3,50,51].

Their cheap availability, bulk and resilience have made the handling of waste tires problematic in a developing country such as Nigeria. Although there is no estimated record for the quantity of waste tires generated in the country is faced with the challenge of shortage landfill space to accommodate the huge volume of waste tires generated annually. These tires are stock piled at different locations in undeveloped lands and are in most cases set ablaze whenever such lands are to be utilized. Another common practice in the country is that these waste tires litter roadsides across the country and are often incinerated on the highways during unrest due to their proximity.

The unwholesome methods of disposal provide excellent breeding space for mosquitoes – the malaria parasite carrier, before being burnt and when burnt, they result in fire hazards as well as environmental pollution. Although there is feasibility in the use of tires as fuel, it is impaired by high initial cost. The large amount of carbon dioxide emitted in the process is also a major source of concerns for the environment. The pyrolysis process which produces carbon black is costly and substandard to the obtained products of petroleum [2–4]. One of the promising ways in which waste tires can be useful is in concrete. This can be done by incorporating the waste tire crumbs as aggregates in concrete and by using the extracted steel fibre component of waste tires as reinforcements in concrete mix. This attempt could be environmental friendly. This study in particular focuses on the use of such extracted steel fibres in concrete.

Balaguru and Shah [5] identified steel as one of the fibres useful in concrete. Other fibres include ceramics, glass, polymers, ceramics, asbestos, carbon. Altun et al. [6] indicated that steel fibre volume fraction of concrete containing should range between 1 and 2.5%, estimated by the absolute concrete volume. Bayramov et al. [7] and Yazici et al. [8] observed that unreinforced cementitious materials are usually brittle and of low tensile strength and that the presence of fibre reinforcement decreases the brittleness of concrete. They identified aspect ratio (length/diameter), volumetric fraction and fibre distribution as factors that influence the performance of steel fibre reinforced concrete (SFRC).

Holschemacher et al. [9] also identified aspect ratio and volumetric fraction of steel fibres as factors which significantly enhance the mechanical properties of concrete. The inclusion of steel fibres into concrete matrix has positive influence on the mechanical properties of concrete such as tensile strength, impact strength and toughness while the aspect ratio and volumetric fraction were identified as important factors for successful design of SFRC. In addition Suriaendy & Horiguchi [10] also observed that the mechanical properties of SFRC depend on fibre volume and length. The authors observed that the tensile strength of SFRC was improved as a result of the bridging action of steel fibres. Likewise, some investigations reported that concrete reinforced waste tire steel fibres exhibited improved mechanical performance similar to industrial steel fibres [11–12].

In addition, concrete as a porous material with discrete and interconnected pores of different sizes and shapes, requires the presence of very finely grounded material of about the same fineness as portland cement for proper pore size refinement and reduced permeability [13]. One of such very finely grounded material is limestone powder. Nehdi et al. [14] observed improved workability and stability of fresh concrete containing limestone powder as fillers. Although it has been identified that the incorporation of limestone powder reduces certain properties of concrete such as compressive strength, flexural strength and split tensile strength as the percentage of limestone powder increases in the concrete [15]. However, according to European Standard [16], the addition of five percent calcareous filler material like limestone powder to concrete mix is acceptable.

Therefore, this study explores the aspect ratio of steel fibres alongside cement content and water cement ratio as independent variables in predicting the compressive strength, flexural strength, split tensile strength, water absorption and slump of waste tire steel fibre reinforced concrete using response surface methodology (RSM). The RSM design identifies both linear interactions and quadratic contributions of the independent variables to the concrete properties. Furthermore, this study optimizes the combined effect of these factors to maximize or minimize desired outputs. The introduction of RSM in defining a suitable mix design for concrete containing steel can enhance the concrete performance in both fresh and hardened states. RSM has the predictive capability to determine properties such as slump, water absorption capacity, compressive strength, flexural strength and split tensile strength, thereby reducing the time and drudgery of repetitive laboratory experiments. The accurate and speedy determination of these properties reduces construction time especially when they are required for fast project execution. Such innovative application of RSM provides opportunity to make essential adjustments in mix proportions of concrete ingredients to achieve design objectives. In addition, such approach prevents circumstances in which either the targeted design strength is not met or concrete with excessive strength is produced. This approach invariably results in cost-effective utilization of raw-material, reduced construction failure and reduced construction cost [17].

RSM provides statistically validated predictive models that can be manipulated for finding optimal process configurations [18]. RSM typically is useful in situations where several factors influence one or more performance characteristics, or
responses. It can also be utilized to optimize one or more responses to meet a given set of specifications. More importantly, RSM, provides sufficient experimental interpretation of the non-linear responses surfaces of experimental results [19].

