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A New Hierarchical $M$-ary DCSK Communication System: Design and Analysis

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ABSTRACT Multiresolution $M$-ary differential chaos shift keying (MR-$M$-DCSK) modulation using non-uniformly spaced phase constellation is a promising technique that can satisfy different bit-error-rate (BER) requirements within one symbol over multipath fading channels. However, the BER performance always deteriorates as the modulation order $M$ becomes larger. To overcome this shortcoming, a new hierarchical square-constellation-based $M$-DCSK communication system using non-uniformly spaced distance constellation is proposed, which can be easily extended to other-constellation scenarios, e.g., rectangular, star, and asymmetrical square constellations. Furthermore, an adaptive transmission scheme is designed by modifying the distance between the two constellation points in the last hierarchical level. In addition, theoretical BER expressions of the proposed systems are derived over multipath Rayleigh fading channels, which are consistent with the simulated results. Analytical and simulated results show that the proposed system not only can provide lower energy consumption as compared with the conventional MR-$M$-DCSK system but also can efficiently adjust BER performance based on the signal-to-noise ratio information. Therefore, the proposed system can serve as a desirable alternative for energy-efficient short-range wireless-communication applications.

INDEX TERMS Hierarchical square-constellation-based $M$-ary differential chaos shift keying ($M$-DCSK), bit error rate (BER), multipath Rayleigh fading channel, adaptive transmission.

I. INTRODUCTION As a promising chaotic modulation technique, differential chaos shift keying (DCSK) has very desirable performance with relatively low complexity, and hence has drawn much attention in the past two decades. In contrast to the conventional differential phase shift keying (DPSK) system, the DCSK system not only inherits the advantage of requiring no channel state information (CSI) for detection, but also is more robust to multipath fading [1]–[5]. More importantly, DCSK modulation is particularly suitable for ultra-wideband (UWB) applications [6]–[8]. To further expand its scope of applications, a large volume of research work has been dedicated to designing network coding schemes [9], [10], cooperative schemes [2], [11]–[13], multi-input multi-output schemes [3]–[5], [14]–[16], and simultaneous wireless-information-and-power transfer schemes [17], under the condition of DCSK modulation. In recent years, DCSK has also played an increasingly important role in short-range wireless-communication applications, such as wireless sensor networks (WSNs).

Unfortunately, conventional DCSK systems require a radio-frequency (RF) delay line at both the transmitter and the receiver, which is rather difficult to be implemented. To address this weakness, a code-shifted DCSK (CS-DCSK) system based on Walsh codes has been conceived to remove the RF delay line at the receiver [18]. Following this work,
a multicarrier DCSK (MC-DCSK) system [19] and a phase-separated DCSK (PS-DCSK) system [20] have been further developed to avoid using delay lines in both the transmitter and the receiver. In the MC-DCSK and PS-DCSK systems, the orthogonal sinusoidal signals (i.e., carriers) instead of the time-delayed signals are transmitted.

In conventional DCSK-related systems, the reference-chaotic signal and information-bearing signal are transmitted successively using the same energy in each transmission period, thus suffering from relatively low energy efficiency and spectral efficiency. To boost the spectral efficiency, some variants have been explored [19]–[26]. Specifically, the generalized CS-DCSK and high-data-rate CS-DCSK systems have been developed in [21] and [22], respectively. These modified CS-DCSK systems can dramatically improve the spectral efficiency, since more than one information-bearing signals are transmitted in each transmission period. Also, an orthogonal multilevel DCSK system has been introduced in [23], which achieves outstanding performance but decreases some spectral efficiency.

With respect to the single-carrier DCSK system, the MC-DCSK system enhances both energy and spectral efficiency [19], while the PS-DCSK system [20] only doubles the data rate. Aiming to reduce the implementation complexity of the MC-DCSK system, an orthogonal-frequency-division-multiplexing DCSK system has been formulated in [24]. As an extension of the quadrature chaos-shift keying (QCSK, i.e., 4-DCSK) modulation [25], a multiresolution M-ary DCSK (MR-M-DCSK) system has been designed based on the non-uniformly spaced phase constellations in [26], which achieves both lower energy consumption and higher spectral efficiency. This system can provide different quality of services (QoSs) for the transmitted bits within a symbol subject to their specific bit-error-rate (BER) requirements. Recently, some research attempt has been devoted to investigating a generalized constellation-based M-DCSK communication system framework [27], in which the performance of both square and circle-constellation-based M-DCSK systems are extensively compared. The results have shown that the former can realize lower energy consumption in comparison with the latter, and thus can better fit to power-constrained WSN applications. As compared with conventional modulation schemes, hierarchical-modulation schemes possess the unequal-error-protection property, and hence are able to satisfy more requirements of wireless-communication applications [28]–[30].

