Non-Coherent Capacity of M-ary DCSK Modulation System over Multipath Rayleigh Fading Channels

Hu, Wei; Wang, Lin; Cai, Guofa; CHEN, Guanrong

Published in: IEEE Access

Published: 01/03/2017

Document Version: Final Published version, also known as Publisher's PDF, Publisher's Final version or Version of Record

License: Unspecified

Publication record in CityU Scholars: Go to record

Published version (DOI): 10.1109/ACCESS.2016.2623798


Citing this paper
Please note that where the full-text provided on CityU Scholars is the Post-print version (also known as Accepted Author Manuscript, Peer-reviewed or Author Final version), it may differ from the Final Published version. When citing, ensure that you check and use the publisher's definitive version for pagination and other details.

General rights
Copyright for the publications made accessible via the CityU Scholars portal is retained by the author(s) and/or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights. Users may not further distribute the material or use it for any profit-making activity or commercial gain.

Publisher permission
Permission for previously published items are in accordance with publisher's copyright policies sourced from the SHERPA RoMEO database. Links to full text versions (either Published or Post-print) are only available if corresponding publishers allow open access.

Take down policy
Contact lbscholars@cityu.edu.hk if you believe that this document breaches copyright and provide us with details. We will remove access to the work immediately and investigate your claim.
Non-Coherent Capacity of $M$-ary DCSK Modulation System Over Multipath Rayleigh Fading Channels

WEI HU$^1$, LIN WANG$^1$, (Senior Member, IEEE), GUOFA CAI$^2$, AND GUANRONG CHEN$^3$, (Fellow, IEEE)

$^1$Department of Communication Engineering, Xiamen University, 361005 Fujian, China
$^2$Department of Information Engineering, Guangdong University of Technology, 510006 Guangzhou, China
$^3$Department of Electronic Engineering, City University of Hong Kong, Hong Kong

Corresponding author: L. Wang (wanglin@xmu.edu.cn)

This work was supported in part by the NSF of China under Grant 61271241 and in part by the Hong Kong Research Grants Council under GRF Grants CityU 11208515.

ABSTRACT Non-coherent capacity bounds for the $M$-ary differential chaos shift keying (DCSK) modulation system are derived and analyzed over multipath Rayleigh fading channels, deriving conditions on the channel state information/non-channel state information (CSI/NonCSI) and soft-decision/hard-decision (SD/HD), respectively. Meanwhile, the inter-symbol interference is modeled and analyzed mathematically. Through numerical simulations and analyses, it is found that: 1) the influences of the spreading factor $\beta$, multipath number $L$, modulation dimension $M$, CSI/NonCSI, and SD/HD on the non-coherent capacity bounds are significant; 2) there is a relatively broad range of code rates corresponding to the optimal system power in U-shaped capacity curves of the non-coherent reception; and 3) the U-shaped non-coherent capacity bounds are proven to exist by investigating the mechanism of the low density parity check coded the $M$-ary DCSK system. These results are useful as benchmarks for designing power-efficient coded $M$-ary DCSK systems.

INDEX TERMS Non-coherent capacity, soft/hard-decision (SD/HD), channel state information/non-channel state information (CSI/NonCSI), inter-symbol interference (ISI).

I. INTRODUCTION

Differential chaos shift keying (DCSK) modulation based ultra-wideband (UWB) technology is in research frontier of wireless personal area networks (WPANs) [1]–[4] because of its unique characteristics of wide-spectral carrier chaotic signals, low power, low correlation and anti-interference ability. This modulation technique offers an outstanding bit error rate (BER) performance on the multipath fading setting [5]–[7], which is attributed to its differential structure and results from overcoming the difficult synchronization problem without requiring channel state information (CSI) by the receiver [8].

Noticeably, it is a drawback that the DCSK system needs the radio frequency (RF) delay lines in the transceiver. Code-shifted DCSK (CS-DCSK) was proposed in [9] to improve the energy efficiency by spreading two chaotic slots with Walsh codes. Multi-carrier DCSK (MC-DCSK) system proposed in [10] can solve the delay line requirement problem and also enhance the spectrum efficiency and energy efficiency. In [11], a phase-separated DCSK (PS-DCSK) scheme was proposed by using the orthogonality of sine signal and cosine signal. Another drawback is that half bit duration is set as reference, resulting in the decreasing of data rate and energy efficiency. A differential DCSK (DDCSK) scheme in [12] and a reference-modulated DCSK (RM-DCSK) scheme in [13] were proposed to achieve a high data rate. The orthogonal multi-level DCSK (OML-DCSK) system was proposed in [14] to achieve a higher data rate and better BER performance. To achieve low power, a double-level multi-resolution DCSK system was proposed in [15] based on MC-DCSK and quadrature chaos shift keying (QCSK) [16]. In [17], $M$-ary DCSK, using chaotic basis functions and dividing the symbol period into four time slots, was designed to improve the data rate. Another scheme of $M$-ary DCSK system in [18] was presented to replace chaotic codes with Walsh codes. The BER derivation of the frequency
modulated (FM) version of the corresponding communication systems was studied in [18] and [19] over additive white Gaussian noise (AWGN) channels and it revealed that the $M$-ary DCSK modulation scheme suits to power-limited communication systems.

![Block diagrams of coding channel and modulation channel.](image)

FIGURE 1. Block diagrams of coding channel and modulation channel.

