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**Published in:**

Electric Power Systems Research

**Published:** 01/03/2025

**Document Version:**

Final Published version, also known as Publisher's PDF, Publisher's Final version or Version of Record

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**Publication record in CityU Scholars:**

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**Published version (DOI):**

[10.1016/j.epsr.2024.111285](https://doi.org/10.1016/j.epsr.2024.111285)

**Publication details:**

Liang, B., Yang, J., Wen, F., Wang, L., & Dong, Z. Y. (2025). Fairness considered aggregation mechanism for consumers and prosumers in electricity distribution networks. *Electric Power Systems Research*, 240, Article 111285. <https://doi.org/10.1016/j.epsr.2024.111285>

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# Fairness considered aggregation mechanism for consumers and prosumers in electricity distribution networks<sup>☆</sup>

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## ARTICLE INFO

### Keywords:

Economic dispatch

Fairness

Equity

Prosumer and consumer

User satisfaction level

## ABSTRACT

Efficiently realizing a decarbonized energy system relies on effectively aggregating and equitably allocating cost to consumers and prosumers in distribution networks. This is crucial for fostering flexibility in transitioning to a sustainable, low-carbon energy system with increased renewable integration. This paper proposed an innovative aggregation mechanism for prosumers and consumers, emphasizing fairness and equality in their aggregation via proper pricing and efficient dispatching. The approach aims to enhance participant retention and attract investments by analysing characteristics, operation models, and operational impact on end users. The proposed two-level model underscores fairness and equity on different scopes. The upper-level model incorporates distributed nodal pricing (DNP) to facilitate fair dispatch among nodes, accounting for transmission congestion impacts. Simultaneously, the lower-level model addresses the efficiency-fairness trade-off, emphasizing equity in the energy sector and utilizing user satisfaction levels as the reference index. To validate the proposed aggregation mechanism, an example system is employed, with numerical studies illustrating its potential to enhance efficiency and fairness in the aggregation. A substantial improvement in the fairness index is observed when the efficiency-vs-fairness ratio changes from 1 to 0.9, indicating that even a modest emphasis on equity indicators can significantly enhance energy fairness.

## 1. Introduction

THE rapidly evolving IoT technology is facilitating the emergence of the Energy Internet (EI), an evolution from traditional power systems to integrated energy. EI functions as a renewable energy-based, distributed, open sharing network, remaining as crucial to daily life and production as in the past. While maintaining requirements for security, reliability, and economy, its organizational structure has become more “internet-based”. The increasing penetration of distributed energy and renewables presents significant challenges to system organization and balance maintenance. The surge in prosumers, driven by a high penetration of renewable energy at the residential level, intensifies the dependence of grid regulation capacity on the demand side. Traditional power systems urgently require digitalization and marketization to

establish a new operational order and transactive mechanisms. This transformation aims to cultivate support and flexibility from end-users, ensuring adaptability in the face of evolving energy landscapes and enhancing the overall resilience and sustainability of the energy infrastructure.

Effectively aggregating power end users is a key concern in current power systems which demand flexibility [1]. Ref. [2] introduces a fog based IoT architecture for transactive energy management systems, examining customer and utility company operations through an IoT lens. Beyond major utilities, attention is given to smaller suppliers, notably prosumers. Actually, modern consumers can be regarded as prosumers without DER integration, and traditional consumers can be regarded as modern consumers missing the flexibility, in general. As traditional load-following consumer allocation is attended to in enormous research and practice, we will mainly go through the new attempts

<sup>☆</sup> This work was jointly supported by Zhejiang Provincial Natural Science Foundation, China under Grant LQ19E070001, and Fundamental Research Funds for Zhejiang University of Science and Technology, China under Grant No. 2023QN043.

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<https://doi.org/10.1016/j.epsr.2024.111285>

Received 26 June 2024; Received in revised form 30 October 2024; Accepted 17 November 2024

Available online 26 November 2024

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### Nomenclature

$l_{ij}$	Load quantity of the $j$ th consumer/prosumer at node $i$	$\chi$	Conversion efficiency ratio
$l_{ij}^0, l_{ij}^1, l_{ij}^2, l_{ij}^3$	Uncontrollable, deferrable, convertible, and diminishable part of load $l_{ij}$	$\rho$	Energy price of natural gas
$L_{ij}^0$	Predicted value of $l_{ij}^0$	$\alpha, \beta, \delta$	Coefficients for deferrable, diminishable and convertible loads in the calculation of the satisfaction level
$\sigma_{ij}$	Deviation of $l_{ij}^0$ from its predicted value	$\gamma$	Coefficients for solar power curtailment in the calculation of the satisfaction level
$L_{ij}^k, X_{ij}^k$	Designated amount and unsatisfied ratio of deferrable ( $k=1$ ), convertible ( $k=2$ ), and diminishable ( $k=3$ ) loads	$F_{\max}$	Channel capacities of distribution lines (in both directions)
$g_{ij}$	Actual integrated generation from equipped PV panels of prosumer $j$ at node $i$	$\lambda$	Electricity purchase price
$G_{ij}, \theta_{ij}$	Predicted generation and its deviation from equipped PV panels of prosumer $j$ at node $i$	$L_{sum}$	Aggregated amount of purchased electricity
$D_{ij}$	Discarded solar generation of prosumer $j$ at node $i$	$P_{loss}$	Network distribution loss
$G_{ij\max}$	Maximum output of equipped PV panels of prosumer $j$ at node $i$	$F_k$	Power flow on distribution line $k$
$R_{ij}$	Energy cost for consumer/prosumer $j$ at node $i$	$H_{ik}$	Admittance of line $k$ starting from $i$
$L_i, L'_i$	Original and suggested aggregated load at node $i$	$\omega, \pi_k, \bar{\kappa}_k, \underline{\kappa}_k$	Lagrange multipliers for the upper model
$p_i$	Energy price at node $i$	$L$	Lagrange function of the upper model
$\eta_{ij}$	User's satisfaction level of prosumer $j$ at node $i$	$F, F^*$	Cost function and the optimal cost solution of the upper model
$c_{ij}$	Additional energy conversion cost for convertible loads for consumer/prosumer $j$ at node $i$	$DNP_i$	Derived distributed nodal pricing at node $i$
		$p_i$	Electricity price at node $i$
		$M$	Penalty factor for equity violation in $p_i$
		$\mu$	Efficiency-vs-fairness ratio
		$\sigma\{\eta_{ij}\}, \sigma\{c_{ij}\}$	Variance of $\{\eta_{ij}\}$ and $\{c_{ij}\}$ ,
		$x_{AB}, x_{BC}, x_{AC}$	Impedance of line A-B, B-C, and C-A