Motivated by the above discussions, a new hierarchical square constellation-based M-DCSK communication system is proposed in this paper, which can meet different BER requirements for different transmitted bits within one symbol. Distinguished from the MR-M-DCSK system, the proposed system exploits the distance vector to realize a more flexible BER service. Moreover, the new system can be extended to the hierarchical other constellations-based M-DCSK systems, e.g., rectangular, star and asymmetrical square constellations. Furthermore, an adaptive transmission scheme for the proposed system is designed by utilizing the last elements of the distance vectors. In addition, the theoretical error performance of the proposed system is carefully analyzed over multipath Rayleigh fading channels. Both analytical and simulated results indicate that this hierarchical M-DCSK system outperforms the conventional MR-M-DCSK system, and the designed adaptive scheme can satisfy a more flexible BER requirement based on the signal-to-noise ratio (SNR) information. Therefore, the proposed system appears to be an excellent candidate for low-complexity and low-power wireless-communication applications.

The major contributions of this work are as follows.

1) A new hierarchical square-constellation-based M-DCSK communication system framework is proposed, which can be generalized to other types of constellations, e.g., rectangular, star and asymmetrical square constellations. Moreover, an adaptive transmission scheme is designed for the proposed system.

2) The BER expressions of the proposed system are derived over multipath Rayleigh fading channels. Furthermore, the corresponding analysis methodology is generalized to the hierarchical rectangular-constellation-based M-DCSK system.

3) Both analytical and simulated results show that the new system accomplishes a noticeable performance gain than the conventional MR-M-DCSK system, and the designed adaptive scheme can efficiently modify the BER performance of the component bits within a symbol based on the SNR information.

The rest of this paper is organized as follows. Section II introduces the hierarchical square-constellation-based M-DCSK communication system. Section III derives theoretical BER expressions of the proposed system over multipath Rayleigh fading channels. Section IV provides an adaptive transmission scheme for the proposed system. Section V performs simulations to demonstrate the superiority of the new design. Finally, Section VI gives some concluding remarks.

II. HIERARCHICAL SQUARE-CONSTELLATION-BASED M-DCSK SYSTEM

In this section, a new hierarchical square-constellation-based M-DCSK communication system is proposed, with the block diagram illustrated in Fig. 1.

A. HIERARCHICAL SQUARE-CONSTELLATION-BASED M-DCSK SYSTEM

Fig. 2 presents the constellation of a hierarchical square-based 64-DCSK signal. As seen from the figure, $d_k^1$ and $d_k^q$ ($k = 1, 2$) denote the distances between the decision boundary in the $k$-th hierarchical level and the decision boundary in the $(k + 1)$-th level, respectively; and $d_1^q$ and $d_2^q$ denote the half distances between the two constellation points in the last hierarchical level, respectively. Define two types of distance
where the vectors as follows:

\[ \mathbf{d}^i = [d_1^i, d_2^i, \ldots, d_m^i], \]

and

\[ \mathbf{d}^q = [d_1^q, d_2^q, \ldots, d_m^q], \]

where \( d_1^i, d_2^i, \ldots, d_{m-1}^i \) and \( d_1^q, d_2^q, \ldots, d_{m-1}^q \) are the decision boundaries for the two distance vectors, respectively, \( d_k^i > d_{k+1}^i, d_k^q > d_{k+1}^q \), and \( m = \log_2 \sqrt{M} \).

Without loss of generality, assume that \( d_k^i \geq d_k^q \). Then, define two priority vectors as

\[ \mathbf{p}^i = [p_1^i, p_2^i, \ldots, p_{m-1}^i] \]

and

\[ \mathbf{p}^q = [p_1^q, p_2^q, \ldots, p_{m-1}^q] \]

Thus, different BER values can be obtained by modifying the distance vectors. The bit in the \( k \)-th position is protected with higher priority than that in the \( (k+1) \)-th position, for a larger ratio \( p_k^i/p_{k+1}^i \) or \( p_k^q/p_{k+1}^q \).

