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Leader-Following Consensus of Multiple Uncertain Euler-Lagrange Systems via Fully Distributed Event-Triggered Adaptive Fuzzy Control

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***Abstract*—This paper deals with the leader-following consensus problem of multiple uncertain Euler-Lagrange systems with unknown nonlinear dynamics. By introducing a dynamic compensator for each agent, a fully distributed control strategy is developed based on the fuzzy approximation approach, which is independent of any priori global information associated with the communication topology. Meanwhile, a distributed event-triggering mechanism is designed such that each agent broadcasts its states only when an event occurs. It is shown that with the proposed event-triggering mechanism, leader-following consensus is achieved with aperiodic intermittent communication and Zeno behavior is excluded by contradiction. Moreover, the consensus tracking errors converge to small sets around the origin. Finally, an example is provided to illustrate the effectiveness of the obtained theoretical results.**

***Index Terms*—Adaptive fuzzy control, multiagent systems, uncertain Euler-Lagrange systems, leader-following consensus, event-triggered control.**

I. Introduction

Euler-Lagrange (EL) systems are a class of typical nonlinear systems and can be used to model lots of practical systems, such as unmanned aerial vehicles (UAVs), robotic manipulators and autonomous surface vehicles (ASVs). Inspired by the broad applications of networked mobile robots, cooperative control of a group of EL systems has been intensively studied by many researchers in the past few years, see, for example, [1]–[13]. This paper focuses on the consensus problem of uncertain EL multiagent systems (MASs).

So far, there have been two types of consensus problems, i.e., the leaderless consensus problem (LCP) and the leader-following consensus problem (LFCP). In the former one, all agents eventually reach an agreement on their common states or outputs by designing distributed control protocols, while in the latter one, there exists a leader that specifies the control objective for all followers. There have been some recent works on consensus of multiple EL systems [3]–[7], [12]. In particular, the author in [4] considered the leaderless consensus problem of multiple EL systems by proposing several distributed control algorithms. Then, leader-following consensus was achieved in [6] for multiple EL systems, where the state of the leader system is not required to be accessible to all followers. Later in [7], the same problem was studied for EL systems subject to switching networks and communication delays. Recently, the coordination problem of multiple ASVs was further investigated in [8] and [13] by employing distributed control protocols.

It is worth noting that the aforementioned literatures focus on control algorithm development in a continuous-time setting. This requires continuous communication among agents, which will inevitably lead to large energy consumption and communication burden, and thus shorten the operation life of the whole system. To improve communication efficiency while maintaining the control performance, event-triggered control has attracted a lot of attention for their superiority in reducing the amount of data transmission [14]. With event-triggered control strategies, the data transmission among agents occurs only when certain predefined conditions are satisfied, which is expected to use resources more efficiently. Over the past decade, a considerable amount of related literatures have been published for single systems [15]–[18] and for multiagent systems with simple agent dynamics such as single- or double-integrator [19], [20]. Then, consensus problems for general linear multiagent systems were discussed in [21]–[27] via event-triggered and self-triggered strategies, respectively. Unfortunately, however, the above literature [19]–[27] cannot be utilized to analyze the LFCP of multiple uncertain EL systems via event-triggered strategies.

More recently, there have been some results on event-triggered consensus of multiple uncertain EL systems [28]–[33]. In particular, the authors in [28] presented model-independent controllers to achieve consensus for a network of EL systems with a stationary leader via distributed event-triggered strategies. A distributed control protocol was developed in [32] to address the event-triggered formation control problem of EL MASs with real experiments. In [33], the event-triggered bipartite consensus problem of uncertain EL MASs was solved in the presence of external disturbances. However, the results in [28]–[33] rely on a common assumption that the nonlinearities in agent dynamics are linearly parameterized. To remove such a restrictive assumption, approximator-based modeling techniques, for instance, neural networks (NNs) [34] and fuzzy logic systems (FLSs) [35]–[38], have been

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employed. In fact, adaptive NN/fuzzy control has been widely utilized for both single nonlinear systems and nonlinear multiagent systems with completely unknown nonlinearities, see, e.g., [39]–[45]. In particular, the authors in [44] used adaptive fuzzy control to solve the event-triggered containment control problem of multiple uncertain EL systems. In [45], a distributed event-triggered NN controller was developed to address the containment control problem for multiple EL systems with dynamic leaders. It is however noted that the existing event-triggered control strategies developed in [28]–[33], [44] and [45] for multiple EL systems still suffer from some limitations. To be specific, with the proposed event-triggering functions in [28]–[33] and [45], each agent has to monitor the triggering condition continuously. Consequently, the communication burden among neighboring agents cannot be reduced as expected. Besides, some global information, in terms of the eigenvalue information of Laplacian matrix, is required in [28]–[33], [44] when designing the distributed controller, which might bring difficulties in synthesizing the control law, especially for large-scale networks. Despite the significance of designing fully distributed event-triggered adaptive fuzzy control strategies for multiple uncertain EL systems, so far, there are few results on this topic, which motivates the current study.