on integration of prosumers. Ref. [1] aggregates prosumers, acting as retailers in the wholesale market and supplying energy to end-users in the retail market. Further exploration of end-users' flexibility potential occurs in [3], utilizing a two-stage stochastic optimization model to simulate aggregator participation in energy and tertiary reserve markets, exploiting load and generation flexibility. Recent publications emphasize market-based organization for suppliers, with pricing and dispatching at the core. Ref. [4] proposes the *SmartPrice*, a dynamic cloud-based pricing scheme fostering cooperation among prosumers. Meanwhile, Ref. [5] introduces a P2P pricing mechanism based on dynamic supply-demand and export-import retail prices ratios.

In [6], prosumers' charging and discharging schedules are optimized using a shared pricing mechanism in the prosumer community. Ref. [7] diversifies service provision, encompassing arbitrage, peak shaving, and regulation. Ref. [8] analyzes prosumer inner operations with a prospect-based stochastic game, revealing a stationary Nash equilibrium where trading policies are independent of time and play histories. The research in [9] and [10] focuses on operators' policies, including energy expenditure incidence in the presence of prosumers and an inclusive retail tariff model capturing net energy metering policies' features. Clearly, with the advancement of the Energy Internet, participant numbers will increase not only in quantity but also in composition and requirements.

In numerous allocation-related studies, fairness, justice, or equity emerges as a recurring theme across various domains. Equitable allocation of dominant resources proves pivotal in resource sharing, reasonable pricing, just dispatching, and more. Fair pricing mechanisms have been proposed for diverse applications, ranging from heterogeneous IoT wireless access networks [11] to spectrum auctions [12] and sustainable life insurance products [13]. Mathematics and operations research scholars delve into the broader problem in [14–16]. This emphasis on fairness extends to specific participants, notably within power and integrated energy systems. Researchers have engaged in debates regarding equity and justice in the clean energy transition [17]. Power system experts express their concerns about defining and realizing fairness in everyday operations and disaster management [18,19], echoing earlier discussions in the dispatch of suppliers [9]. Recently, even in the realm of demand response, a branch of transactive energy,

implementers focus on achieving “just and reasonable” prices. Ref. [20] proposes a shareholding-based resource sharing mechanism for promoting energy equity in peer-to-peer energy trading, while [21] introduces a trading aiding tool based on a Nash non-cooperative game model to ensure trading fairness.

Amidst various studies, the concept of DLMP (distributional locational marginal price) is widely adopted in cultivating flexibility at the distribution system level [22], which mainly applied to EV dispatch [23]. The derivation of DLMP varies significantly with distinctive focuses on transactive mechanisms [24], market interaction [25], or operational flexibility [26]. It is also referred to as congestion price in many publications, as the flexibility is employed eventually to solve the congestion in distribution power system [27]. Geographical location of users [28] or grid topology [29] are also found meaningful to generate better congestion price to enhance the distribution system operation performance. A DSO flexibility platform for the European CoordiNet project based on an optimal congestion management model is also designed, carried out and tested [30]. Professionals in the area are devoted to generating different network tariff schemes to address emerging challenges in power system planning and operation as well as electricity market evolution [31]. Among these attempts, fairness or justice is involved occasionally, in a rather simple and preliminary way. Specifically, fairness is addressed in bi-level transactive energy mechanism in distribution systems in [24], which describes a novel mechanism to charge users in a more equitable manner, by adding Jain's index of fairness to DSO social welfare objective function. Current DLMP or congestion price derivations closely resemble traditional nodal pricing, without adequately addressing to the consideration of massive prosumers and their energy justice. As massive prosumers and consumers coexist within the same node, adjustments become inevitable, planning with consideration of cooperation, flexibility, and other social features of various stakeholders, including consumers, prosumers, and microgrids becomes quite essential [32].

While fair allocation of limited resources is not commonplace in the energy sector due to its unique nature, the operation research field provides valuable insights [33,34]. In [35], the topics of energy equity are comprised of two parts: normal operation scenarios which focuses on long-term distributional equity; disaster management on short-term

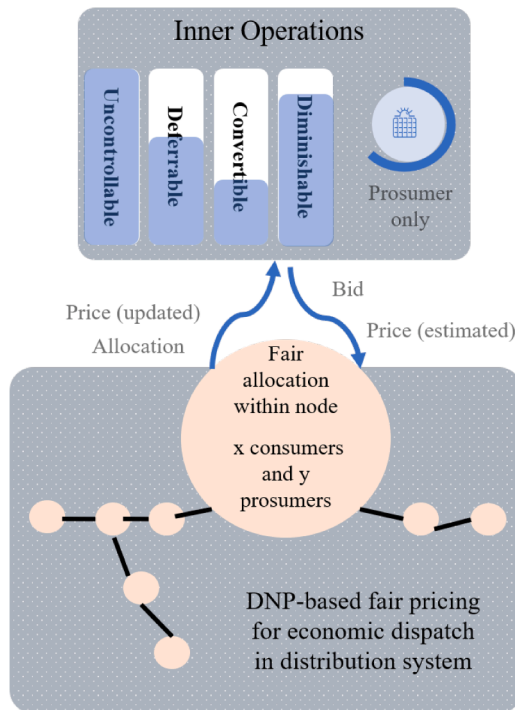


Fig. 1. Interaction procedure within the distributed system.

restorative equity. In [36], Farley noted the three cores of energy justice, distributional justice, procedural justice, and restorative justice. Among which, the equitable distribution of undesirable outcomes, or the rectification of harms already done falls under the umbrella of restorative equity. Due to the gap between disaster dispatch and daily dispatch, the paper will focus on daily dispatch, pursuing energy equity in normal operation scenarios. In ordinary power economic dispatch, fairness entails ensuring equal opportunities and treatment for all participants in the power market. This involves not only “gaining more from valuable resources” but also ensuring the “same sacrifice/disturbance under the same condition.” This paper reflects the former by pricing each node differently according to system constraints and the latter by guaranteeing equal rights for all prosumers and consumers within each node. Investigating the characteristics and operation models of prosumers and consumers, considering individual users’ satisfaction levels, the paper proposes a two-level economic dispatch model emphasizing fairness at both the distribution system and node levels. The upper model employs distributed nodal pricing to conduct economic dispatch among different nodes, factoring in system transmission constraints. Simultaneously, the lower model incorporates an efficiency-vs-fairness trade-off, utilizing users’ satisfaction levels as the reference index. Fig. 1 illustrates the interaction procedure of the distributed system.