Response Surface Methodology (RSM) is an effective statistical tool for experimental design, model building, factors effects evaluation and optimum condition search [20–23]. In RSM, several factors which vary simultaneously are fitted to quadratic function [24]. RSM offers several advantages for optimization over the one factor at a time approach which is tedious and also does not take into account the interaction of factors [25]. RSM proportions the constituent material to obtain an optimum mix proportions used as a mathematical model for the prediction of the desired properties [26].

Even though RSM has been applied to numerous cement and concrete researches [27–30], its application to steel fibre reinforced concrete is very limited. Therefore, this study focuses on application of RSM in predicting and optimizing the compressive strength, flexural strength, split tensile strength, water absorption and slump of waste tire steel fibre reinforced concrete. The objectives of this study are (i) to evaluate the interactive effects of water-cement ratio, aspect ratio of waste tire steel fibres and cement content on the mechanical properties of waste-tire steel fibre reinforced concrete (ii) develop and assess predictive models for the mechanical properties (iii) Optimize the waste tire steel fibre reinforced concrete mixtures for construction applications [49].

2. Experimental programme, material and procedure

2.1. Experimental programme and model efficiency assessment

A total of twenty experimental runs were generated using the Central composite rotatable design (CCRD) of response surface methodology. CCRD can be used in predicting dependent variables also known as response by means of a small number of experimental data, with all parameters varied in preferred range. Each numerical factor is varied over five (5) levels. They are plus Alpha (+α), minus Alpha (-α), (axial/star points) +1(high level), -1(low level) (factorial points) and the center point (mid-level). Table 1 and Fig. 1 give a representation of the above explanation. The three parameters also known as independent variables considered in the design are aspect ratio, water cement ratio and cement. They are represented as A, B and C in coded terms (independent variables) while R1– R5 represent the water absorption/intake and compressive strength, flexural strength, split tensile strength and slump (dependent variables) respectively. The coding of variable was done using Eq. (1).

\[
\text{coded value} = \frac{r - c}{h - c}
\]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Code</th>
<th>Unit</th>
<th>Coded parameter levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aspect ratio</td>
<td>A</td>
<td>50</td>
<td>95 140</td>
</tr>
<tr>
<td>Water cement ratio</td>
<td>B</td>
<td>0.25</td>
<td>0.33 0.40</td>
</tr>
<tr>
<td>Cement</td>
<td>C</td>
<td>%</td>
<td>25 32.25 40</td>
</tr>
</tbody>
</table>

Fig. 1. A Pictorial representation of CCRD.
Where \( r \) is the runs number to be coded, \( c \) is the center point of the factor to be coded and \( h \) is the highest value of the factor being considered as given by the design of experiment.

The independent variables ranged between 50–140, 0.25–0.4 and 25%–40% for the aspect ratio of steel fibre extracted from waste tires (A), water cement ratio (B) and percentage of cement content (C) respectively.

The predictive efficiency of the RSM model in terms of output error/differences between observed (experimental) values and predicted values were measured using the following indices: mean prediction error (MPE), mean square error (MSE), root-mean-square error (RMSE), and Nash & Sutcliffe coefficient of efficiency (NSE) as given in Eqs. (2) to (6).

\[
MPE = \frac{\sum (Y_i - O_i)}{n}
\]  

\[
MSE = \frac{\sum (Y_i - O_i)^2}{n}
\]  

\[
RMSE = (MSE)^{1/2}
\]  

\[
NSE = 1 - \left( \frac{1}{\eta_t + 1} \right)^2
\]  

Where \( \eta_t = \frac{SD}{RMSE} - 1 \)

(Source: [31]; Sojobi et al. [32])

Where \( Y_i \) is the predicted value, \( O_i \) is the observed (experimental) value, \( n \) is the number of data set, \( n_t \) is the number of times the standard deviation is greater than RMSE [31]. In terms of NSE values, a model is classified as very good, good, acceptable and unsatisfactory if the NSE values are in the range of \( \geq 0.90, 0.8–0.9, 0.65–0.8 \) and \( < 0.65 \) respectively [31]. Model efficiency was also classified as very good, good, acceptable and unsatisfactory for \( SD \geq 3.2 \) RMSE, 2.2 RMSE < SD < 3.2 RMSE, 1.2 RMSE < SD < 2.2 RMSE and SD < 1.7 RMSE [31].