It is now well known that the reliability of communication channels can be enhanced by some forward error correcting (FEC) codes for low power design [20]. The coding channel is described in Fig. 1, constituting of the modulator, the communication channel and the demodulator, which also shows the influence of the modulator-demodulator on the capacity bound. The product accumulate (PA) codes were combined with the single input multiple output (SIMO) FM-DCSK systems to obtain an outstanding BER performance in [21]. In [22] and [23], the low density parity check (LDPC) codes were incorporated to improve the overall performance of the DCSK system. Although these schemes are proposed to enhance the performances, they cannot be used to analyze and calculate the theoretical capacity limits directly for coding design.

Several works have been devoted to the study of the capacity, including systems with multipath channels. In [23], the achievable rate (output extrinsic mutual information) is simulated by numeric statistics on the log likelihood ratios (LLRs) for the $M$-ary DCSK communication system. Although it is not accurate, the nonlinear capacity exists, which is different from the conventional modulation systems. In [24], the capacity is analyzed and calculated in term of non-CSI demodulation over single-path Rayleigh fading channel for binary DCSK system, but ignoring the influence of inter-symbol interference, the soft/hard-decision, and the channel state information, the dimension $M$ and the fading path number $L$. The capacity and mutual information of a broadband fading channel consisting of a finite number of time-varying paths were investigated in [25]. In [26], the capacities of Gaussian and Webb channels were computed for modeling an optical channel using an avalanche photodiode detector (APD) and pulse-position modulation (PPM). In [27], the capacity of coherent reception of the $M$-ary DCSK system over AWGN channels was derived, which is independent of the spreading factor and different from the model of non-coherent reception to be studied in this paper.

But the above designs ignore the influences of multipath effect on the capacity bound. In [28], the capacity of a discrete-time Gaussian channel with inter-symbol interference (ISI) was derived, where the channel was mapped to an $N$-circular Gaussian channel. The ISI channel and the capacity limit were normalized in [29]. In [30] and [31], approximations of the capacity limit for ISI channels were proposed, introducing a Markovian achievable scheme. And a simulation-based technique was proposed in [32] and [33] to compute the information rates of soft-input soft-output (SISO) ISI channels with inputs selected from a finite alphabet.

In this paper, the non-coherent theoretical capacity bounds for the $M$-ary DCSK system are calculated and analyzed over multipath Rayleigh fading channels, taking into account the influences of soft/hard-decision and channel/non-channel state information. The aim is to achieve an energy-efficient design for coded chaotic modulation systems.

The main contributions of this paper are summarized as follows:

1) The general non-coherent capacity expressions of the $M$-ary DCSK modulations without inter-symbol interference over multipath Rayleigh fading channels are derived and analyzed accurately, and conditions on the SD/HD and CSI/NonCSI are established, respectively.

2) The capacity expressions of the $M$-ary DCSK modulations with inter-symbol interference are derived under various scenarios by considering the parameters of modulation dimension $M$, path number $L$, SD/HD and CSI/NonCSI, where the interference is modeled mathematically. These results can guide the designers to improve the performance of $M$-ary DCSK communication systems.

3) It is proven that U-shaped capacity bounds exist in the LDPC coding mechanism of the $M$-ary DCSK system at different code rates: $R_c = \frac{1}{4}, \frac{1}{2}, \frac{3}{2}$. Meanwhile, the capacity bounds of infinite fading path ($L \rightarrow \infty$, AWGN channels) and single-path ($L = 1$) are simulated and analyzed.

The organization of this paper is as follows. The influences of SD/HD, including CSI/NonCSI and ISI/NonISI interference, on capacity bounds are analyzed in Section II. In Section III, the non-coherent capacity bounds in various cases are derived and analyzed, along with some numerical simulations and discusses, in Section IV, where the U-shaped capacity bounds are proved to exist. Finally, Section V provides some concluding remarks.

II. SYSTEM MODEL AND NOTATIONS

The $M$-ary DCSK system model is shown in Fig. 2, which includes the modulator and non-coherent demodulator [23].

A. $M$-ary DCSK SYSTEM

Transmitter: the transmitter of the $M$-ary DCSK system is shown in Fig. 2(a). The delayed copies of the chaotic signal are multiplied by the elements of a Walsh function. These products are used to construct the transmitted signal. The Walsh functions are defined by the following recursive relation: $W_{2n} = [W_{2n-1} \cdot W_{2n-1}]$ and $W_2 = [1]$, for $M = 2^n$, $n \geq 1$, and the transmitted signal is defined as a constant energy. Assuming that symbol $m$ is transmitted, the transmitted signal...
(b) Non-coherent demodulator. 

**FIGURE 2.** Model of the M-ary DCSK modulation system. (a) M-ary DCSK system modulator. (b) Non-coherent demodulator.

for the M-ary DCSK modulation system is defined as follows:

\[ S_m(t) = \sum_{k=1}^{M} \sum_{j=1}^{\beta} w_{m,k} x_{m,j} \text{rect}(t - k \beta T_c - jT_s). \]  

(1)

where \( \text{rect}(\cdot) \) is named by the rectangular wave function; \( w_m = [w_{m,1}; w_{m,2}; \ldots; w_{m,M}] \) and \( w_j = [w_{j,1}; w_{j,2}; \ldots; w_{j,J}] \) denote two arbitrarily chosen Walsh functions, which satisfy \( \frac{1}{M} \sum_{k=1}^{M} w_{m,k} w_{j,k} = 1 \) when \( m = l \) or 0 when \( m \neq l \); \( \beta \) is defined as the spreading factor; \( x(t) \) denotes the chaotic signal, which satisfies \( \int_{0}^{T_c} x(t)^2 dt = \frac{1}{M} \); \( T_c \) and \( \beta T_s \) present the chip duration and the bit duration of a basic chaotic signal, respectively.