### B. HIERARCHICAL SQUARE-CONSTELLATION-BASED M-DCSK SYSTEM

Suppose that a normalized chaotic reference signal \( \mathbf{c}_s = [c_k^x, c_{k+1}^x, \ldots, c_M^x] \) is generated by a chaotic generator, e.g., the logistic map

\[ x_{k+1} = 1 - 2x_k^2, \quad k = 1, 2, \ldots, \beta, \]

where \( \beta \) is the spreading factor.

When the signal \( \mathbf{c}_s \) is passed through a Hilbert transform, a quadrature signal \( \mathbf{c}_q \) will be obtained. The two independent orthogonal signals are used to constitute the encoded information. Referring to Fig. 1, the encoded information is expressed as \( \mathbf{m}_s = a_s \mathbf{c}_s + b_s \mathbf{c}_q \), where the index \( s = 1, 2, \ldots, M \) presents the symbol, and \( a_s \) and \( b_s \) denote the \( x \)-axis coordinate value and the \( y \)-axis coordinate value corresponding to a constellation point, respectively, with \( a_s^2 + b_s^2 \neq 1 \). In particular, \( a_s \) and \( b_s \) can be computed by the distance vectors \( \mathbf{d}^i \) and \( \mathbf{d}^q \). Moreover, the reference and information signals are expressed as \( x_1(t) = \sum_{k=1}^\beta c_{k}^x p(t - kT_c) \) and \( x_2(t) = \sum_{k=1}^\beta c_{k}^y p(t - kT_c) \), respectively, where \( T_c \) is the chip time, \( p(t) \) is the square-root-raised-cosine filter, \( f(t) = \int_{(k+1)T_c}^{(k+1)T_c} p(t - kT_c)^2 dt = E_p \), \( \sum_{k=1}^\beta c_{k}^x c_{k}^y = 0 \), and \( \sum_{k=1}^\beta c_{k}^x c_{k}^y = 1 \). Consequently, the transmitted signal for the hierarchical square-constellation-based M-DCSK system is formulated as

\[ s(t) = \sqrt{2} x_1(t) \cos(2\pi f_0 t) - \sqrt{2} x_2(t) \sin(2\pi f_0 t), \]
where \( f_0 \) is the frequency of the sinusoidal carriers, with \( f_0 \gg 1/T_c \).

The impulse response function of a multipath fading channel is given by \( h(t) = \sum_{l=1}^{L} \alpha_l \delta(t - \tau_l) \), where \( L \) is the number of paths, and \( \tau_l \) and \( \alpha_l \) are the path delay of the \( l \)th path and the channel coefficient, respectively. Thus, the received signal can be written as

\[
    r(t) = h(t) \otimes s(t) + n(t),
\]

where \( \otimes \) is the convolution operator and \( n(t) \) is wideband additional white Gaussian noise (AWGN) with zero mean and the variance \( N_0 / 2 \).

At the receiver end, the in-phase and quadrature carriers are utilized to separate the received signal \( r(t) \). The separated signals are first processed by matched filters, and subsequently the filtered signals are sampled every \( kT_c \) time units. The sampled reference signals will be correlated with the sampled information signal or their Hilbert-transformed version. The decision vector \( z = (z_a, z_b) \) can be obtained after the correlation operation. It should be noted that the CSI, i.e., \( h = \sum_{l=1}^{L} \alpha_l^2 \), can be estimated by a method in [27], [31]. Here, we simply assume that the receiver knows a perfect CSI. Prior to the decision process, the original decision vector should be first converted to \( z' = z/(hE_b) = (z_a', z_b') \).

Generally, a search algorithm should be used to obtain the transmitted bits. To be specific, the distances between the decision vector and all constellation points, i.e., \((z_a - a_i)^2 + (z_b - b_i)^2\), are compared to find the bits for the position of the minimum distance corresponding to a constellation point. However, as the value of \( M \) increases, the algorithm complexity will become higher. In fact, the receiver needs to perfectly know all elements of the distance vectors to make a decision.