2.2. Experimental material

Steel fibres with specific gravity of five (5) and diameter 0.25 mm were used in the study. These fibres were extracted from waste tires by shredding. The indentation microscope was used to ensure a constant diameter was maintained. The fibres were cut into different lengths depending on the required aspect ratio as presented in Fig. 2. The aspect ratio is usually defined as ratio of the length of fibre to diameter \( l/d \). The tensile strength of the fibre was determined at a cross head speed of 20 mm/min. The test piece was properly gripped and mounted before testing. The average tensile strength was determined to be 700 N/mm\(^2\) with a strain at failure of 0.097. The volumetric fibre content was kept constant at 1% as recommended by Chan [33].

Ordinary Portland cement (OPC) of grade 42.5 was used in the study. The specific gravity, initial and final setting times in accordance to BS 12, [34] were determined and presented in Table 2. A pictorial view of the cement and limestone powder is presented in Fig. 3. Lime stone powder (LSP) with specific gravity of 2.48 was used as fillers. The percentage composition of LSP was kept constant at 5% of the weight of concrete. The chemical composition of the cement and limestone powder is presented in Table 3. River sand with specific gravity of 2.58 was used as fine aggregate. Granite passing through sieve size 20 mm was used as coarse aggregate. The particle size gradations of the aforementioned materials are presented in Fig. 4. The high range water reducing admixture (HRWRA) also known as superplasticizer conforming to ASTM C 494 [35] kindly provided by Advanced Chemical Technology, Ikeja was used to enhance the workability of the concrete, and lastly potable water was used for mixing.

Fig. 2. Different length of steel fibre.
2.3. Experimental procedure

The concrete mix design for all the twenty (20) experimental runs generated by RSM is presented in Table 4. The Table also presents details of all mix proportions in kilogram per cubic meter. Concrete mixing was done with the aid of a laboratory mixer at 60 revolutions/minute. Cement and limestone were first poured into the mixer and mixed for one (1) minute followed by addition of sand (fine aggregate) and granite (coarse aggregate). Subsequently, water mixed with HRWRA was poured into the mixer before the steel fibres were gradually added to the concrete mix through the wire mesh on the top of the mixer. This method was used to ensure uniform dispersion of the steel fibres in the concrete matrix. The mixing was
stopped after proper blending of all constituent materials was achieved. The duration of concrete mixing for each batch (run) was approximately 5–7 minutes.

Thereafter, the fresh concrete mix was poured into already lubricated moulds. The moulds used for the compression strength, split tensile strength and water absorption tests were 100 mm x 100 mm x 100 mm. The moulds used for the four-point bending flexural test was 100 mm x 100 mm x 400 mm. Each mould was filled in three layers, with each layer receiving 35 strokes of the tampering rod. This was done to ensure uniform compaction. The specimens were covered with damp sacks before demoulding after 24 h. All the specimens for compressive strength, flexural strength and split tensile strength tests were water-cured for 28 days after demoulding. Sample of specimens in slump, compressive, flexural and split tensile strengths testing respectively are presented in Fig. 5.

2.3.1. Slump test

The test was done in accordance to [36] to know the consistency of freshly mixed concrete. The consistency of a concrete mix is closely related to workability. A slump cone of bottom diameter 10 cm, top diameter 20 cm and height 30 cm was used. The concrete was filled in three layers with each layer receiving approximately twenty five strokes (25) of the tampering rod (the tampering rod is a steel of 16 mm diameter and 600 mm long). On filling the third layer excess concrete was removed using a hand trowel. The mould was immediately raised slowly in a vertical direction. The slump measured in centimeter (cm) is the difference between the height of the mould and the height point of the highest point of the specimen being tested.

2.3.2. Water Absorption/ intake

The test was done in accordance with ASTM C 642-06 [46]. Test specimens were immersed in water at temperature of 29 °C for twenty four (24) hours. Subsequently, samples were drained with towel for ten (10) minutes to remove excess

---

**Table 4**

Details of Mix proportion.

<table>
<thead>
<tr>
<th>Run</th>
<th>A</th>
<th>B</th>
<th>C (%)</th>
<th>Fibre Length (mm)</th>
<th>Cement (kg/m³)</th>
<th>Water (kg/m³)</th>
<th>Fibre (kg/m³)</th>
<th>SP (kg/m³)</th>
<th>LSP (kg/m³)</th>
<th>Sand (FA) (kg/m³)</th>
<th>Granite (CA) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95(0)</td>
<td>0.33(0)</td>
<td>32.5(0)</td>
<td>24</td>
<td>780</td>
<td>257.4</td>
<td>24</td>
<td>15.6</td>
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<td>546.75</td>
<td>668.25</td>
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<td>240.0</td>
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<td>473.58</td>
<td>578.82</td>
</tr>
<tr>
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<td>24</td>
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<td>578.82</td>
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<tr>
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<td>24</td>
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<td>546.75</td>
<td>668.25</td>
</tr>
<tr>
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<td>257.4</td>
<td>24</td>
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</tr>
<tr>
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<td>15.6</td>
<td>120</td>
<td>546.75</td>
<td>668.25</td>
</tr>
</tbody>
</table>

A = Aspect ratio; B = Water cement ratio; C = Cement; SP = Super plasticizer; LSP = limestone powder; FA = Fine Aggregate; CA = Coarse Aggregate.