This paper aims to study the LFCP of uncertain EL MASs subject to a dynamic leader system via distributed event-triggered adaptive fuzzy control. Due to the existence of non-identical nonlinearities, the uncertain EL multiagent system considered in this paper is essentially heterogeneous. A novel distributed event-triggered control scheme is developed, which is composed of a distributed event-triggering mechanism (ETM) and a distributed adaptive fuzzy controller. It is shown that the LFCP of uncertain EL multiagent systems can be solved with the proposed control approach. The main contributions are summarized as follows.

First, a novel Zeno-free distributed event-triggered adaptive fuzzy controller and a distributed ETM with an adaptive threshold are developed. With the proposed event-triggered control protocol, the information exchange among agents is significantly reduced. Moreover, FLSs are employed to approximate the unknown nonlinear functions so that unstructured nonlinearities can be handled.

Second, in comparison to the existing works on event-triggered consensus of multiple uncertain EL systems, such as [28]–[33], [44], both the distributed control protocol and the distributed ETM developed in this paper are fully distributed. No global information of the network is required to construct the distributed control law or the distributed ETM.

Third, compared with the most recent works [45], an event-triggered dynamic compensator with an adaptive coupling gain is designed for each agent. With this design, the distributed event-triggered adaptive fuzzy control scheme proposed in this work does not need continuous communications among neighboring agents to either update the controller or monitor the event-triggering condition. The frequency of data transmission is thus reduced.

II. Preliminaries and Problem Statement

A. Notations and Algebraic Graph Basics

The symbol $\otimes$ denotes the Kronecher product. $\mathbf{I}_n$ and $\mathbf{0}_{m\times n}$ denote the $n\times n$ identity matrix and $m\times n$ zero matrix, respectively. $\|\theta\|$ denotes the Euclidean norm of a vector θ. $\|\Theta\|$ denotes the Euclidean norm of a matrix Θ. $\lambda_{\min}(\Theta)$ and $\lambda_{\max}(\Theta)$ represent the minimum eigenvalue and the maximum eigenvalue of Θ, respectively. Denote the upper Dini derivative of $\mathbf{v}(t)$ as $\mathrm{D}^+\mathbf{v}(t) = \limsup_{\Delta\to 0^+}\frac{\mathbf{v}(t+\Delta)-\mathbf{v}(t)}{\Delta}$. For simplicity, define $\dot{\mathbf{v}}(t) = \mathrm{D}^+\mathbf{v}(t)$ if no confusion arises.

$\mathcal{G}=(\mathcal{V},\mathcal{E})$ is adopted to represent a graph, where $\mathcal{V}=\{1,\cdots,\mathscr{N}\}$ represents the node set of $\mathscr{N}$ agents, and the edge set is denoted by $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{\mathscr{N}\times\mathscr{N}}$ denotes the adjacency matrix corresponding to the graph $\mathcal{G}$, where $a_{ij}=1$ means agent i can receive data from agent j (i.e., $(j,i)\in\mathcal{E}$); otherwise, $a_{ij}=0$. The Laplacian matrix is defined as $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{\mathscr{N}\times\mathscr{N}}$, where $l_{ij}=-a_{ij}$ if $i\neq j$; otherwise, $l_{ii}=\sum_{j=1}^{\mathscr{N}}a_{ij}$. Agent i's neighbor set is represented by $\mathscr{N}_i=\{j\in\mathcal{V},(j,i)\in\mathcal{E}\}$.

To study the LFCP, define another graph $\bar{\mathcal{G}}$ consisting of one leader (node 0) and $\mathscr{N}$ agents. If agent i can access the leader directly, then $a_{i0}=1$, $i\in\mathcal{V}$, otherwise, $a_{i0}=0$. Denote $\mathcal{D}=\mathrm{diag}\{a_{10},\cdots,a_{\mathscr{N}0}\}\in\mathbb{R}^{\mathscr{N}\times\mathscr{N}}$ and $\mathcal{H}=\mathcal{L}+\mathcal{D}$ [46].