**Non-Coherent Reception:** the non-coherent demodulator of the M-ary DCSK system is shown in Fig. 2(b), which has \( M \) Walsh functions and \( M - 1 \) delay elements. In this demodulator, the received signal is likewise required to multiply with all the \( M \) Walsh functions. Differing from the coherent reception, the received signal is first fed into a delay line with delay \( T_c \) for non-coherent system. Then, the output signal of every delay element is multiplied with the corresponding element in each Walsh function. The products corresponding to one Walsh function are then summarized. The sum values are used to compute the energies \( y_1, \ldots, y_M \), the received signal \( y_m(t) \) is given by

\[ y_m(t) = \sum_{l=1}^{L} \alpha_{l,m} S_m(t - \tau_l) + n(t) \]  

(2)

where \( L \) is the path number, \( \alpha_{l,m} \) and \( \tau_l \) are the channel coefficient and the time delay of \( l \)-th path respectively.

Thus, for the non-coherent M-ary DCSK system, the observation value of the \( m \)-th auto-correlator is obtained as

\[ z_m = \sum_{j=1}^{\beta} \left( \sum_{l=1}^{L} \alpha_{l,m} w_{l,m,k} x_{m,(j-\tau_l)} \right) w_{m,k} + \sum_{k=1}^{M} w_{m,k} n_{k,j} \]  

(3)

where \( \alpha_{l,m} \) presents the \( l \)-th path channel coefficient when transmitting the \( m \)-th symbol; and \( n_{k,j} \) are Gaussian noise variables with zero mean and variance \( \sigma^2 \).

**B. CAPACITY ANALYSIS**

In Fig. 1, the input \( X \) is modulated through a modulator and is then transmitted over the modulation channel. At the output side of the channel, the received signals are processed by the demodulator, thus a scalar or vector \( Y \) is obtained. The observation of the channel output \( Y \) provides an average bits of information about the input \( X, I(X;Y) \), which is the mutual information between \( Y \) and \( X \). For M-ary DCSK system, the \( Y \) is the observation of the auto-correlator for non-coherent demodulation. The capacity of the modulator over the modulation channel is the maximum amount of information that can be transmitted reliably, given by

\[ C = \max_{p(X)} I(X;Y), \]  

(4)

where the maximum mutual information is taken over the input probability distribution \( p(X) \), providing the capacity for the communication system.

For soft-decision, the capacity of the channel with input signals \( X = \{x_1, x_2, \ldots, x_M\} \) restricted to an M-ary signal constellation without restriction on the demodulator, i.e., the analog output \( Y = (-\infty, +\infty) \), is given by

\[ C = \max_{p(X)} \int_{-\infty}^{+\infty} p(y|x_i) p(x_i) \log_2 \frac{p(y|x_i)}{p(y)} dy, \]  

(5)

where \( x = (x_1, x_2, \ldots, x_M) \) is the transmitted signal vector, \( y = (y_1, y_2, \ldots, y_M) \) is the received signal vector, \( p(y|x_i) \) is the probability density function (PDF) of \( y \) conditioned on \( x_j \), and \( p(y) = \sum_{j=1}^{M} p(x_j)p(y|x_j) \) is the marginal PDF.

Thanks to the symmetry of the considered channel, the capacity is achieved with an equi-probable M-ary data distribution, \( p(x = x_i) = \frac{1}{M} \), thus the soft-decision (SD) capacity
can be expressed as

\[
C_{SD} = \log_2 M - E_{y|x_1} \left\{ \log_2 \left[ \sum_{y=1}^{M} \frac{p(y|x_i)}{p(y|x_1)} \right] \right\},
\]

(6)

where \(E_{y|x_1}\) is the expectation operator with respect to \(y\) conditioned on \(x_1\), and \(C_{SD}\) denotes the SD capacity.

For hard-decision, the capacity with \(M\)-ary input, \(M\)-ary output and symmetric channel, is written as

\[
C_{HD} = \log_2 M + p_M \log_2 \left[ \frac{p_M}{M-1} \right] + (1 - p_M) \log_2 [1 - p_M],
\]

(7)

where \(p_M\) is the probability of incorrect symbol detection, and \(C_{HD}\) denotes the HD capacity. As soon as the value \(p_M\) is obtained, the HD capacity can be determined. Assuming that the signal \(x_1\) is transmitted, the probability of incorrect symbol detection can be obtained, as \(p_M = 1 - \text{pr}\{y_j < y_1, y_j \neq 1|x_1\}\), where \(\text{pr}\{\cdot\}\) is the probability operator. Since the events are not independent due to the existence of the random variable \(y_1\) in all of them, one can add a condition on \(y_1\) to make these events independent:

\[
p_M = 1 - \int_{-\infty}^{\infty} (\text{pr}\{y_j < y_1|y_1\})^{M-1} p(y_1|x_i)dy_1.
\]

(8)

Therefore, to obtain the HD capacities, the PDFs \(p(y_i|x_i)\) of \(y_i\) and \(p(y_i|x_j)\) of \(y_j\), conditioned on \(x_j\), must be calculated.