---

![Fig. 5. Samples of test specimen.](image_url)
water. The water absorption/intake was determined using Eq. (7). Three samples were tested for each concrete mix to obtain a reliable average value.

\[
\frac{W_2 - W_1}{W_1} \times 100
\]

\( W_1 = \) Weight of sample before immersion in water
\( W_2 = \) Weight of sample after immersion in water

2.3.3. Compressive strength

The concrete specimens were prepared according to [47] and tested at twentyeight (28) days. Compressive strength values obtained were determined by taking the average of three specimens. Testing was done using compression testing machine. The bearing surface of the supporting and loading rollers was wiped clean before placing specimen. Positioning of specimen was done with load applied on the uppermost surface of specimen as cast in the mould. Care was taken to ensure that specimen aligned with the loading device. Loading was done gradually and continuously increased until the dial stopped moving and the maximum load was recorded. The compressive strength of the specimen was expressed as the maximum crushing load in Newton (N) divided by the effective surface area of the tested specimen in millimeter (mm).

2.3.4. Flexural strength test

The beams at 28 days were tested under four point loading until failure using flexural testing machine in line with [48]. A constant loading configuration with shear span of 300 mm and shear span depth ratio of 3.0 was applied. Specimens were placed on the supporting bearing blocks with side in respect to its position when moulded. The upper surface of the test specimen was brought in contact with the load-applying block at one quarter distance from the end of supports. Following this action, the load applying block is brought in full contact with the beam surface. The beam was checked to ensure it had uniform contact with both the bearing and the load applying blocks. Loading was done continuously until the specimen failed and the dial stopped moving. The maximum applied load indicated by the testing machine was recorded. The flexural strength is calculated using Eq. (8).

\[
R = \frac{3FL}{4bd^2}
\]

\( R = \) Flexural strength (N/mm²)
\( F = \) Applied load at failure
\( l = \) beam span measured in millimeter
\( b = \) beam breadth measured in millimeter
\( d = \) beam depth measured in millimeter

2.3.5. Split tensile strength test

The samples at twentyeight (28) days were tested using compression testing machine according to BS EN 12390-6 [37]. Samples were placed between the loading surfaces of a compression testing machine. The compressive line loads applied along a vertical symmetrical plane. This loading arrangement sets up normal tensile stress along the loading axis of two equal and opposed loads. The splitting of the specimen at maximum load was recorded and Eq. (9) was used to determine the split tensile strength

\[
F_t = \frac{2P}{(\pi DL)}
\]

\( F_t = \) Tensile Strength (N/mm²)
\( P = \) Load at failure (N)
\( D = \) Diameter of cylinder or side of the cube (mm)
\( L = \) Length of the cylinder/ cube (mm)

3. Result and discussion

3.1. Perturbation plots and predictive efficiency of the derived models

Perturbation plots in RSM design revealed significant parameters by displaying changes in response of each factor as each factor moves from the reference point, which is the zero coded level of each factor, with all other factors held constant at the reference value.

Perturbation plot for water intake presented in Fig. 6a revealed that all the three factors have significant influence on the water intake. However water intake reduces with increased water-cement ratio (B), while it increases with increased cement content (C). For fibres, water intake was found to be lower with long fibres signifying high aspect ratio when compared to
short fibres signifying low aspect ratio. This corroborates earlier research finding that the presence of fibres prevent ingress of external substances [10].

Perturbation plot for compressive strength (CS) in Fig. 6b revealed that only water cement ratio has significant influence on CS as higher CS was obtained in the region close to the reference point. This corroborates earlier research reports that fibres do not contribute significantly to the static compressive strength of concrete and even concrete structures with conventional reinforcements [33].

Fig. 6. (a) Perturbation plot for water intake (b) Perturbation plot for compressive strength (c) Perturbation plot for flexural strength (d) Perturbation plot for split tensile strength (e) Perturbation plot for slump.
Perturbation plot of flexural strength (FS) in Fig. 6c revealed that water-cement ratio had the most significant influence on flexural strength. The influence of factors such as aspect ratio and cement were less significant when compared to water cement ratio. It also revealed that flexural strength will slightly reduce with increasing aspect ratio. This implies that short fibres with low aspect ratio have positive contribution on flexural strength.