The major contributions of this study are summarized as follows:

a) The prosumer/consumer model introduced in this study frames the energy operation problem as a decision-making process, incorporating uncertainties related to household demands, PV generation, and environmental influences. An analysis of both external and internal operations on end-users is conducted, considering factors such as users’ unsatisfactory levels and renewable curtailment. These considerations are later employed as reference indices. The original problem is formulated as a continuous linear programming problem, which transforms into a quadratic convex programming problem when incorporating additional factors and variances.

b) A fair pricing strategy, based on distributed nodal pricing (DNP), is proposed to account for the congestion impact of each node and guide load adjustments to alleviate transmission constraint impacts. DNP is designed for economic dispatch among different nodes in the upper

model. Simultaneously, the lower model incorporates a trade-off between efficiency and fairness, emphasizing fairness in the energy sector and utilizing users’ unsatisfactory levels as a reference index.

c) The coefficient efficiency-vs-fairness ratio  $\mu$  is utilized in deriving the local allocation plan at each node to reflect varying emphasis levels on efficiency and fairness. The economic solution is derived based on node composition and state, while the fairness solution considers energy equity among all prosumers/consumers. The sensitivity parameter  $\mu$  is employed to coordinate the trade-off between efficiency and fairness. Individual participants’ extra energy costs and satisfaction are evaluated, along with their opportunities to optimize their welfare further.

The rest of the paper is structured as follows: Section 3 details the energy operation model for individual prosumer/consumer, considering both inner and outer uncertainties. The proposed aggregation mechanism is implemented via the optimization of a bi-level model for a proper pricing and an efficient dispatching scheme, which seeks balance between economy and fairness is described in Section 4. Moving on to Section 5, the derivation and solution of the proposed mechanism are demonstrated using an example system. Section 6 then unfolds the numerical results and engages in a comprehensive discussion. Lastly, Section 7 summarizes the key conclusions drawn throughout the paper.

## 2. Discussion and comparison with relevant literature

As mentioned initially, embedding fairness or justice in power distribution systems has been minimal and sporadic. On one hand, the push for fairness has only recently gained traction, while traditional energy management has primarily focused on efficiency and safety. On the other, the growing large-scale integration of distributed resources is accelerating power system decentralization. The rise of prosumers and flexibility providers has made fairness considerations more prominent. Simply adding Jain’s fairness index to the DSO’s social welfare objective, as done in [24], is insufficient to address the range of emerging challenges. Instead, diverse scenarios and technical nuances must be considered for effective implementation of this transformation.

Deriving Distribution Locational Marginal Pricing (DLMP) or congestion pricing can follow either a direct or active power flow approach. The former, as in [23], is computationally efficient, while the latter provides a more accurate system representation, as seen in [24]. The proposed Distribution Network Pricing (DNP) adopts the former approach and differs from corresponding nodal pricing in three key aspects. Firstly, objectives vary: [24] focuses on maximizing social welfare and agent fairness (using Jain’s index), while [23] seeks to minimize electricity consumption costs. In the context of substantial DER integration and distribution system decentralization, our DNP minimizes extra purchase costs. Secondly, iteration setups differ, related nodal pricing in prior models is derived in a single calculation, while our method allows iterative interaction between upper and lower models, introducing a penalty parameter  $M$  to promote operational flexibility. Thirdly, unlike prior research that treats each node as an independent aggregator or agent, our approach allows aggregation, managing variations in key metrics and facilitating a fairer distribution.

While few energy sector examples are available, fair allocation of limited resources has been explored in other fields. For instance, [33] shows that a balanced focus on equity in nonprofit resource allocation enhances welfare among low-income families. Studies such as [37] explore fairness-accuracy trade-offs in machine learning applications in resource-constrained programs, while [38] highlights fairness as a critical factor in public opinion on climate policies. As fairness gains importance across sectors, the insights gained could be highly valuable to the energy field.

## 3. Operational models of consumers and prosumers

In this research, the focus is primarily on households, encompassing both consumers and prosumers, while excluding large commercial and

industrial loads. The study aims to delve into the aggregation and operation of crowdsourcing end-users, with and without Distributed Energy Resources (DERs). The loads of these households typically fall into four categories: uncontrollable, deferrable, convertible, and diminishable. Prosumers contribute to the generation aspect, utilizing their equipped Photovoltaic (PV) panels. Additionally, within the scope of this study, convertible loads are conceptualized as heating loads that have the potential to be substituted with natural gas alternatives.

### 3.1. Modeling of consumers and prosumers

For the  $j$ th consumer/prosumer at node  $i$ , their load  $l_{ij}$  is composed of four distinct categories:

$$l_{ij} = l_{ij}^0 + l_{ij}^1 + l_{ij}^2 + l_{ij}^3 \quad (1)$$

Let  $\sigma_{ij}$  the variance of uncontrollable load to reflect its uncertainty and  $x_{ij}^k$  ( $k = 1, 2, 3$ ) the unsatisfied ratio to reflect the operation decision. Obviously,  $x_{ij}^k$  is a decision variable of prosumer/consumer, which may greatly influence its revenue/cost, user satisfactory level and interaction with counterparts. The loads of various categories can be delineated as follows:

$$l_{ij}^0 = L_{ij}^0 + \sigma_{ij} \quad (2)$$

$$l_{ij}^k = x_{ij}^k L_{ij}^k, k = 1, 2, 3 \quad (3)$$

$$0 \leq x_{ij}^k \leq 1, k = 1, 2, 3 \quad (4)$$

For another prosumer  $j'$  connecting to node  $i$ , the intention behind should be minimizing the discarded solar generation  $D_{ij'}$ , thereby enhancing overall utilization.