Because square-based \( M \)-DCSK constellations can be referred to as the superposition of several QCSK constellations based on the distance vectors \( d\) and \( d' \). QCSK de-mapper and mapper in [25] are used to make the decision in this paper, where the decision mechanism is shown in Fig. 3. As an example, the decision process for a hierarchical square-based 16-DCSK constellation is shown in Fig. 4. It is assumed that the bits “0000” are transmitted and the decision vector is presented by \((z_1, z_2)\), as shown in Fig. 4 (a). Based on the QCSK de-mapping rule, one can easily obtain the two high-priority (HP) bits “00” because \( z_1 > 0 \) and \( z_2 > 0 \). After that, the two HP bits are re-mapped by the QCSK constellation with the distance vector \((d'_1, d'_2)\). Based on \((z_1, z_2)\) and \((d'_1, d'_2)\), one can obtain the decision vector for the low-priority (LP) bits, denoted by \((z_1 - d'_1, z_2 - d'_2)\). According to the resultant vector \((z_1 - d'_1, z_2 - d'_2)\) as shown in Fig. 4(b), one can finally get the two LP bits, as “01”.

C. EXTENSION TO OTHER TYPES OF CONSTELLATIONS

The proposed system can be generalized to other types of constellations, such as rectangular, star and asymmetric square constellations. In the following, two examples are used to illustrate the flexibility of the distance vectors.

In the first example, consider the asymmetric hierarchical square-based 16-DCSK constellation, as shown in Fig. 5(a). In this case, the HP bit \( b_1 \) is determined by the first cluster, while the HP bit \( b_2 \) is determined by the second cluster, which is involved in the first cluster. Furthermore, the LP bit \( b_3 \) is determined by the third cluster, while the LP bit \( b_4 \) is determined by the specific signal point involved in the third cluster. The asymmetric hierarchical square-based 16-DCSK has three embedded sub-constellations and thus it can provide four levels of BERs.
In the second example, consider the hierarchical star-based 8-DCSK constellation, as shown in Fig. 5(b). This type of constellation efficiently combines the distance vector with the phase vector, and hence it inherits both advantages of the hierarchical square-based and phase-based constellations. The HP bit $b_1$ is determined by the distance vector, while the two LP bits $b_2$ and $b_3$ are determined by the phase vector.

### III. PERFORMANCE ANALYSIS

In this section, the theoretical BER performance for the hierarchical square-constellation-based $M$-DCSK system is carefully analyzed over multipath fading channels. Besides, the corresponding analytical methodology is generalized to the hierarchical rectangular-constellation-based $M$-DCSK system.

#### A. ENERGY AND CHANNELS

The average symbol energy for the hierarchical square-based $M$-DCSK constellation can be computed as

$$E_s = \left( \text{d}^T (\text{d}^T)^T + \text{d}^T (\text{d}^T)^T + 1 \right) E_p.$$  

($8$)

#### B. BER PERFORMANCE

To derive the BER expression of the hierarchical square constellation-based $M$-DCSK system, we first consider the constellations shown in Fig. 6. To simplify the exposition, we use $\text{d} = [d_1, d_2, \ldots, d_m]$ to represent $\text{d}^1$ or $\text{d}^2$ here.

For the hierarchical square-based 16-DCSK constellation shown in Fig. 6(a), the average BER for bit $b_1$, conditioned on $\gamma_s$, is given by ($12$), as shown at the top the next page, where $\rho = \frac{2\gamma_s}{4\gamma_s + 2}$. Additionally, the average BER for bit $b_2$, conditioned on $\gamma_s$, is given by ($13$), as shown at the top the next page.

On the other hand, considering the hierarchical square-based 64-DCSK constellation shown in Fig. 6(b), the average BER for the bit $b_1$, conditioned on $\gamma_s$, can be calculated as ($14$), as shown at the top the next page, where $\text{d}_1 = [d_1, d_2 + d_3]$ and $\text{d}_2 = [d_1, d_2 - d_3]$. Likewise, the average BERs for the bits $b_2$ and $b_3$, conditioned on $\gamma_s$, can be formulated as ($15$) and ($16$), as shown at the top the next page, respectively.

---

1. The expression of $E_s$ is dependent on the constellation. Precisely speaking, considering the square-based constellation, $E_s$ should be calculated by ($8$). Especially, if one only considers the constellations shown in Fig. 5, the average energy reduces to $E_s = (\text{d}^T \text{d}^T + 1) E_p$. 