Perturbation plot in Fig. 6d revealed that the highest split tensile strength (STS) can be achieved close to the reference point (middle region) of both aspect ratio (A) and w/c ratio (B) while the influence of cement on STS was observed to be negligible.

Perturbation plot for Fig. 6e revealed that all the factors have effect on slump. Slump was observed to reduce with increased aspect ratio, increased with increasing w/c ratio and cement. It can be inferred that short fibres with low aspect ratio provide high acceptable slump while long fibres contributes to slump reduction. This will enhance proper blending and mixing with the aggregate in freshly prepared concrete mix and also translates to improved workability. This implies short fibres are applicable where high slump is required while long fibres are more effective where slump reduction is required.

3.2. Predictive efficiency of the derived models

In terms of predictive efficiency (Table 5), the RSM model can be classified as ‘very good’ for Compressive strength (CS), Split tensile strength (STS), Slump and water absorption/ Intake (WA) while it was classified as ‘acceptable’ for flexural strength (FS). The NSE values of 0.97, 0.96, 1.0 and 0.95 for CS, STS, slump and WA fell within > 0.9 NSE specification required for very good model while NSE value of 0.70 for FS was within NSE specification for acceptable model.

Comparative evaluation of the model results with respect to SD and RMSE revealed that RSM model was very good for CS, STS, slump and WA but acceptable for FS. This is because the SD values of 7.42, 1.30, 9.35 and 0.37 were found greater than 3.2 times their respective RMSEs. On the other hand, the SD for FS fell within the range of 1.2–2.2 RMSE required for ‘acceptable’ model classification [31].

3.3. ANOVA and regression models equations for investigated properties

The analysis of variance (ANOVA) for all properties investigated is presented in Tables 6A–6E. From the table, it was observed that the following linear terms were statistically insignificant according to the Student’s t-test (p-value < 0.05). The terms A and B for water absorption, terms A and C for flexural strength and terms A, B and C for both compressive and split tensile strengths. However, all linear terms were significant for slump judging by the p-values obtained. The interaction and quadratic effect were also evaluated using same parameter.

Table 5

SD, MPE, MSE, RMSE and NSE for CS, FS, STS, Slump and water absorption.

<table>
<thead>
<tr>
<th>Test</th>
<th>SD</th>
<th>MPE</th>
<th>MSE</th>
<th>RMSE</th>
<th>NSE</th>
<th>Model classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>7.42</td>
<td>−0.007</td>
<td>2.49</td>
<td>1.58</td>
<td>0.97</td>
<td>Very good</td>
</tr>
<tr>
<td>FS</td>
<td>1.16</td>
<td>−0.1317</td>
<td>0.41</td>
<td>0.64</td>
<td>0.70</td>
<td>Acceptable</td>
</tr>
<tr>
<td>STS</td>
<td>1.30</td>
<td>−0.0015</td>
<td>0.10</td>
<td>0.31</td>
<td>0.96</td>
<td>Very good</td>
</tr>
<tr>
<td>Slump</td>
<td>9.35</td>
<td>−0.18</td>
<td>0.0764</td>
<td>0.28</td>
<td>1.0</td>
<td>Very good</td>
</tr>
<tr>
<td>WA</td>
<td>0.37</td>
<td>0.0006</td>
<td>0.01</td>
<td>0.11</td>
<td>0.95</td>
<td>Very good</td>
</tr>
</tbody>
</table>

Table 6A

ANOVA for Water intake/absorption.