$$g_{ij'} = G_{ij'} + \theta_{ij'} - D_{ij'} \geq 0 \quad (5)$$

$$0 \leq G_{ij'} \leq G_{ij' \max} \quad (6)$$

### 3.2. Cost and user unsatisfactory level analysis

The decision variables,  $x_{ij}^k$ , for the prosumer/consumer play a crucial role in shaping their interactions and cost. The cost for each consumer and prosumer in a single round  $R_{ij}$  is calculated in (7). PV generation  $g_{ij}$  is set to zero for consumers.

$$R_{ij} = p_i (l_{ij} - g_{ij}) \quad (7)$$

$$L_i = \sum_j (l_{ij} - g_{ij}) \quad (8)$$

Except for the cost, various other factors come into play. For deferrable and diminishable loads, the decision to defer or diminish contributes to a reduction in the user's satisfaction level  $\eta_{ij}$ , since the deferred/diminished portion will introduce some inconvenience. In the case of convertible loads, opting for conversion entails an additional cost to the energy cost  $c_{ij}$ , which calculated as follows.

$$c_{ij} = \chi \rho x_{ij}^2 l_{ij}^2 \quad (9)$$

$$\eta_{ij} = \alpha (1 - x_{ij}^1) + \beta (1 - x_{ij}^3) + \gamma D_{ij} / G_{ij \max} + \delta (1 - x_{ij}^2) \quad (10)$$

In addition to cost and user satisfaction levels, the power system's sensitivity to the net interaction of each node is critical, reflecting the burden imposed on the distribution system. This aspect falls within the realm of congestion management. The subsequent section of the research aims to derive an efficient and equitable pricing and bidding strategy, drawing insights from congestion alleviation and nodal pricing.

## 4. Fairness and economy in pricing and dispatch

Equity involves the fair and impartial distribution of resources or costs, considering factors such as market conditions and regulatory policies. Distributive equity, often regarded as a facet of fairness and justice, will be collectively referred to as 'fairness' herein, emphasizing the advocacy for fair rights and interests. The allocation within an efficient market is inherently economical. In power dispatch, we commonly adopt the economic solution as the efficient choice, comparing it with analogous situations in other resource allocations. To ensure its effectiveness, lowering thresholds, clarifying rules, and encouraging sustained positive participation are imperative.

Simultaneously, fair prices typically entail an 'equitable distribution of benefits' resulting from exchanges between consumers and firms. In the architecture of the distributed energy market, the exchange occurs among consumers, prosumers, and the Distribution System Operator (DSO), responsible for local energy stability and system-level interactions. The system operator of the area then formulates a global optimal solution based on the system state and characteristics reported by all prosumers/consumers.

### 4.1. Upper-level model: DNP-based fair pricing

When the capacity of prosumers and flexible loads is substantial, the scheduling strategy within a node can potentially lead to an upsurge in load during low-price periods. This situation may give rise to new load peaks and, in severe instances, result in congestion on distribution lines, consequently impacting the safety and economic efficiency of the system. In response to these challenges, this study draws inspiration from node pricing and congestion management strategies and introduces a DNP model. This model aims to achieve a relatively fair and efficient economic scheduling while considering the implications of node systems.

Under this design, prosumers and consumers within the same node are assigned identical electricity prices, while the electricity price across different nodes varies based on their respective impacts on the system. The operator initiates congestion verification based on the initially reported load/generation from each node. If congestion is anticipated, the operator determines the DNP using the optimal power flow method, based on the reported initial load/generation. Subsequently, congestion prices for each node are calculated based on the determined DNP. Charges are then applied at nodes where congestion is likely, according to the results of the optimal power flow model. Following this, a secondary adjustment is conducted within the node to mitigate additional costs, naturally alleviating system congestion.

Assuming there are  $N$  nodes and  $K$  distribution lines in the distribution system, with the substation node marked as node 1, DC power flow model is employed to derive the node price of the distribution system. The optimization model is formulated as follows. Given that the power systems under study share the same level within the region, it is assumed that they utilize distribution lines of uniform material and capacity. Consequently, their channel capacities (in both directions) are set to the same as  $F_{\max}$  to simplify the calculation model. Even if capacities differ, the model remains effective with only a modification to Eq. (14).

$$\min F = \lambda L_{\text{sum}} \quad (11)$$

$$\text{s.t. } L_{\text{sum}} = \sum_{i=2}^N L_i + P_{\text{loss}} \quad (12)$$

$$F_k = \sum_{i=1}^N H_{ik} L_i, \forall k \quad (13)$$

$$-F_{\max} \leq F_k \leq F_{\max}, \forall k \quad (14)$$

Construct its Lagrange function, namely

$$L = F + \omega \left( L_{sum} - \sum_{i=2}^N L_i - P_{loss} \right) + \sum_{k=1}^K \left\{ \pi_k \left( F_k - \sum_{i=1}^N H_{ik} L_i \right) + \bar{\kappa}_k (F_k - F_{max}) + \underline{\kappa}_k (F_k + F_{max}) \right\} \quad (15)$$

By taking the partial derivatives of  $L_i$  and following the Karush-Kuhn-Tucker (KKT) optimality conditions, a set of equations is derived, as presented in (16). Additionally, another series of equations is also included, as indicated in (12) and (13).

$$\frac{\partial L}{\partial L_i} = 0, \forall i \quad (16)$$

Namely,

$$(\lambda + \omega) \frac{\partial L_{sum}}{\partial L_i} - \omega - \omega \frac{\partial P_{loss}}{\partial L_i} - \sum_{k=1}^K \pi_k H_{ik} = 0, \forall i \quad (17)$$

Denoting the optimal dispatch with a star, we can derive the corresponding DNP for each node using the equations in (17). Consequently, the optimal solution is achieved when each node incurs the same cost to adjust its aggregate load, as illustrated in (18).

$$\omega = \frac{\sum_{k=1}^K \pi_k H_{ik} - \lambda \frac{\partial L_{sum}}{\partial L_i}}{\left( \frac{\partial L_{sum}}{\partial L_i} - 1 - \frac{\partial P_{loss}}{\partial L_i} \right)}, \forall i \quad (18)$$

where  $\omega$  represents the marginal cost associated with altering the aggregate load, and when these costs are equal, there exists no opportunity for further improvement.