---

![FIGURE 5. Constellations of (a) a asymmetric hierarchical square-based 16-DCSK signal, and (b) a hierarchical star-based 8-DCSK signal.](image)

![FIGURE 6. Constellations of (a) the row/column for a hierarchical square-based 16-DCSK signal, and (b) the row/column for a hierarchical square-based 64-DCSK signal.](image)
\[
P(4, \mathbf{d}, b_1|\gamma_s) = \frac{1}{4} \left( \text{erfc} \left( \frac{2(d_1 + d_2) h E_p}{\sqrt{4E_i h N_0 + 2\beta N_0^2}} \right) + \text{erfc} \left( \frac{2(d_1 - d_2) h E_p}{\sqrt{4E_i h N_0 + 2\beta N_0^2}} \right) \right) \\
= \frac{1}{4} \left( \text{erfc} \left( \frac{2(d_1 + d_2) h E_p}{\mathbf{d}^T \mathbf{d}^T + \mathbf{d} \mathbf{d}^T + 1} \right) + \text{erfc} \left( \frac{2(d_1 - d_2) h E_p}{\mathbf{d}^T \mathbf{d}^T + \mathbf{d} \mathbf{d}^T + 1} \right) \right)
\]

\[
P(4, \mathbf{d}, b_2|\gamma_s) = \frac{1}{4} \left( 2\text{erfc} \left( \frac{(d_1 + d_2) h E_p}{\mathbf{d}^T \mathbf{d}^T + \mathbf{d} \mathbf{d}^T + 1} \right) - \text{erfc} \left( \frac{2(d_1 + d_2) h E_p}{\mathbf{d}^T \mathbf{d}^T + \mathbf{d} \mathbf{d}^T + 1} \right) + \text{erfc} \left( \frac{2(d_1 - d_2) h E_p}{\mathbf{d}^T \mathbf{d}^T + \mathbf{d} \mathbf{d}^T + 1} \right) \right)
\]

\[
P(8, \mathbf{d}, b_1|\gamma_s) = \frac{1}{8} \left( \text{erfc} \left( \frac{(d_1 - d_2 - d_3) h E_p}{\mathbf{d}^T \mathbf{d}^T + \mathbf{d} \mathbf{d}^T + 1} \right) + \text{erfc} \left( \frac{(d_1 - d_2 - d_3) h E_p}{\mathbf{d}^T \mathbf{d}^T + \mathbf{d} \mathbf{d}^T + 1} \right) \right)
\]

Based on the recursive algorithm described in [32] and the above expressions, the following expressions can be obtained. When \( i < m \), the average BER \( b_i \) is given by

\[
P(\sqrt{M}, \mathbf{d}, b_i|\gamma_s) = \frac{1}{2} \left( P \left( \frac{\sqrt{M}}{2}, \mathbf{d}_+, b_i|\gamma_s \right) + P \left( \frac{\sqrt{M}}{2}, \mathbf{d}_-, b_i|\gamma_s \right) \right),
\]

where \( \mathbf{d}_+ = [d_1, d_2, \ldots, d_{m-1}, d_{m-1} + d_m] \) and \( \mathbf{d}_- = [d_1, d_2, \ldots, d_{m-1}, d_{m-1} - d_m] \).

When \( i = m \), the average BER \( b_m \) is expressed as

\[
P(\sqrt{M}, \mathbf{d}, b_m|\gamma_s) = \frac{1}{2^m} \left( \sum_{i=1}^{2^m-1} \sum_{j=1}^{2^m-1} \frac{1}{2} (-1)^{j+1} \text{erfc} \left( \frac{(d_0(i) - B(j)) h E_p}{\mathbf{d}^T \mathbf{d}^T + \mathbf{d} \mathbf{d}^T + 1} \right) \right)
\]

where \( \mathbf{d}_0 \) and \( \mathbf{d}_1 \) are the position vectors of the constellation points for LP bits 0 and 1, respectively, \( B(j) = 0.5 (x_{j-1} + x_j), j = 1, 2, \ldots, m \), and \( x_j \) is the coordinate for the \( j \)th symbol. The method for generating these parameters is available in [32].