<table>
<thead>
<tr>
<th>SoD</th>
<th>SoS</th>
<th>DoF</th>
<th>MS</th>
<th>F-value</th>
<th>P-value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>1.43</td>
<td>10</td>
<td>0.14</td>
<td>5.08</td>
<td>0.0152</td>
<td>SD = 0.17</td>
</tr>
<tr>
<td>A</td>
<td>2.103E-003</td>
<td>1</td>
<td>2.103E-003</td>
<td>0.075</td>
<td>0.7915</td>
<td>Mean = 0.99</td>
</tr>
<tr>
<td>B</td>
<td>0.077</td>
<td>1</td>
<td>0.077</td>
<td>2.73</td>
<td>0.1370</td>
<td>R² = 0.8640</td>
</tr>
<tr>
<td>C</td>
<td>0.34</td>
<td>1</td>
<td>0.34</td>
<td>12.18</td>
<td>0.0082</td>
<td>Adj. R² = 0.70</td>
</tr>
<tr>
<td>AB</td>
<td>0.084</td>
<td>1</td>
<td>0.084</td>
<td>2.97</td>
<td>0.1230</td>
<td>AP = 10.901</td>
</tr>
<tr>
<td>AC</td>
<td>0.053</td>
<td>1</td>
<td>0.053</td>
<td>1.88</td>
<td>0.2075</td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>1.125E-003</td>
<td>1</td>
<td>1.125E-003</td>
<td>0.040</td>
<td>0.8465</td>
<td></td>
</tr>
<tr>
<td>A²</td>
<td>0.16</td>
<td>1</td>
<td>0.16</td>
<td>5.81</td>
<td>0.0425</td>
<td></td>
</tr>
<tr>
<td>C²</td>
<td>0.25</td>
<td>1</td>
<td>0.25</td>
<td>9.05</td>
<td>0.0169</td>
<td></td>
</tr>
<tr>
<td>ABC</td>
<td>0.22</td>
<td>1</td>
<td>0.22</td>
<td>7.86</td>
<td>0.0231</td>
<td></td>
</tr>
<tr>
<td>A² B</td>
<td>0.28</td>
<td>1</td>
<td>0.28</td>
<td>10.03</td>
<td>0.0133</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>0.23</td>
<td>8</td>
<td>0.028</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lack of Fit</td>
<td>0.19</td>
<td>4</td>
<td>0.049</td>
<td>6.36</td>
<td>0.0503</td>
<td></td>
</tr>
<tr>
<td>Pure Error</td>
<td>0.031</td>
<td>4</td>
<td>7.640E-003</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conversely, the interaction effects AB and AC for flexural strength and AC for split tensile strength were statistically significant while for the other properties all interaction effects were insignificant. The significant quadratic terms are $A^2$ and $C^2$ for water absorption, terms $A^2$ and $B^2$ for split tensile strength and term $B^2$ for slump, and compressive and flexural strengths while all other quadratic terms were insignificant. In order to improve the performance of the models, some cubic terms were introduced. The introduction of both significant and insignificant cubic terms for all properties investigated
improved the p-values of the models and the coefficient of determination (R²). The aim was to ensure that all R² values obtained were greater than 0.8 since the closer this value is to one/unify the better the predictive efficiency of the models.

The significance of the models obtained from ANOVA (Tables 6A–6E was discussed in descending order; the model for slump was most significant at 95% confidence level, F-value of 33.17 and a p-value < 0.0001. Although the lack of fit (LOF) was significant, it did not invalidate the model for predictive purpose because R² was approximately 0.98. This high value for R² shows that the regressors in the model did not explain only about 5% of the total variability. The flexural strength is next, at 95% confidence level, F-value of 19.91, p-value of 0.0008 and an insignificant LOF of 0.0527 were obtained. This implies that the specified model fits the data satisfactorily [38].

With respect to compressive strength at same confidence level, F-value was 7.32, p-value was 0.0193 and a significant LOF was obtained. It is worth noting that while LOF was significant, R² was approximately 0.95 which implies that approximately 95% of total variation of outcomes was explained by the model. As regards the split tensile strength, and at the same confidence level of 95%, R² of 0.93, F-value of 8.15 and a p-value 0.0052 were attained. With respect to water intake, at the same confidence level of 95%, R² of 0.86, F-value of 5.08, p-value of 0.0152 and an insignificant LOF of 0.0650 was obtained, signifying good fit of the derived model.

Adequate precision (AP) was also used in evaluating the performance of the model. It compares the range of values predicted at design point with the average prediction error. In this study, the AP values of the models were 10.90, 12.11, 19.61, 10.8 and 18.38 for water intake/absorption, compressive strength, flexural strength, split tensile strength and slump respectively. All values of AP obtained were greater than 4 which indicate that the model can be used to navigate the space defined by the CCRD [39]. By applying multiple regression analysis on the experimental data, the following regression equations namely (Eqs. 10a–10e) were derived for all responses.

\[
\text{Water Absorption/intake} = +0.16 + 0.062A - 0.14B - 0.14C + 0.098AB + 0.1AC + 0.17BC - 0.036A^2 \\
+ 0.047B^2 - 0.082C^2 + 0.14ABC + 0.25A^2B + 0.25A^2C
\] (10a)

\[
\text{Compressive strength} = +32.99 - 1.15A - 4.46B - 6.24C - 1.24AC - 5.04BC + 0.19B^2 - 5.11C^2 + 0.67B^2C \\
- 1.93BC^2
\] (10b)

\[
\text{Flexural strength} = +6.82 + 0.24A + 1.23B + 1.17C - 0.19AB - 0.38AC - 0.80BC - 1.01B^2 - 0.37C^2 \\
- 0.69B^2C - 1.04BC^2
\] (10c)