B. Problem Statement

Consider a group of $\mathscr{N}$ EL systems with their dynamics in the following form [47],

$$M_i(q_i)\ddot{q}_i + N(q_i,\dot{q}_i) = \tau_i,\ i=1,\cdots,\mathscr{N}, \tag{1}$$

where $q_i\in\mathbb{R}^n$ and $\dot{q}_i\in\mathbb{R}^n$ are the generalized position vector and velocity vector, respectively, $\tau_i\in\mathbb{R}^n$ is the generalized control input vector. $M_i(q_i)=M_i^T(q_i)>0\in\mathbb{R}^{n\times n}$ is the inertia matrix, $N(q_i,\dot{q}_i)=V_{mi}(q_i,\dot{q}_i)\dot{q}_i+F_{fi}(q_i,\dot{q}_i)+G_i(q_i)$ with $V_{mi}(q_i,\dot{q}_i)\dot{q}_i$ being the Coriolis/centripetal vector, $F_{fi}(q_i,\dot{q}_i)$ being the friction and disturbance vector, and $G_i(q_i)$ being the gravity vector, all of which are smooth and unknown due to the uncertain environment or unknown physical parameters. The matrix $\dot{M}_i(q_i)-2V_{mi}(q_i,\dot{q}_i)$ in EL system (1) is skew symmetric.

The reference signal is generated by the following linear autonomous system,

$$\begin{bmatrix}\dot{v}_0\\ \ddot{v}_0\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{n\times n} & \mathbf{I}_n\\ F_1 & F_2\end{bmatrix}\begin{bmatrix}v_0\\ \dot{v}_0\end{bmatrix} = S\begin{bmatrix}v_0\\ \dot{v}_0\end{bmatrix}, \tag{2}$$

where $v_0\in\mathbb{R}^n$ is the generalized position vector and $\dot{v}_0\in\mathbb{R}^n$ is the generalized velocity vector. F_1 and F_2 are pre-given matrices.

Let $x_i=\mathrm{col}(q_i,\dot{q}_i)$, $i\in\mathcal{V}$ and $x_0=\mathrm{col}(v_0,\dot{v}_0)$. Our objective is to design a fully distributed event-triggered adaptive fuzzy control strategy to solve the LFCP defined as follows:

Definition 1. Given the uncertain EL multiagent systems consisting of (1), (2), as well as the digraph $\bar{\mathcal{G}}$, for any $x_i(0)\in\Omega$, a compact subset of $\mathbb{R}^{2n}$, if the agents' states satisfy

$$\lim_{t\to\infty}\|q_i-v_0\|\leq\varepsilon_0,\ \lim_{t\to\infty}\|\dot{q}_i-\dot{v}_0\|\leq\delta_0,\ \forall i\in\mathcal{V} \tag{3}$$

where ε_0 and δ_0 are adjustable parameters, then the LFCP is said to be solved.

The following assumptions are necessary:

Assumption 1. There exist positive constants d_m, d_M such that $d_m\mathbf{I}_n \leq M_i(q_i) \leq d_M\mathbf{I}_n$.

Assumption 2. Graph $\mathcal{G}$ is undirected and node 0 is globally reachable in $\bar{\mathcal{G}}$.

Assumption 3. The matrix S of the leader system (2) has no eigenvalues with negative real parts.

Remark 1: Assumptions 1-2 are fairly standard in the coordination of multiple Euler-Lagrange system [5]–[7], [47]. We use $\lambda_1, \cdots, \lambda_{\mathscr{N}}$ to denote the eigenvalues of $\mathcal{H}$, then all the eigenvalues of $\mathcal{H}$ have positive real parts. Since graph $\mathcal{G}$ is undirected, the matrix $\mathcal{H}$ is positive definite.

Remark 2: In this paper, a linear autonomous system under Assumption 3 is adopted as the leader system, which is more general than the assumption that the leader system is marginally stable or $\dot{v}_0$ is a constant as required in [5]–[7]. Assumption 3 is made only for convenience and without loss of generality. In fact, if Assumption 3 is violated, the responses related to the eigenvalues of S with negative real parts will delay to zero exponentially and will not have any impact on the consensus errors in steady state.

C. FLSs

In what follows, the universal approximation capability of FLSs is introduced, which is summarized in Lemma 1.