The partial derivative of the optimal cost function  $F^*$  for the aggregated load at node  $i$ , denoted as  $L_i$ , can be interpreted as its marginal cost, or DNP.

$$DNP_i = \frac{F^*}{\partial L_i} = \frac{\lambda}{\lambda + \omega} \left[ \omega \left( 1 + \frac{\partial P_{loss}}{\partial L_i} \right) + \sum_{k=1}^K \pi_k H_{ik} \right], \forall i \quad (19)$$

In a distribution system, the purchase price cost and load response cost carry similar weight, represented by the coefficient  $\lambda/(\lambda+\omega)$  within the entire expression. This is in contrast to traditional nodal pricing results. The DNP is structured into three components: the first signifies the marginal load adjustment cost, the second represents the marginal system power loss, and the third denotes the congestion factor. In specific scenarios where power loss can be disregarded, the optimal condition in (18) and DNP in (19) can be further simplified, as demonstrated in (20) and (21).

$$\omega = \frac{\sum_{k=1}^K \pi_k H_{ik} - \lambda \frac{\partial L_{sum}}{\partial L_i}}{\frac{\partial L_{sum}}{\partial L_i} - 1}, \forall i \quad (20)$$

$$DNP_i = \frac{F^*}{\partial L_i} = \frac{\lambda}{\lambda + \omega} \left( \omega + \sum_{k=1}^K \pi_k H_{ik} \right), \forall i \quad (21)$$

Besides, the inequality constraints outlined in (14) also need to be assessed. If no violations are detected, the electricity price at node  $i$ ,  $p_i$ , will be set to the DNP. However, if violations are identified, a penalty factor  $M$  will come into effect.

$$p_i = \begin{cases} DNP_i, & \text{no violation} \\ DNP_i + M, & \text{otherwise} \end{cases} \quad (22)$$

#### 4.2. Lower-level model: fairness integrated dispatching

Distributional equity advocates for equitable access to in-

frastructures, services, and resources for all individuals, irrespective of their background, ethnicity, and education [35]. This concept extends beyond mere accessibility and revenue/cost, encompassing undesirable outcomes or obligations, such as load curtailment, which can impact the satisfaction levels of users. When prioritizing efficiency, the optimization of the lower-level model, specifically within a node, aims to minimize the sum of costs associated with the nodes, as outlined in (7).

$$\min p_i L_i = p_i \sum_j (l_{ij} - g_{ij}) \quad (23)$$

In this scenario, when the penalty factor  $M$  is not effective (indicating no blocking), there is no requirement for flexible load adjustments from end-users. In other words, the proposed method aims to preserve the user experience unless necessary. Consequently, the objective in (23) is established in such a way that the electricity price  $p_i$  is derived from the upper level, maintaining the load at its original curve. However, in cases where blocking or congestion occurs, we must rely on internal operations within the node to bridge the gap between the suggested load  $L_i$  and the original load  $L_i$ .

Under these conditions, we formulate the lower-level model for an integrated and fair dispatch. For each node, once the load quantity and price are set, the dispatch becomes a straightforward deterministic problem. Importantly, the dispatch does not directly impact on the total revenue or cost of the node. Nevertheless, it significantly influences the user experience for each prosumer or consumer within the node, reflected in the user satisfaction level  $\eta_{ij}$  and the additional energy cost  $c_{ij}$ , as defined in Section 3.2.

The optimal solution for the overall system is achieved by minimizing the aggregate user satisfaction level and the extra energy cost. Assuming the coefficients of  $\eta_{ij}$ , as defined previously, have monetized the satisfaction level, making it additive to the extra energy cost, the objective of the problem is to minimize their algebraic sum. Simultaneously, the fairest or most equitable solution should aim to minimize the variance in dispatch. By assigning a weight  $\mu$  to the most efficient solution and the weight  $1-\mu$  to the latter, representing the one with the least controversy, the objective of the lower-level model under these new circumstances is as follows.

$$\min \mu \sum_j (\eta_{ij} + c_{ij}) + (1 - \mu) (\sigma\{\eta_{ij}\} + \sigma\{c_{ij}\}) \quad (24)$$

$$s.t. \quad \sum_j (l_{ij}^0 + l_{ij}^1 + l_{ij}^2 + l_{ij}^3 - g_{ij}) = L_i \quad (25)$$

Eqs. (1)–(10)

Suppose there are totally  $J$  prosumers and consumers at node  $i$ , then

$$\sigma\{\eta_{ij}\} = \frac{1}{J} \sum_{j=1}^J (\eta_{ij} - \bar{\eta}_i)^2 \quad (26)$$

$$\sigma\{c_{ij}\} = \frac{1}{J} \sum_{j=1}^J (c_{ij} - \bar{c}_i)^2 \quad (27)$$

where  $\bar{\eta}_i = \frac{1}{J} \sum_{j=1}^J \eta_{ij}$ ,  $\bar{c}_i = \frac{1}{J} \sum_{j=1}^J c_{ij}$ .

When the supply of natural gas is abundant, the problem in (24) can be further streamlined to:

$$\min \mu \sum_j \eta_{ij} + (1 - \mu) \sigma\{\eta_{ij}\} \quad (28)$$

The coefficient  $\mu$  is utilized to strike a balance between efficiency and fairness. Altering the value of  $\mu$  allows for different dispatch decisions. Specifically, when  $\mu=1$ , the dispatch prioritizes efficiency, while  $\mu=0$  results in a purely fair dispatch.

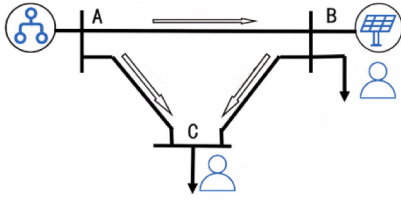


Fig. 2. Three node simple system.