\[
\text{Split tensile} = 4.65 + 0.095A + 1.22B + 1.07C + 0.21AB + 0.016AC + 0.16BC - 0.25A^2 - 0.56B^2 - 0.42C^2 \\
+ 0.75ABC - 1.54A^2B - 0.71A^2C
\] (10d)

\[
\text{Slump} = +10.95 - 0.54A + 8.62B + 7.73C - 1.62AB + 5.88AC + 2.63BC - 4.04A^2 + 2.27B^2 + 1.74C^2 \\
- 4.88ABC - 6.25A^2B - 6.85A^2C
\] (10e)
3.4. Normal probability plot

Data were analyzed to check the normality of residuals as well as actual versus predicted plots for all properties investigated. Fig. 7a–e display the normal probability plot of both residuals and the actual versus predicted for all responses. From these figures it was observed that for water intake, all the plotted points fell very close to the distribution fitted line while for flexural strength, split tensile strength and slump, majority of the plotted points fell very close to the distribution fitted line. However for compressive strength, the plotted points fell at a far distance from the distribution fitted line [40]. In all, it was observed that the normal distribution plots generally was a better choice for analyzing interested responses (properties) compared to predicted vs actual plots disparity between prediction values and actual experimental values.

3.5. Contour and 3D plots for all responses

Assessment of the interactive relationship between the mix design parameters and the properties of steel fibre reinforced concrete was done and displayed using contours and 3D plots of RSM. These plots for all investigated responses were presented in Fig. 8a–e. Fig. 8a–e display the contour and 3D plots of dependent variables drawn as function of two independent variables while the third independent variable was held constant. From the contour and 3D plots of water intake shown in Fig. 8a, it was observed that there is no clear interaction between the factor A and B with C held constant. Reduced water intake was observed with reduced B while A had reduced water intake for both low and high values of A.

The interaction between factors A and C shows elliptical contours and this is the pattern obtained when there are perfect interactions between factors (independent variables) [41]. The observations from the contour and 3D plots imply that the water intake will reduce with increase in factor A and reduction in factor C. For the interaction between C and B, reduction in factor C and increase in factor B will also reduce the water intake. From the contour and 3D plots of compressive strength presented in Fig. 8b, a clear relationship was observed for the interaction between A and B as increase in A and reduction in B will increase the compressive strength. For the interaction between A and C, increase in these factors increased the compressive strength. For the interaction between B and C, it was observed that the compressive strength increased with increase in C and at reduced value of B. This implies that an increase in factor C and a reduction in factor B will increase the compressive strength.

The contour plots for Flexural strength presented in Fig. 8c show distorted contours which is the pattern obtained when interactions between independent variables are few [41]. From the contour plots of the interaction between the factors A and B with C held constant, no clear trend was observed, however from the 3D plot it was observed that an increase in flexural strength can be achieved with increased A and reduced B. From the interaction between A and C it could be observed for both plots that changes in A and C have little effect on the Flexural strength. From the contour 3D plots for the interaction between B and C, increase in both factors will enhance the Flexural strength.

From the contour and 3D plots presented in Fig. 8d, it was observed that an increase in A and reduction in B will increase the split tensile strength. For the interaction between A and C, it was observed that increase in C and reasonable increase in A will enhance split tensile strength. From the interaction between B and C it was observed that at B of 0.33 with a reasonable increase in C, the split tensile strength can be enhanced. From the contour 3D plots for flexural and split tensile strengths it could be deduced that both the flexural and split tensile strengths do not really require as much cement for improved strength when compared to compressive strength. Reduced slump was observed for all interactions at reduced factors B and C, and increased factor A as presented in Fig. 8e.

Overall, it was observed that long fibre length obtained with higher aspect ratio will reduce the water absorption, slump and compressive strength properties of steel fibre reinforced concrete which implies that longer fibre length has a positive filling effect which increases the compactness of the concrete and hence reduces porosity. For concrete containing fibres, workability measured in terms of slump was not really affected by the presence of fibres which may be ascribed to the fact that the fibres generally had low surface area and are impermeable to water absorption. With respect to other responses, shorter fibre length obtained from lower aspect ratios will increase the flexural and split tensile strengths at reduced water/cement ratio.

3.6. Optimum conditions for the mix design components

The optimization process considered all responses simultaneously in order to achieve a concrete mix design that will be favorable for all investigated responses. According to Oehlert [42], when there is more than one response, then it is important to find the compromise optimum that does not optimize only one response. The process was carried out to determine the optimum values for aspect ratio (A), water-cement ratio (B) and cement content (C) required to achieve desirable values for the dependent parameters coded R1–R5. Parameters R1, R2, R, R4 and R5 represent water intake, compressive strength (CS), flexural strength (FS), split-tensile strength (STS) and slump respectively.