Finally, the BER expressions for the hierarchical square-constellation-based \( M \)-DCSK system can be derived. Specifically, the BER of the \( k \)-th bit, i.e., \( b_k \) \((k = 1, 2, \ldots, m-1)\), in the hierarchical square-constellation-based \( M \)-DCSK system, is given by

\[
P_b(M, \mathbf{d}^i, b_k) = \int_0^{\infty} P \left( \sqrt{M}, \mathbf{d}^i, b_k|\gamma_s \right) f(\gamma_s) d\gamma_s.
\]

Furthermore, the BER of the \( n \)-th bit, i.e., \( b_n \) \((n = 2, 4, 6, \ldots, 2m)\), is given by

\[
P_b(M, \mathbf{d}^q, b_n) = \int_0^{\infty} P \left( \sqrt{M}, \mathbf{d}^q, b_n|\gamma_s \right) f(\gamma_s) d\gamma_s.
\]
C. EXTENSION TO HIERARCHICAL RECTANGULAR-CONSTELLATION-BASED M-DCSK SYSTEM

The above BER results can be immediately extended to the rectangular-constellation scenario. Fig. 7 shows the constellation of a hierarchical rectangular-based 8-DCSK signal. To guarantee the consistency with the definition for the hierarchical square-based 64-DCSK constellation, one should set $m = \log_2 \sqrt{M/2}$. The two distance vectors can be written as $\mathbf{d}^i = [d_1^i, d_2, \ldots, d_{m+1}^i]$ and $\mathbf{d}^q = [d_1^q, d_2^q, \ldots, d_m^q]$. In this scenario, the average transmitted energy is also given by $E_t = (\mathbf{d}^i \mathbf{d}^i + \mathbf{d}^q \mathbf{d}^q + 1)E_p$.

Using Eqs. (12)-(18) as the root for this algorithm, the BER of the $k$-th bit, i.e., $b_k$ ($k = 1, 3, 5, \ldots, 2m - 1$), in the hierarchical rectangular-constellation-based $M$-DCSK system, is given as

$$P_b^q(M, \mathbf{d}^i, \mathbf{d}^q, b_k) = \int_0^\infty P \left( \sqrt{2M}, \mathbf{d}^i, b_k \gamma_s \right) f(\gamma_s) d\gamma_s,$$

while the BER of the $n$th bit, i.e., $b_n$ ($n = 2, 4, 6, \ldots, 2m$), is expressed by

$$P_b^q(M, \mathbf{d}^i, \mathbf{d}^q, b_n) = \int_0^\infty P \left( \sqrt{\frac{M}{2}}, \mathbf{d}^q, b_n \gamma_s \right) f(\gamma_s) d\gamma_s.$$

(21)

(22)

Note that, based on Eqs. (12)-(18) and Eqs. (21)-(22), the distance vectors $\mathbf{d}^i$ and $\mathbf{d}^q$ for the hierarchical rectangular constellations should be utilized to calculate the BERs of the hierarchical rectangular-constellation-based $M$-DCSK system.

IV. AN ADAPTIVE TRANSMISSION SCHEME

As discussed in Section II, the receiver does not need the values of $d_m^i$ and $d_m^q$ when demodulating the received signal. Based on this property, an adaptive transmission scheme is designed in this section.

To begin with, in order to achieve different BERs for different transmitted bits within a symbol and to ensure the transmitted energy be no more than that of the MR-$M$-DCSK system [26], the distance vectors $\mathbf{d}^i$ and $\mathbf{d}^q$ should be subject to the following constraints:

$$d_k^i \geq 2d_{k+1}^i, \quad k = 1, 2, \ldots, m - 1,$$

$$d_k^q \geq 2d_{k+1}^q, \quad k = 1, 2, \ldots, m - 1,$$

$$\mathbf{d}^i \mathbf{d}^i^T + \mathbf{d}^q \mathbf{d}^q^T \leq \frac{1}{2}.$$

(23)

Then, to effectively realize adaptive transmission, an example for the hierarchical square-based 64-DCSK constellation is presented in Fig. 8, showing the design of the distance vectors. In this example, it is assumed that $\mathbf{d}^i = \mathbf{d}^q = \mathbf{d}$. The principle of the design is adjusting the distance between the two constellation points in the last hierarchical level while keeping the remaining distances unchanged. By this operation, the decision boundaries remain the same but the constellation points move farther from or closer to the decision boundaries. As shown in Fig. 8, the decision boundaries remain unvaried as the distance $d_3$ increases or decreases. At the receiver, one can obtain the transmitted bits correctly without using $d_3$.