C. CHANNEL STATE INFORMATION

As is well known, channel state information can have a substantial influence on the fading channel capacity, which the CSI can use to optimize the transmitted power at the transmitter so as to increase the information rate. However, the difference between the capacity curves under transceiver CSI\(^1\) and receiver CSI\(^2\) are negligible in all cases \([34]\). So, in the following, it is assumed that the system is modeled based only on the receiver CSI.

D. INTER-SYMBOL INTERFERENCE ANALYSIS

As analyzed in Part A, for a multipath Rayleigh fading channels, the observation value of the \(\hat{m}\)-th auto-correlator is obtained as shown in (3). Generally, the ISI-noise has a significant impact on the performance, especially in the calculation of the capacity bounds. So, one cannot ignore the ISI, where the largest multipath time delay should not be set shorter than the chaotic signal duration, thus

\[
z_{\hat{m}} = \begin{cases} 
\sum_{j=1}^{\beta} \left( \sum_{l=1}^{L} (M\alpha_{l,m,x_0,\hat{m},m}(j-\tau_l))^2 \right) 
\end{cases} + N + Q_m, \quad \hat{m} = m \\
\sum_{j=1}^{\beta} \left( \sum_{k=1}^{M} (\hat{m},k)^2 \right), \quad \hat{m} \neq m,
\]

(9)

\(^1\)transceiver CSI, both for transmitter and receiver.  \(^2\)receiver CSI, only for receiver.

\[
FIGURE 3. Data statistics of ISI-noise \(Q_m\).
\]

where \(M_l = \sum_{k=1}^{M} w_{l,m,k} w_{\hat{m},k}, \tau_l\) is the delay time of the \(l\)-th path and \(N\) is the channel noise, satisfying

\[
N = 2 \sum_{j=1}^{\beta} \left( \sum_{l=1}^{L} M_l \alpha_{l,m,x_0,\hat{m},m}(j-\tau_l) \right) \left( \sum_{k=1}^{M} W_{\hat{m},k} k_{n,j} \right) \\
+ \sum_{j=1}^{\beta} \left( \sum_{k=1}^{M} W_{\hat{m},k} k_{n,j} \right)^2 \\
Q_m = \sum_{j=1}^{\beta} \sum_{l=1}^{L} \left( \alpha_{l,m,x_0,\hat{m},m}(j-\tau_l) \right) \left( \alpha_{s,m,x_0,\hat{m},s}(j-\tau_l) \right) \\
Y_m = \sum_{j=1}^{\beta} \sum_{l=1}^{L} \left( M_l \alpha_{l,m,x_0,\hat{m},m}(j-\tau_l) \right)^2 + N,
\]

(10)

where \(Q_m\) is the ISI-noise, which results from the multipath-effect but is useless for available signals, and \(Y_m\) is defined by the output of the auto-correlator without ISI. For the receiver, the ISI-noise and \(N\) are both interference noises.

III. NON-COHERENT CAPACITY OF THE \(M\)-ARY DCSK SYSTEM

In this section, the non-coherent capacity bounds of the \(M\)-ary DCSK with/without ISI are calculated and analyzed under the conditions of CSI-SD, CSI-HD, NonCSI-SD and NonCSI-HD, respectively.

Because of the serious influence of the ISI on the performance of the \(M\)-ary DCSK system, although the PDF is difficult to calculate, one can analyze the ISI-noise individually to obtain data statistics of the ISI-noise \(Q_m\), as shown in Fig. 3. It is found that the envelope curve of the statistics is similar to a Gaussian distribution:

\[
f_0(y_m) = \frac{1}{\sqrt{2\pi} \sigma_0} \exp \left( -\frac{y_m^2}{2\sigma_0^2} \right),
\]

(11)
where $\sigma_0$ is the variance of ISI-noise. Then, it can be verified that the transition probability $p_{\text{ISI}}(z|x_{\alpha}, \alpha_{l,m})$ of the receive signal is the convolution value of the traditional channel transition probability and the ISI-probability, given by

$$p_{\text{ISI}}(z|x_{\alpha}, \alpha_{l,m}) = p_{\text{NonSI}}(z|x_{\alpha}, \alpha_{l,m}) \ast f_0(y_{\alpha}).$$  \hspace{1cm} (12)

where $p_{\text{NonSI}}(z|x_{\alpha}, \alpha_{l,m})$ is generated for the traditional Non-ISI system, when $\sum_{j=1}^{\beta} x_{\alpha}(j-t_j)x_{\alpha}(j-t_j) \neq 0, l \neq s$ at $0 < t_j \ll \beta T_c$. So, the PDF of the multipath NonISI fading channels of the $M$-ary DCSK system is

$$\overline{p}(z|x_{\alpha}, \alpha_{l,m}) = p_{\text{NonSI}}(z|x_{\alpha}, \alpha_{l,m}).$$ \hspace{1cm} (13)

where $\overline{p}(z|x_{\alpha}, \alpha_{l,m})$ is defined as the NonISI-existed, differing from the PDF of ISI-existed. Alternatively, the influence of $\beta$ on the capacity bounds will be considered in the $M$-ary DCSK system over multipath Rayleigh fading channels on all conditions.

### A. SOFT-DECISION DEMODULATION WITH CSI

Mainly benefited from anti-multipath fading, the chaotic communication system is built with a spreading spectrum scheme. So, one should also analyze the effect of the spreading factor $\beta$ on the non-coherent capacity bounds with ISI.