## 5. Efficiency-vs- fairness aggregation method: An illustrative example

### 5.1. DNP-based fair pricing: a three-node system

In the three-node simplified system (as shown in Fig. 2), Node A serves as the substation node, while Nodes B and C function as end-user nodes. Assuming, without loss of generality, the presence of two prosumers, one flexible consumer at Node B, and two consumers at Node C. Power loss is neglected for simplicity.

At the distribution system level (upper model), the operator endeavors to optimize the power flow within the distribution system by solving the following optimization problem.

$$\min F = \lambda L_{sum} \quad (29)$$

$$s.t. \quad L_{sum} = L_B + L_C \quad (30)$$

$$F_1 = H_{1A}L_A + H_{1B}L_B + H_{1C}L_C \quad (31)$$

$$F_2 = H_{2A}L_A + H_{2B}L_B + H_{2C}L_C \quad (32)$$

$$F_3 = H_{3A}L_A + H_{3B}L_B + H_{3C}L_C \quad (33)$$

$$-F_{max} \leq F_1, F_2, F_3 \leq F_{max} \quad (34)$$

Formulate its Lagrange function and compute the partial derivatives with respect to variables and introduced factors. This yields the optimal conditions for the problem, expressed as (35), and the DNP expression as (36).

$$\frac{\sum_{k=1}^3 \pi_k H_{Bk} - \lambda \frac{\partial L_{sum}}{\partial L_B}}{\frac{\partial L_{sum}}{\partial L_B} - 1} = \frac{\sum_{k=1}^3 \pi_k H_{Ck} - \lambda \frac{\partial L_{sum}}{\partial L_C}}{\frac{\partial L_{sum}}{\partial L_C} - 1} \quad (35)$$

$$DNP_i = \frac{\lambda}{\lambda + \omega} \left[ \omega + \sum_{k=1}^K \pi_k H_{ik} \right], i = B, C \quad (36)$$

The inequality constraints in (34) also needs to be assessed. If no violations are detected, the electricity price at node  $i$ , denoted as  $p_i$ , will be set to  $DNP_i$ . However, in the presence of violations, penalty factor  $M$  will come into effect.

### 5.2. Efficiency-vs-fairness dispatch within nodes: taking nodes B and C as example

Similarly, an analysis of the conditions at nodes B and C within the 3-node system will be conducted. Commencing with node C, which exclusively features flexible loads, the scenario is relatively straightforward. When a discrepancy exists between  $L_C$  and  $L'_C$ , the operations within node C can be expressed through the following representation.

$$\min \sum_{j=1}^2 \left[ \mu \eta_{Cj} + (1 - \mu) \frac{(\eta_{Cj} - \bar{\eta}_C)^2}{2} \right] \quad (37)$$

$$s.t. \quad \sum_{j=1}^2 \left( L_{Cj}^0 + \sigma_{Cj} + \sum_{k=1}^3 x_{Cj}^k L_{Cj}^k \right) = L'_C \quad (38)$$

In the case of node B, when prosumers are present, the internal operations are similar, with consideration given to incorporating PV generation, transforming it into an energy-providing entity.

$$\min \sum_{j=1}^3 \left[ \mu \eta_{Bj} + (1 - \mu) \frac{(\eta_{Bj} - \bar{\eta}_B)^2}{3} \right] \quad (39)$$

$$s.t. \quad \sum_{j=1}^3 \left( L_{Bj}^0 + \sigma_{Bj} + \sum_{k=1}^3 x_{Bj}^k L_{Bj}^k \right) = L'_B + \sum_{j=1}^2 g_{Bj} \quad (40)$$

The aforementioned problem, encompassing its generalized formulation, is inherently convex and can be efficiently solved using optimization solvers, such as Gurobi.

## 6. NUMERICAL STUDIES

In this section, the proposed two-level aggregation framework is demonstrated via numerical studies. Various scenarios are considered at the prosumer side, and the numerical analysis underscores the potential of the proposed method in enhancing and effectively balancing fairness and efficiency.

### 6.1. Simulation settings

The depicted structure and network are presented in Fig. 1, with detailed load characteristics and generation capacity information for each consumer/prosumer available in Table 1. The line capacity  $F_{max}$  is set as 200 kW, with the impedance set to 1 (per unit system). Additionally, the day-ahead price  $\lambda$  is set as 0.5 \$/kWh.

The case studies focus on a single-time period when evaluating the proposed mechanism for maintaining energy fairness and equality. To illustrate this, the actual PV output is set in increments of 500 kW, ranging from 0 to its maximum capacity. This variation aims to emphasize the impact on DNP-based fair pricing and the trade-off between fairness and economic dispatch within nodes. Future work can incorporate uncertainty from the power output of PV generation, as calculated in [2], especially when multi-period valuation is introduced. The coefficients  $\alpha$  and  $\beta$  are uniformly set to 10, without specific attention to deferrable or diminishable loads, while  $\gamma$  is set as 100 to

Table 1  
Load characteristics and generation capacity.

Consumer /Prosumer	Load(kW)				PV Capacity
	Uncontrollable	Deferrable	Convertible	Diminishable	
B1(P)*	50	50	50	50	500
B2(P)	150	150	150	150	1000
B3(C)	0	50	50	50	-
C1(C)	75	75	75	75	-
C2(C)	25	25	25	25	-

\* (P) stands for prosumer; (C) stands for consumer.

**Table 2**  
Load transfer factors under different settings.

$H_{ik}$		Line flow		
		Line1(A→B)	Line2(A→C)	Line3(B→C)
Source	A	2/3	1/3	-1/3
	B	-1/3	1/3	2/3
	C	1/3	2/3	1/3

**Table 3**  
Typical PV generation and corresponding dispatch.

PV generation(kW)	0	250	500	750	1000	1250	1500
$L_B^*(kW)$	200	200	200	200	-50	-300	-300
$L_C^*(kW)$	200	200	200	200	325	400	400

mitigate solar curtailment. Assuming a non-restrictive natural gas supply and no additional compensation for convertible loads,  $\delta$  is set as 1 to prevent unnecessary conversion.