Finally, the effect of the distance $d_m^i$ and $d_m^q$ on the BER is further illustrated. According to Eqs. (14)-(16), the BER performance is changed when modifying $d_3$. Therefore, based on the SNR information, one can adjust the distances $d_m^i$ and $d_m^q$ to achieve the target BERs for HP bits, while allowing the LP bits to be transmitted as reliably as possible. In actual implementation, to satisfy different target BER requirements for transmitted bits, the initial elements of $\mathbf{d}^i$ and $\mathbf{d}^q$ can be obtained by numerically solving the BER expressions (12)-(20).

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the BER performance of the hierarchical square-constellation-based $M$-DCSK system is presented over AWGN and multipath Rayleigh fading channels, thus verifying the superiority of the design and the accuracy of the analysis. Moreover, the feasibility of the proposed system for UWB transmission environments is discussed via simulations. Unless otherwise stated, the parameters used for the multipath channel and proposed systems in simulations are set as follows: number of paths is $L = 3$, channel gains are
FIGURE 9. BER curves of the hierarchical square-constellation-based (a) 16-DCSK system, and (b) 64-DCSK system over an AWGN channel. The spreading factors used are $\beta = 5$ and 80.

$E(\alpha_1^2) = E(\alpha_2^2) = E(\alpha_3^2) = 1/3$, time delays are $\tau_1 = 0$, $\tau_2 = 2$, $\tau_3 = 5$, and the distance vectors are $d_1 = d_2 = d$. Note also that, for the hierarchical 16-DCSK system, $b_1$ and $b_2$ denote the HP bits while $b_3$ and $b_4$ represent the LP bits. For the hierarchical 64-DCSK system, $b_1$ and $b_2$ denote the HP bits while $b_5$ and $b_6$ represent the LP bits.

A. BER COMPARISON BETWEEN THEORETICAL ANALYSIS AND SIMULATED RESULTS

Fig. 9 compares the theoretical and simulated BER curves of the hierarchical square-constellation-based 16-DCSK system and 64-DCSK system over an AWGN channel, where the spreading factors are set to $\beta = 5$ and 80, and the distance vectors for $M = 16$ and $M = 64$ are set to $d = [\sqrt{10}/5, \sqrt{10}/10]$ and $d = [2\sqrt{2}/21, \sqrt{2}/21, \sqrt{2}/42]$, respectively. Referring to Fig. 9(a), the theoretical BER curves match well with the simulated BER curves for both spreading factors, which validates the performance analysis. Similar observations can be obtained from the results for $M = 64$ (see Fig. 9(b)).

In Fig. 10, the theoretical and simulated BER curves of the hierarchical square-constellation-based 16-DCSK system and 64-DCSK system are compared over a multipath Rayleigh fading channel, where the spreading factors are set to $\beta = 80$ and 160, and distance vectors are the same as that in Fig. 9. As can be seen, the theoretical result is also highly consistent with the simulated result, irrespective of the modulation order and spreading factor.

B. PERFORMANCE COMPARISON BETWEEN THE PROPOSED SYSTEM AND THE MR-M-DCSK SYSTEM

Fig. 11 shows the BER curves of the hierarchical square-constellation-based 16-DCSK system and the MR-16-DCSK system [26] over a multipath Rayleigh fading channel, where the parameters are set to $\beta = 160$, $d = [\sqrt{10}/5, \sqrt{10}/10]$ and $\theta = [\pi/4, \pi/8, \pi/16]$. For a fair comparison, assume that the two systems have the same transmitted energy. It can be observed that the proposed system provides better BER performance than the MR-16-DCSK system. For example, under a fixed channel condition (i.e., a fixed fading profile and a fixed noise variance $\sigma_n^2$), the proposed system requires a relatively lower transmitted energy to achieve the same BER level (e.g., $BER = 10^{-4}$) with

$\theta$ The distance vector $d = [\sqrt{10}/5, \sqrt{10}/10]$ and the phase vector $\theta = [\pi/4, \pi/8, \pi/16]$ are used for the proposed system and the MR-16-DCSK system, respectively.
respect to the MR 16-DCSK system. Moreover, at $E_b/N_0 = 30$ dB, the HP bits $(b_1, b_2)$ in the proposed system achieves a BER of $2 \times 10^{-4}$, while that in the MR-16-DCSK system only accomplishes a BER of $2 \times 10^{-3}$. Thanks to the above advantage, the proposed system should be more suitable for energy-efficient wireless-communication applications, e.g., WSNs.