Lemma 1 [35]: Let Σ be a compact set in $\mathbb{R}^g$ and let $f(\zeta)$ be a real continuous function defined on Σ. Then for any $\epsilon^* > 0$, there exists an FLS $W^T\varphi(\zeta)$ such that

$$\sup_{\zeta\in\Sigma} |f(\zeta) - W^T\varphi(\zeta)| \leq \epsilon^*, \tag{4}$$

where $\zeta = [\zeta_1, \cdots, \zeta_g]^T \in \mathbb{R}^g$ is the input of an FLS, $W^T = [W_1, \cdots, W_r]$ is the weighting vector, and $\varphi(\zeta) = [\varphi_1, \cdots, \varphi_r]^T$ is the vector of fuzzy basis functions with r being the number of fuzzy "IF-THEN" rules. In addition, the fuzzy basis function satisfies

$$\varphi_l(\zeta) = \frac{\prod_{i=1}^{g} \mu_{\mathcal{F}_i^l}(\zeta_i)}{\sum_{l=1}^{r}\left[\prod_{i=1}^{g} \mu_{\mathcal{F}_i^l}(\zeta_i)\right]}, l \in \mathscr{L} := \{1, \cdots, r\}$$

with $\mu_{\mathcal{F}_i^l}(\zeta_i)$ being the Gaussian membership functions.

Remark 3: As introduced in Lemma 1, the function approximation capability of FLSs is established in a compact set, which implies the boundedness of the initial conditions. Thus, it is worth mentioning that the obtained stability results of this work are semi-global. It is also true for other approximation-based adaptive control approaches. For simplicity, the concept of semi-global boundedness will be omitted in the rest of this paper.

III. Main Results

A distributed event-triggered adaptive fuzzy control law will be presented in this section to solve the LFCP of the MAS consisting of (1) and (2).

A. Controller Design

To design the distributed controller, a dynamic compensator of the following form is developed:

$$\begin{aligned}
&\begin{bmatrix} \dot{v}_{i1}(t) \\ \dot{v}_{i2}(t) \end{bmatrix} = S\begin{bmatrix} v_{i1}(t) \\ v_{i2}(t) \end{bmatrix} - \mu_i(t)\begin{bmatrix} \hat{\eta}_{i1}(t) \\ \hat{\eta}_{i2}(t) \end{bmatrix}, \\
&\begin{bmatrix} \hat{\eta}_{i1}(t) \\ \hat{\eta}_{i2}(t) \end{bmatrix} = \\
&\begin{bmatrix} \sum\limits_{j\in\mathscr{N}_i} a_{ij}(\hat{v}_{i1}(t) - \hat{v}_{j1}(t)) + a_{i0}(\hat{v}_{i1}(t) - v_0(t)) \\ \sum\limits_{j\in\mathscr{N}_i} a_{ij}(\hat{v}_{i2}(t) - \hat{v}_{j2}(t)) + a_{i0}(\hat{v}_{i2}(t) - \dot{v}_0(t)) \end{bmatrix}, \\
&\dot{\mu}_i(t) = \gamma_i\hat{\eta}_i(t)^T\hat{\eta}_i(t) - \sigma_i\mu_i(t),
\end{aligned} \tag{5}$$

where $v_{il}(t) \in \mathbb{R}^n$, $l = 1, 2$, are the states of the dynamic compensator, $\hat{v}_{i1}(t)$, $\hat{v}_{i2}(t)$, $\hat{v}_{j1}(t)$, and $\hat{v}_{j2}(t)$ are the estimates of $v_{i1}(t)$, $v_{i2}(t)$, $v_{j1}(t)$, and $v_{j2}(t)$, respectively. $\hat{\eta}_i(t) = \mathrm{col}(\hat{\eta}_{i1}(t), \hat{\eta}_{i2}(t))$, $\mu_i(t)$ is the adaptive coupling gain satisfying $\mu_i(0) \geq 0$, γ_i and σ_i are two scalars, and σ_i serves the same role as "σ- modification" to avoid parameter drift [48]. Let $v_i(t) = \mathrm{col}(v_{i1}(t), v_{i2}(t))$ and $\hat{v}_i(t) = \mathrm{col}(\hat{v}_{i1}(t), \hat{v}_{i2}(t))$ for $i = 1, \cdots, \mathscr{N}$.