### 6.2. DNP-based fair pricing

With the above configurations, then  $x_{AB}=x_{BC}=x_{AC}=1$ . Solving the optimization problem specified in (29–34) and substituting the corresponding values into (35) and (36) yields the results presented in Table 2.

When the aggregated PV output is zero, the substation node becomes the sole source, resulting in congestion in the incoming direction. The boundary condition is as follows: it is evident that the distribution system can fulfill the requirements of uncontrollable components.

$$\frac{1}{3}L_C + \frac{2}{3}L_B \leq F_{\max} \quad (41)$$

$$\frac{1}{3}L_B + \frac{2}{3}L_C \leq F_{\max} \quad (42)$$

$$-F_{\max} \leq \frac{1}{3}L_C - \frac{1}{3}L_B \leq F_{\max} \quad (43)$$

Together with  $L_C+L_B=L_{\text{sum}}$ , then  $L_C=L_B=200$  kW.

When the aggregated PV output is 500 kW, the substation node remains the sole source, and PV serves the users within Node B. Despite this, a gap persists between the original curve and actual fulfillment. The distribution system proves capable of meeting the needs of uncontrollable components, as well as many deferrable, diminishable, and convertible components. However, congestion still occurs in the incoming direction, resulting in  $L_C = L_B = 200$  kW.

With an aggregated PV output of 1000 kW, both the substation node and Node B can provide power. The load within B is fully satisfied, and  $L_B = -50$  kW represents its surplus. The line capacity limits are as follows:

$$\frac{1}{3}L_{\text{sum}} + \frac{1}{3}L_B \leq F_{\max} \quad (44)$$

$$\frac{1}{3}L_{\text{sum}} - \frac{2}{3}L_B \leq F_{\max} \quad (45)$$

$$\frac{2}{3}L_{\text{sum}} - \frac{1}{3}L_B \leq F_{\max} \quad (46)$$

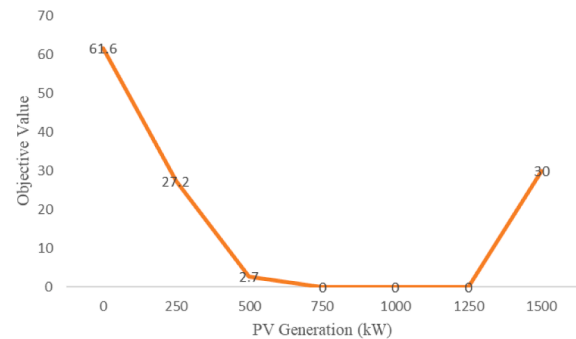
Hence,  $L_{\text{sum}} \leq 275$  kW, along with  $L_C = L_{\text{sum}} - L_B$ , the maximum  $L_C$  is 325 kW. Table 3 presents some typical PV generation values along with corresponding  $L_B$  and  $L_C$ . When end users adjust their load according to the proposed dispatch, no congestion cost will be incurred.

**Table 4**  
Dispatch in C under different  $L_C^*$  ( $\mu = 0.9$ ).

$L_C^*(kW)$	Decisions of consumer load adjustment in node C						Objective value
	$x_{C1}^1$	$x_{C1}^2$	$x_{C1}^3$	$x_{C2}^1$	$x_{C2}^2$	$x_{C2}^3$	
200	0.5	0	0.5	1	0	0	19.8
325	0.92	0	0.92	1	0	0.83	4.8
400	1	0.33	1	1	0.33	1	1.2

**Table 5**  
Dispatch in B under different  $L_B^*$  ( $\mu = 0.9$ ).

$L_B^*(kW)$	Decisions of consumer load adjustment in node B								
	$x_{B1}^1$	$x_{B1}^2$	$x_{B1}^3$	$x_{B2}^1$	$x_{B2}^2$	$x_{B2}^3$	$x_{B3}^1$	$x_{B3}^2$	$x_{B3}^3$
200	0	0	0	0	0	0	0	0	0
450	1	0	0.58	0	0	0.61	1	0	0.58
700	1	0	1	1	0	1	1	0	1
950	1	1	1	1	1	1	1	1	1



**Fig. 3.** Relationship between objective value and PV generation at node B.

### 6.3. Efficiency-vs-fairness dispatch within nodes and sensitivity analysis of $\mu$

As analyzed above, varying PV generations lead to distinct recommended  $L_B^*$  and  $L_C^*$ . In managing each node, it is crucial to distribute load adjustments among users (both prosumers and consumers) in a judicious manner. This involves minimizing the impact on aggregate social welfare while maintaining a sense of justice and fairness to encourage continued participation. The coefficient  $\mu$  is introduced to balance the tradeoff between efficiency and fairness. Initially setting  $\mu = 0.9$ , prioritizing efficiency, we derive the load adjustment plan under different PV generations.

For consumers 1 and 2 in C, we compare their dispatch plans under three potential aggregate dispatch values from Table 3, namely 200, 325, and 400. The load adjustment results are presented in Table 4.

For prosumers 1 and 2, as well as consumer 3 in B, we compare their dispatch plans under four potential aggregate dispatch values for the net load part from Table 3, namely 200, 450, 700, and 950. The load adjustment results are detailed in Table 5. Node B transitions into generator mode upon reaching its full load capacity, maintaining an objective value of 0 until solar curtailment occurs. The objective value of the dispatch model at this point can be observed in Fig. 3. Notably, solar curtailment occurs when PV output is 1500 kW. To minimize variance, curtailment is dispatched in inverse proportion to their capacity ( $D_{B1} = 83.3$  kW,  $D_{B2} = 166.7$  kW), as evident from Table 2.

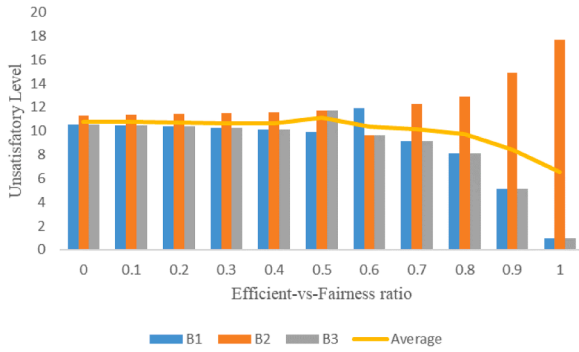
Different values of  $\mu$  result in distinct dispatch decisions. In this comparison, three representative PV generations and various  $\mu$  values are utilized. Without loss of generality,  $L_B^*$  and  $L_C^*$  are set to 450 and 325 kW, respectively, when the adjustment becomes necessary and competitive.