Assuming that $m = 1$. The decision vector can be expressed as $y = (y_1, y_2, ..., y_M)$, where $y_1$ is a statistically independent non-central chi-square random variable with $\beta$ degrees of freedom and non-centrality parameter $\beta^2 = ME_0$, and $y_2, y_3, ..., y_M$ are all statistically independent central chi-square random variables with $\beta$ degrees of freedom. Given a transmitted signal $x_\alpha$, the decision variables are written respectively as follows: if the signal is present, $y_m \sim \chi^2(\beta, \beta^2)$; if the signal is absent, $y_m \sim \chi^2(\beta)$, where $\chi^2(\cdot)$ denotes the chi-square distribution. Thus, the conditional PDFs of $y_m$ are given by

$$p(y_m|x_{\alpha}, \alpha_{l,m}) = \left( \frac{1}{M\sigma_0} \right) \left( \frac{y_m}{ME_0\beta_{l,m}} \right)^{\frac{\beta^2}{2}} \exp \left( -\frac{ME_0\beta_{l,m} + y_m}{M\sigma_0} \right) \cdot I_{\frac{\beta}{2}} \left( 2\sqrt{y_m ME_0\beta_{l,m}} \right), \hspace{1cm} y_m \geq 0, \hspace{0.2cm} m = \hat{m},$$

and

$$p(y_m|x_{\hat{m}}, \alpha_{l,m}) = \left[ \frac{\beta^2}{(M\sigma_0)^{\frac{\beta}{2}} \Gamma \left( \frac{\beta}{2} \right)} \right] \exp \left( -\frac{y_m}{M\sigma_0} \right), \hspace{0.2cm} y_m \geq 0, \hspace{0.2cm} m \neq \hat{m},$$

where $I_n(\cdot)$ denotes the $n$-th Bessel function of the first kind, and the fading factor $\beta_{l,m} = \sum_{j=1}^{\beta} \alpha_{l,m}^2 \alpha_{j,m}^2$. So, the conditional PDF for the received vector $y$ can be written as

$$p(y|x_{\alpha}, \alpha_{l,m}) = p(y_{\hat{m}}|x_{\hat{m}}, \alpha_{l,m}) \prod_{m=1}^{M} p(y_m|x_{\alpha}, \alpha_{l,m}).$$ \hspace{1cm} (16)

Attentively, if the AWGN channels are selected for the non-coherent $M$-ary DCSK, equation (16) becomes

$$p(y|x_{\alpha}, \alpha_{l,m}) = \left( \frac{1}{M\sigma_0} \right) \left( \frac{y_m}{ME_0} \right)^{\frac{\beta^2}{2}} \exp \left( -\frac{ME_0\beta_{l,m} + y_m}{M\sigma_0} \right) \cdot \prod_{m=1}^{M} \left[ \frac{2\sqrt{y_m ME_0\beta_{l,m}}}{(M\sigma_0)^{\frac{\beta}{2}} \Gamma \left( \frac{\beta}{2} \right)} \right] \exp \left( -\frac{y_m}{M\sigma_0} \right).$$

where the AWGN channels capacity can be judged by the multipath Rayleigh fading channels when $L$ is infinite. Thus, the non-coherent capacity of $M$-ary DCSK system can be considered as a particular case of the general multipath channels of DCSK system when the path number is increasing to infinity.

So, for the general multipath fading channels, we obtain

$$p_{\text{SD}}(z|x_{\alpha}, \alpha_{l,m}) = p_{\text{NonSI}}(z|x_{\alpha}, \alpha_{l,m}) \ast f_0(y_{\alpha}) = p(y|x_{\alpha}, \alpha_{l,m}) \ast f_0(y_{\alpha}) = \left[ \frac{1}{M\sigma_0} \right] \left( \frac{y_m}{ME_0\beta_{l,m}} \right)^{\frac{\beta^2}{2}} \exp \left( -\frac{ME_0\beta_{l,m} + y_m}{M\sigma_0} \right) \cdot \prod_{m=1}^{M} \left[ \frac{2\sqrt{y_m ME_0\beta_{l,m}}}{(M\sigma_0)^{\frac{\beta}{2}} \Gamma \left( \frac{\beta}{2} \right)} \right] \exp \left( -\frac{y_m}{M\sigma_0} \right) \ast f_0(y_{\alpha}).$$

Considering equation (6), based on the soft-decision demodulation and the receiver CSI, the receiver can get the CSI and the transceiver can also obtain the channel distribution information (CDI). In this case, one can derive the non-coherent capacity of ISI channel as follows:

$$C_{\text{CSI-SD}} = \log_2 M - E_{z|x_1} \left\{ \log_2 \left[ \sum_{m=1}^{M} \frac{p(z|x_m)}{p(z|x_1)} \right] \right\}$$

$$= \log_2 M - E_{z|x_1} \left\{ \log_2 \left[ \sum_{m=1}^{M} \sum_{l=1}^{L} \sum_{m=1}^{M} p_{\text{ISI}}(z|x_{\alpha}, \alpha_{l,m}) f(\alpha_{l,m}) d\alpha_{l,m} \right] \right\}.$$  \hspace{1cm} (19)
B. HARD-DECISION DEMODULATION WITH CSI