To validate the feasibility of the design in UWB transmission scenarios, the hierarchical square-constellation-based 16-DCSK system and the MR-16-DCSK system are further examined over an IEEE 802.15.4a CM1 channel, with corresponding results shown in Fig. 12. In the simulation, the parameters used for the UWB channel model are set as follows: the pulse duration time is $T_c = 10$ ns, the guard interval is $T_g = 90$ ns, and the sample frequency is $f_s = 8$ GHz. Furthermore, the distance vector and the phase vector used here are the same as that in Fig. 11. As can be seen, the proposed system also achieves better BER performance than the MR-16-DCSK system over UWB channels.

### C. BER PERFORMANCE OF THE PROPOSED ADAPTIVE TRANSMISSION SCHEME

Fig. 13 presents the BER results of the adaptive transmission schemes in the hierarchical square-constellation-based (a) 16-DCSK system, and (b) 64-DCSK system, over a multipath Rayleigh fading channel. The spreading factors used are $\beta = 160$. To implement the adaptive transmission schemes, the distances $d_2$ and $d_3$ are adjusted for the cases of $M = 16$ and $M = 64$, respectively.

16-DCSK system and 64-DCSK system over a multipath Rayleigh fading channel, where the spreading factor is set to 160. The distance vectors in the case of $M = 16$ are assumed as $d = [\sqrt{10}/5, \sqrt{10}/10]$ and $d = [\sqrt{10}/5, \sqrt{10}/20]$, while those in the case of $M = 64$ are assumed as $d = [2\sqrt{42}/21, \sqrt{42}/21, \sqrt{42}/42]$ and $d = [2\sqrt{42}/21, \sqrt{42}/21, \sqrt{42}/84]$. For a given $M$, it can be observed that different BER values can be obtained by adjusting $d_m$. Accordingly, by using such an adaptive transmission scheme, the HP bits can be transmitted as reliably as possible when the channel condition is bad, while both HP and LP bits can be reliably transmitted when the channel condition is good.

Next, the effect of the distance $d_2$ on the BER performance of the hierarchical square-constellation-based 16-DCSK system is discussed. The result is shown in Fig. 14, where $d = [d_1, d_2] = [\sqrt{10}/5, d_2]$, $\beta = 80$, and SNR = 25 dB, 30 dB, 35 dB. Referring to Fig. 14(a), given a fixed SNR, there is intersection between the BER curves of the HP bits and the LP bits. Let $d_{2,1}$ be the value of $d_2$ that make the BER of HP bits be equal to that of LP bits. one can observe that $d_{2,1} > d_1/2$ as SNR increases from 30 dB to 35 dB.
Hence, when \( d_2 \leq d_1/2 \), one can find that there is no intersection between the BER curves of the HP and LP bits and the LP bits, thus ensuring that different bits have different BERs. This result verifies the accuracy of the constraints (23). In addition, according to Fig. 14(b), there exists an optimal value of \( d_2 \) that can accomplish the minimum average BER for a given SNR. For a generalized adaptive system, one can estimate the optimal value of \( d_m \), i.e., \( d_{m,\min} = \arg \min_{d_m} P_{b}^{\text{avg}}(d) \), based on a simple computer search.

**VI. CONCLUSIONS**

In this paper, a new hierarchical square constellation-based \( M \)-DCSK communication system has been proposed, which is also applicable to other constellations, such as rectangular, star, and asymmetrical square constellations. To realize a more flexibility BER requirement, an adaptive transmission scheme has been designed by adjusting the distance vector based on the SNR information. Furthermore, the BER expressions of the proposed system have been derived over multipath Rayleigh fading channels. Both theoretical and simulated results have been given to demonstrate the superiority of the new design and the accuracy of the analysis.

Compared with the MR-\( M \)-DCSK system, the proposed system can achieve remarkably better error performance. As a result, the proposed system stands out as a good candidate for energy-efficient wireless-communication applications.

**REFERENCES**


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