The ‘maximum’ condition (goal) was selected for CS, FS and STS in order to achieve highest strength possible while ‘minimum’ condition was selected for water absorption to ensure low penetration capacity of external substances. However, ‘in range’ condition was selected to achieve desirable slump. The optimization process gave several solutions but the solution with the highest CS is presented in Table 7.
Fig. 7. (a) Normal probability distribution plot for water absorption/intake (b) Normal probability distribution plot for compressive strength (c) Normal probability distribution plot for flexural strength (d) Normal probability distribution plot for split tensile strength (e) Normal probability distribution plot for slump.
Furthermore, an additional experiment was carried out to validate the optimum design proportions obtained by the RSM model and the result is also presented in Table 7. The standard deviation (SD) and absolute relative percent error (PE) of the RSM model with respect to the experimental results were observed to be low. Therefore, the model predicted the desired responses with good accuracy. The estimation of PE was done using Eq. (11).

\[
\text{Absolute relative percent error (PE)} = \left(1 - \frac{\text{Predicted value}}{\text{Experimental value}}\right) \times 100
\]  

(Source: [43])

From the optimization process, it could be inferred that fibres with higher aspect ratio will withstand more split, flexural and compressive loadings with acceptable slump and water absorption capacity than lower aspect ratio. This agrees with Chanh [33] who observed that higher aspect ratio enhances the performance of hardened concrete. He also pointed out the adverse effect of higher aspect ratio on the workability of fresh mix concrete. However from this study, acceptable workability measured in terms of slump has been achieved with higher aspect ratio.

It was also observed that the aforementioned properties are inversely proportional to water cement ratio. Cihan et al. [44] also observed that at certain water cement ratio during optimization, increase in workability does not significantly reduce cohesion in the concrete due to the addition of superplasticizer. According to Neville [45], the presence of superplasticizers
could reduce the water content for a given workability by 25–35%. In addition, the presence of limestone powder used as filler in this study was observed to absorb low quantity of water during concrete mixing. The advantage of having both materials (superplasticizer and LSP) included in the concrete mix for this study at water cement ratio of 0.26 will provide a workable concrete with improved fresh and hardened properties.

**Fig. 8.** (a) Contour and 3D plots for water absorption/intake (b) Contour and 3D plots for compressive strength (c) Contour and 3D plots for Flexural strength (d) Contour and 3D plots for Split tensile strength (e) Contour and 3D plots for Slump.
b: Contour and 3D plots for compressive strength

Fig. 8. (Continued)
c: Contour and 3D plots for Flexural strength

Fig. 8. (Continued)
d: Contour and 3D plots for Split tensile strength

Fig. 8. (Continued)
Fig. 8. (Continued)

**e: Contour and 3D plots for Slump**

*Fig. 8. (Continued)*
4. Conclusion

The following conclusions were derived from the study:

i) The statistical error analysis used for evaluating the RSM model shows the accuracy of the obtained models and as well the significant contributions made by the independent variables.

ii) RSM model predictive efficiency was classified as very good for CS, STS, slump and WA and acceptable for FS in terms of Nash & Sutcliffe coefficient of model efficiency.

iii) The practical approach described by the study for prediction provides a dependable tool for evaluating both the design strength and performance of fibre reinforced concrete.

iv) Utilization of steel fibres from waste tires in concrete will encourage and help curb the menace arising from poor disposal and low recycling of waste tires in developing countries.

v) Waste tire fibres can be safely utilized to improve the fresh and hardened properties of concrete especially at optimum conditions.

vi) RSM models can be used to show the interactions of the investigated factors such as aspect ratio, water-cement ratio and cement content.

vii) RSM is useful in optimization of concrete mixtures to achieve desired experimental and project objectives. Owing to its high predictive efficiency, RSM is useful in predicting the fresh and hardened properties of steel fibre reinforced concrete, thereby eliminating the drudgery of repetitive laboratory tests and facilitates prompt decision making for construction applications.

viii) Appropriate collection and recycling schemes with incentives should be put in place in developed and developing countries to discourage poor disposal and burning of waste tires in landfills and unapproved open dumps. (vii) Owing to its high predictive efficiency, RSM is useful in predicting the fresh and hardened properties of steel fibre reinforced concrete, thereby eliminating the drudgery of repetitive laboratory tests and facilitates prompt decision making for construction applications.

Conflict of interest

The authors declare no conflict of interest.

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