For the hard-decision case, the coding channel can be mapped to a symmetric channel of $M$-ary input and $M$-ary output for simplification, but its performance can be downgraded, as

$$
C_{\text{CSI-HD}} = \log_2 M + p_{\text{CSI}} \log_2 \left[ \frac{p_{\text{CSI}}}{M-1} \right] + (1 - p_{\text{CSI}}) \log_2 [1 - p_{\text{CSI}}],
$$

where $p_{\text{CSI}}$ is the probability of having an incorrect symbol, so the HD capacity should be based on the PDF of $p_{\text{CSI}}$. When the hard-decision is employed, the detector can operate the DCSK symbol decision on the demodulator, but not on individual soft counts. Assuming that symbol $x_1$ is transmitted into the channel, the probability of incorrect symbol detection is given by $p_{\text{CSI}} = 1 - \text{pr} \{ z_m < z_1; \forall m = 1 \} \text{in} \{ z_m \}$ used for determining the capacities of the multipath Rayleigh fading channels. So, the incorrect symbol PDF is given by

$$
p_{\text{CSI}} = 1 - \int_0^\infty \left\{ \sum_{l=1}^L \int_0^\infty \int_0^\infty \rho_{\text{NonCSI}}(z_m | x_m, \alpha_{l,m}) d\alpha_{l,m} dz_m \right\} d\alpha_{l,m} \int_0^\infty \rho_{\text{NonCSI}}(z_1 | x_1, \alpha_{l,m}) dz_1
$$

$$
= 1 - \int_0^\infty \left\{ \sum_{l=1}^L \int_0^\infty \int_0^\infty \rho_{\text{NonCSI}}(y_m | x_m, \alpha_{l,m}) d\alpha_{l,m} dy_m \right\} d\alpha_{l,m} \int_0^\infty \rho_{\text{NonCSI}}(y_1 | x_1, \alpha_{l,m}) dy_1,
$$

where $\alpha_{l,m}$ denotes the fading factor of the $l$-th path.

C. SOFT-DECISION DEMODULATION WITHOUT CSI

For this case without CSI, the receiver cannot get the fading coefficient $\alpha_{l,m}$ when the symbols are output from the channel, so its PDF $p_{\text{NonCSI}}(z_m | x_m)$ should be defined as

$$
p_{\text{NonCSI}}(z_m | x_m) = \left\{ \begin{array}{ll}
p(y_m | x_m, \alpha_{l,m}) f(\alpha_{l,m}) & \text{for } \alpha_{l,m} = m = \hat{m}, \\
p(y_m | x_m, \alpha_{l,m}) & \text{for } \alpha_{l,m} = m \neq \hat{m}, \end{array} \right.
$$

where the PDF does not include the fading coefficient $\alpha_{l,m}$ when $m \neq \hat{m}$, so the integral of the chi-square distribution based on $\alpha_{l,m}$ is itself.

In summary, the PDF of the entire sequence is

$$
p(z | x) = p_{\text{NonCSI}}(z_m | x_m) * f_0(y_m)
$$

$$
= [p(y_m | x_m)]_{m=1}^M * f_0(y_m)
$$

$$
= \left[ \frac{1}{M \sqrt{2\pi} \sigma} \exp \left( -\frac{y_m^2}{2\sigma^2} \right) \right]_{m=1}^M * f_0(y_m).
$$

D. HARD-DECISION DEMODULATION WITHOUT CSI

As analyzed in Parts B and C, Section II, the probability of having an incorrect symbol in the NonCSI-receiver is given by

$$
p_{\text{NonCSI}} = 1 - \int_0^\infty \left\{ \sum_{l=1}^L \int_0^\infty \rho(y_m | x_m, \alpha_{l,m}) dy_m \right\} d\alpha_{l,m} \int_0^\infty \rho(y_1 | x_1, \alpha_{l,m}) dy_1,
$$

So, the capacity of HD demodulation without CSI is given by

$$
C_{\text{NonCSI-HD}} = \log_2 M + p_{\text{NonCSI}} \log_2 \left[ \frac{p_{\text{NonCSI}}}{M-1} \right] + (1 - p_{\text{NonCSI}}) \log_2 [1 - p_{\text{NonCSI}}].
$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerical results on the capacity bounds of the $M$-ary DCSK modulation over multipath Rayleigh fading channels are presented via Monte-Carlo simulation. It shows that U-shaped non-coherent capacity bounds exist by investigating the mechanism of the LDPC coded $M$-ary DCSK system.

A. NON-COHERENT CAPACITY OF THE M-ARY DCSK SYSTEM

The non-coherent capacity bounds of the $M$-ary DCSK system over a multipath Rayleigh fading channels in Fig. 4 and Fig. 5, where they show the ISI capacities and NonISI ones, respectively.

Fig. 4 shows the capacity bounds of the $M$-ary DCSK system without ISI. Figs. 4(a)-(d) show the capacities under the conditions of CSI-SD, CSI-HD, NonCSI-SD and NonCSI-HD. Firstly, the capacity bounds are left-shifted with the increasing of the modulation dimension $M$, and the overall
power of the DCSK system becomes lower. By examining the vertical lines (including the solid lines and dashed lines), it can be concluded that the dimension’s influence on the SD capacity is much stronger than in the HD case. For the CSI-existing capacity bounds shown in Fig. 4(a), the distance of lowest SNRs between $M = 4$ and $M = 16$ at spreading factor $\beta = 1$ is about 2.8dB, and 2.6dB for $\beta = 20$, where the distances are almost equal. But, in Fig. 4(b), the distances are about 1.2dB and 1.8dB, where the distance is larger when the spreading factor $\beta$ is increasing. The same for the capacity bounds without NonCSI in Fig. 4(c) and Fig. 4(d): the distances are about 2.8dB and 2.7dB for NonISI-NonCSI-SD $M$-ary DCSK in Fig. 4(c), while they are about 1dB and 2.4dB for NonISI-NonCSI-HD $M$-ary DCSK in Fig. 4(d). On the contrary, the capacity curves move to the right (the capacity performance is worsened), when the spreading factor $\beta$ is higher.