For node B, with  $L_B^* = 450$  kW, the dispatch plan within B for both



**Table 6**  
Dispatch in B under different  $\mu$ .

Decision	$\mu$										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$x_{B1}^1$	1	1	1	1	1	1	1	1	1	1	1
$x_{B1}^2$	0	0	0	0	0	0	0	0	0	0	0
$x_{B1}^3$	0.05	0.055	0.06	0.07	0.09	0.11	0.14	0.19	0.29	0.58	1
$x_{B2}^1$	0	0	0	0	0	0	0	0	0	0	0
$x_{B2}^2$	0	0	0	0	0	0	0	0	0	0	0
$x_{B2}^3$	0.97	0.96	0.96	0.95	0.94	0.93	0.91	0.875	0.81	0.61	0.33
$x_{B3}^1$	1	1	1	1	1	1	1	1	1	1	1
$x_{B3}^2$	0	0	0	0	0	0	0	0	0	0	0
$x_{B3}^3$	0.05	0.055	0.06	0.07	0.09	0.11	0.14	0.19	0.29	0.58	1



**Fig. 4.** Trend of  $\eta$  (separate and average) upon  $\mu$  in B.

consumers and prosumers is examined, varying  $\mu$  from 0 to 1 in increments of 0.1. The dispatch results under different  $\mu$  values are detailed in Table 6, and the trend of  $\eta$  (both separate and average values) based on  $\mu$  is illustrated in Fig. 4. Simultaneously, the dispatch plan within C among consumers is evaluated, with  $L_C^* = 325$  kW, employing the same  $\mu$  settings. The corresponding dispatch results under different  $\mu$  values are presented in Table 7, and the trend of  $\eta$  (both separate and average values) based on  $\mu$  is depicted in Fig. 5.

Examining the trend in the load adjustment plan, as illustrated in Tables 3 and 4, it becomes apparent that the efficiency-versus-fairness ratio has a quasi-linear or quasi-quadratic impact on  $\eta$  within a specific interval. Hence, it is reasonable to infer that this ratio plays a pivotal role as a primary determinant of the boundary within the feasible region of the optimal solution.

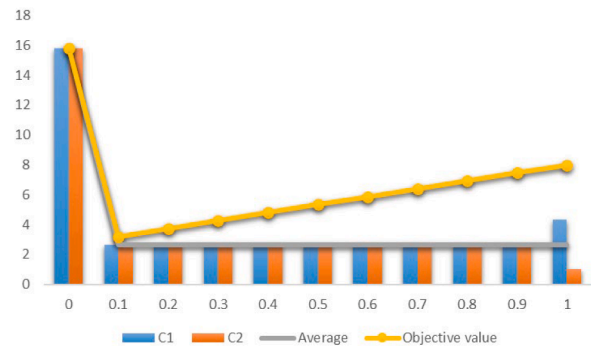
The values of  $x_{B1}^1 + x_{B1}^3$  and  $x_{B3}^1 + x_{B3}^3$  decrease from 2 to 1.05, while that of  $x_{B2}^1 + x_{B2}^3$  increases from 0.33 to 0.97, as  $\mu$  varies from 1 to 0. This indicates that the increase in emphasis on fairness enhances the fairness index for smaller users, while the efficiency-focused plan prioritizes users with larger capacity by its nature.

The gradient of the average unsatisfactory level and the objective value also reveals noteworthy characteristics. As evident from both Figs. 4 and 5, a substantial improvement in the fairness index is observed when the value of  $\mu$  changes from 1 to 0.9. A similar trend appears when

$1 - \mu$  varies from 1 to 0.9 due to its inherent functional relationships. However, the significant improvement in fairness index at the initial step (without continuing to the next step) holds particular importance in system planning and operation considerations for equity and fairness. This highlights a crucial feature, indicating that a minimal input on fairness can result in a profound improvement in energy justice.

### 7. Conclusion

This paper proposed an effective and equitable aggregation mechanism for practices of both energy suppliers and consumers in energy utilization, cultivating flexibility in both prosumers and consumers. The proposed DNP-based pricing effectively evaluates the difference in location and congestion impact, with interaction with the actual dispatch in the lower level. Traditional dispatch is normally directly determined by the corresponding locational price, but in this paper, we have taken in the consideration of fairness or justice, with the factor efficiency-vs-fairness ratio as other significant attempts in other fields. Sensitivity analysis on efficiency-vs-fairness ratio further reveals that even a modest emphasis on equity indicators can significantly enhance energy fairness, benefiting numerous end-users without causing a substantial impact on system efficiency. This underscores the importance of striking a balance between fairness and efficiency in the realm of energy



**Fig. 5.** Trend of  $\eta$  (separate and average) upon  $\mu$  in C.

**Table 7**  
Dispatch in C under different  $\mu$ .

Decision	$\mu$										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$x_{C1}^1$	0.25	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	1
$x_{C1}^2$	0.23	0	0	0	0	0	0	0	0	0	0
$x_{C1}^3$	0.25	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.67
$x_{C2}^1$	0.27	1	1	1	1	1	1	1	1	1	1
$x_{C2}^2$	0.23	0	0	0	0	0	0	0	0	0	0
$x_{C2}^3$	0.22	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.83	0.83	1

public utilities. These insights contribute to the cultivation and retention of prosumers and flexible end-users, concurrently elevating energy justice and equity. Moreover, this study primarily focuses on a single period and a single higher-level power grid. Future work aims to expand this focus to encompass multiple time periods and more intricate grid structures.

### CRedit authorship contribution statement

**Bomiao Liang:** Writing – original draft, Methodology, Formal analysis, Conceptualization. **Jiajia Yang:** Writing – review & editing, Investigation, Formal analysis. **Fushuan Wen:** Writing – review & editing, Methodology, Conceptualization. **Licheng Wang:** Validation, Resources. **Zhao Yang Dong:** Validation, Resources.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Data availability

Data will be made available on request.

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