Besides, it is found that the influence of the spreading factor $\beta$ on the HD capacity is much heavier than that is on the SD one. For example, in Fig. 4(a), the distances of lowest SNRs between $M = 4$ and $M = 16$, about 2.8dB and 2.6dB, are nearly identical, while the distance is larger from 1.2dB to 1.8dB in Fig. 4(b), with the increasing of the spreading factor $\beta$, so the difference between SD and HD is obvious.

More importantly, with the increasing of dimension $M$, it is clear that the capacity bound based on CSI-SD demodulation outperforms that of the schemes in NonCSI-SD and CSI-HD demodulations, where the NonCSI-HD system has the highest power consumption. When $M = 16, L = 3, \beta = 20$, the lowest bit-signal noise ratios (SNRs) corresponding to the optimal rate are about 1.5dB and 6.8dB for CSI-SD and NonCSI-HD, respectively, meanwhile the lowest for NonCSI-SD is about 3.2dB, and is about 3.5dB for the CSI-HD case.

Moreover, for the SD-existing capacity bounds shown in Fig. 4(a) and Fig. 4(c), the optimal rate is larger when $\beta$ is larger, but the optimal rate is still at about 0.5bits/sym for the HD-existing capacity shown in Fig. 4(b) and Fig. 4(d), owing to the symmetric channel of the $M$-ary input and $M$-ary output system. Lastly, it can be seen that the optimal rate is irrelevant to the enlargement of dimension $M$.

Fig. 5 presents the capacity bounds of the $M$-ary DCSK system with ISI. Because of the influence of ISI, the BER performance of ISI $M$-ary DCSK system can be worse, and the capacity performance is right-shifted comparing with the NonISI one. The same for Fig. 4: the U-shaped capacity curves are left-shifted (the power consumption is lower) with the increasing of the dimension $M$, but the dimension’s influence on SD/HD capacity limits is much heavier than that is in the NonISI setting.

In Fig. 5(a), the distances of the lowest SNRs between $M = 4$ and $M = 16$ with different values of the spreading factor $\beta$,
about 3.4dB and 4.1dB, are larger than the cases when they are about 1.2dB and 2dB, as shown in Fig. 5(b). Meanwhile, the influence of the dimension $M$ on the capacity bounds is heavier in this case. In Fig. 5(a), the distance is about 3.4dB between $M = 4$ and $M = 16$ at $\beta = 1$, while it is about 4.1dB at $\beta = 20$.

Also, the U-shaped bound is right-shifted (the power is increased) with a larger spreading factor $\beta$. Eventually, the optimal rate of the ISI capacity bounds is not affected by the dimension $M$ and the spreading factor $\beta$. For pursuing the lowest power over multipath Rayleigh fading channels in practice, the NonISI-SD strategy for the capacity bounds should be chosen as a reference, where the optimal rate may stay at around $R_c = \frac{1}{2}$ for a broad range of code rates.

### B. SIMULATION PROOFS AND DISCUSSIONS

To verify the existence of the U-shaped capacity bounds, the $M$-ary DCSK modulation system with low-density parity-check (LDPC) codes is simulated with three kinds of protograph LDPC as shown in TABLE 1.

Fig. 6(a) shows the BER performance of the verification mechanism of the LDPC coded $M$-ary DCSK system over multipath fading channels without ISI and CSI, and distances of performance at BER=10$^{-6}$. The average power gains of the three paths are $\rho_1 = \frac{4}{7}$, $\rho_2 = \frac{1}{7}$ and $\rho_3 = \frac{3}{7}$ with time delays $\tau_1 = 0$, $\tau_2 = 3$ and $\tau_3 = 5$, respectively. Meanwhile, the modulation scheme is selected with $M = 16$ and spreading factor $\beta = 80$ (which ignores the interference of ISI, $0 < T_c \ll \beta T_c$). It is found that the BER performance at $R_c = \frac{1}{3}$ outperforms the other two rates at BER=10$^{-6}$, where a gain of 0.6dB can be obtained comparing with $R_c = \frac{1}{3}$, and 0.55dB with $R_c = \frac{2}{3}$.

Fig. 6(b) presents a broad range of code rates and the relationship between capacity bound and BER performance. The solid straight lines with “△”, “∗” and “◦” denote the system powers based on the different rates at BER=10$^{-6}$, $(\frac{1}{3}, 14.8dB)$, $(\frac{1}{2}, 14.2dB)$, $(\frac{2}{3}, 14.75dB)$.

And the curves with “∗”, “□”, and “+” express the theoretical capacity bound, iterative receiver system (IR) capacity bound and non-iterative receiver system (NonIR) capacity bound, respectively. It can be seen that the difference of BER performance between the code rate at $R_c = \frac{1}{3}$ and $R_c = \frac{2}{3}$ is negligible in the 16-ary DCSK system, where
the performance at $R_c = \frac{1}{2}$ outperforms the other code rates. Hence, a U-shaped capacity bound exists. As shown in Fig. 6(b), there is a large range of optimal code rates below $1\text{dB}$, $0.18$–$0.80\text{bits/sym}$. Meanwhile, compared with the IR and the NonIR, the derived bound is much more accurate and quantitative for the uniqueness of the capacity. When $R_c = \frac{1}{2}$, the capacity performance of the 16-ary DCSK system is about $0.2\text{dB}$ better than the IR system, and is about $2.2\text{dB}$ compared with the NonIR one. When $R_c = 0.9$, the theoretical capacity bound is much more accurate than the others, about $4.0\text{dB}$ gain for the IR system and about $7.2\text{dB}$ for the NonIR system.

Fig. 7 presents the relationship between the capacity bound and the multipath number $L$ for the $M$-ary DCSK modulation system, where solid curves with “⋆”, “◦”, “□”, “△” and “∗” denote the capacity bounds based on different multipath numbers $L$, over AWGN channels, respectively. It is found that the optimal rate is larger when the multipath number $L$ is increasing under the same conditions: dimension $M = 16$, spreading factor $\beta = 80$. Meanwhile, the optimal code rate (corresponds to the lowest SNR of multipath fading channels) is increasing with the heightening of $L$. When $L$ is large enough ($L \rightarrow \infty$), the capacity bound coincides with the case of the AWGN channels, and the optimal code rate is the largest, where it begins with the case of $L = 1$.

C. COMPLEXITY OF M-ARY DCSK SYSTEM

A high-dimensional DCSK modulation for coherent systems achieves only a little better than that for non-coherent ones, but it is at the cost of having greatly higher complexity. Non-coherent reception system is much simpler, which offers a good tradeoff between complexity and performance, and the capacity bounds are worsened with the increase of the spreading factors. In the case of requiring low power and low complexity, the non-coherent high-dimensional DCSK systems with small spreading factors can be considered, so as to avoid the complicated chaos synchronization requirement.

V. CONCLUSIONS

The non-coherent capacity bounds of the $M$-ary DCSK modulation system have been derived mathematically over multipath Rayleigh fading channels in the following scenarios: CSI-SD, CSI-HD, NonCSI-SD and NonCSI-HD. Firstly, it reveals that the capacity bounds of the non-coherent reception are left-shifted (the capacity performance is improved) with the increasing of the dimension $M$. And the dimension’s influence on the soft-decision capacity is much stronger than that is on the hard-decision one. Compared with NonISI channels, the influence of the dimension $M$ on SD/HD capacity bounds over ISI channels is more significant. In addition, the U-shaped bounds are right-shifted (the capacity performance is worsened) as the spreading factor $\beta$ increases. Besides, the influence of $\beta$ on the hard-decision capacity is more significant than that is on the soft-decision over NonISI channels. It is found that there is a relatively broad range of optimal code rates in U-shaped capacity bounds. Finally, it has been shown that U-shaped capacity bounds exist by investigating the mechanism of the LDPC coded $M$-ary DCSK system at different code rates. When the multipath number $L$ is larger, the optimal code rate of the $M$-ary DCSK system is
increasing. When \( L \) is large enough (\( L \rightarrow \infty \)), the capacity bound coincides with the case of the AWGN channels, meanwhile the optimal code rate is the largest, where it begins with the case of the single-path Rayleigh fading channel.

In the future, it is important to investigate the coded performance and coding optimization of \( M \)-ary DCSK/FM-DCSK systems based on some known capacity bounds over multi-path fading channels.

REFERENCES


LIN WANG (S’99–M’03–SM’09) received the M.Sc. degree in applied mathematics from the Kunming University of Technology, China, in 1988, and the Ph.D. degree in electronics engineering from the University of Electronic Science and Technology of China, China, in 2001. From 1984 to 1986, he was a Teaching Assistant with the Mathematics Department, Chongqing Normal University. From 1989 to 2002, he was a Teaching Assistant, a Lecturer, and then an Associate Professor in applied mathematics and communication engineering with the Chongqing University of Post and Telecommunication, China. From 1995 to 1996, he was with the Mathematics Department, University of New England, Armidale, NSW, Australia, for one year. In 2003, he was a Visiting Researcher with the Center for Chaos and Complexity Networks, Department of Electronic Engineering, City University of Hong Kong, for three months. In 2013, he was a Senior Visiting Researcher with the Department of ECE, University of California at Davis, Davis, CA, USA. From 2003 to 2012, he was a Full Professor and an Associate Dean with the School of Information Science and Technology, Xiamen University, China. He has been a Distinguished Professor since 2012. He holds 14 patents in the field of physical layer in digital communications. He has authored over 100 journal and conference papers. His current research interests are in the area of channel coding, joint source and channel coding, chaos modulation, and their applications to wireless communication and storage systems.

GUOFA CAI received the B.Sc. degree in communication engineering from Jimei University, Xiamen, China, in 2007, the M.Sc. degree in circuits and systems from Fuzhou University, Fuzhou, China, in 2012, and the Ph.D. degree from the Department of Communication Engineering, Xiamen University, Xiamen, in 2016. He is currently with the School of Information Engineering, Guangdong University of Technology, Guangzhou, China. His primary research interests include information theory, chaotic communications, channel coding, UWB, and MIMO communications.

GUANRONG (RON) CHEN (M’89–SM’92–F’97) was a Tenured Full Professor with the University of Houston, Houston, TX, USA. He has been a Chair Professor and the Director of the Center for Chaos and Complex Networks with the City University of Hong Kong since 2000. He received the 2011 Euler Gold Medal, Russia, and conferred Honorary Doctorate by the Saint Petersburg State University, Russia, in 2011 and by the University of Le Havre, France, in 2014. He is a member of the Academy of Europe and a fellow of The World Academy of Sciences. * * *