For each agent i, denote $\{t_k^i, k \in \mathbb{Z}\}$ as its k-th triggering instant with $\mathbb{Z} := \{0, 1, 2, \cdots\}$, then $\hat{v}_i$ is governed by

$$\begin{aligned} &\dot{\hat{v}}_i(t) = S\hat{v}_i(t), \ t \in [t_k^i, t_{k+1}^i), \\ &\hat{v}_i(t_k^i) = v_i(t_k^i). \end{aligned} \tag{6}$$

Once an event occurs for each agent i at $t = t_k^i$, the open-loop estimate of its compensator state, $\hat{v}_i$, is immediately updated based on the sampled compensator state $v_i(t_k^i)$. For $t \in [t_k^i, t_{k+1}^i)$, define the measurement error e_i for agent i as $e_i(t) = \hat{v}_i(t) - v_i(t)$. Then the next triggering time instant is determined by

$$t_{k+1}^i = \inf\left\{t > t_k^i | g_i\left(e_i(t), \mu_i(t), t\right) \geq 0\right\}, \tag{7}$$

$$g_i\left(e_i(t), \mu_i(t), t\right) = (\mu_i(t) + 1)e_i^T(t)e_i(t) - m_1\mathrm{e}^{-m_2 t}, \tag{8}$$

where $m_1 > 0$ and $m_2 > 0$ are positive constants.

Let

$$\dot{z}_i = v_{i2} - \alpha(q_i - v_{i1}), \tag{9}$$

where the scalar $\alpha > 0$, and let

$$s_i = \dot{q}_i - \dot{z}_i = \dot{q}_i - v_{i2} + \alpha(q_i - v_{i1}). \tag{10}$$

Then, the distributed event-triggered adaptive fuzzy control law of agent i is proposed in the following form, for $t \in [t_k^i, t_{k+1}^i)$,

$$\tau_i = -l_i s_i - \frac{\hat{\theta}_i\varphi_i^T\varphi_i s_i}{4b_i^2}, \tag{11}$$

$$\dot{\hat{\theta}}_i = \frac{\rho_i\varphi_i^T\varphi_i s_i^T s_i}{4b_i^2} - \phi_i\hat{\theta}_i, \tag{12}$$

where $l_i > 0$, $\rho_i > 0$, $\phi_i > 0$, and b_i are design parameters. $\hat{\theta}_i$ is the estimate of the optimal parameter θ_i^* to be given later. Moreover, $\hat{\theta}_i(t) \geq 0$ as long as $\hat{\theta}_i(0) \geq 0$.

Remark 4: As compared to the most recent literature [45], a dynamic compensator of the form (5) is designed for each follower to help construct the control law, which utilizes the compensator state v_i, and the estimates $\hat{v}_i$ and $\hat{v}_j$, $j \in \mathscr{N}_i$, instead of the neighboring states v_j. With the proposed ETM, agent i broadcasts signals to its neighbors only when the triggering condition (7) is satisfied, thus communication among neighbors are aperiodic intermittent, which significantly reduces the amount of information exchange among them.

Remark 5: Instead of using the fixed gain, an adaptive coupling gain $\mu_i(t)$, partly inspired by [23], is introduced in the compensator. However, the designed compensator (5) is different from that in [23] which cannot be used to tackle unstructured uncertainties and external disturbances under consideration. By designing a dynamic gain, no global information, such as $\min R(\lambda_i)$ in [44] or the network's scale, is required when designing the distributed control protocol, and thus the proposed control law (11) is implemented in a fully distributed manner.

B. Stability Analysis

The main result of this paper is provided as follows.

Theorem 1: Suppose Assumptions 1-2 hold. Consider the graph $\bar{\mathcal{G}}$, the multiple uncertain EL systems (1), and the leader system (2). With the proposed distributed ETM (7) and the distributed adaptive fuzzy control law (11), the LFCP of uncertain EL multiagent systems can be solved, and the boundedness of all signals in the closed-loop system is guaranteed.

Proof: Define the state transformation

$$\bar{v}_i = v_i - x_0,\ i = 1, \cdots, \mathscr{N}. \tag{13}$$

Substituting the controller (11) into (1), for $t \in [t_k^i, t_{k+1}^i)$, the following closed-loop system is obtained,

$$\begin{aligned}\dot{\bar{v}}_i =& S\bar{v}_i - \mu_i \sum_{j\in\mathscr{N}_i} a_{ij}(\bar{v}_i - \bar{v}_j) + a_{i0}\bar{v}_i \\ & - \mu_i \sum_{j\in\mathscr{N}_i} a_{ij}(e_i - e_j) + a_{i0}e_i, \\ M_i\dot{s}_i =& - V_{mi}s_i - l_is_i - \frac{\hat{\theta}_i\varphi_i^T\varphi_is_i}{4b_i^2} + f_i(X_i), \\ \dot{\mu}_i =& \gamma_i\hat{\eta}_i^T\hat{\eta}_i - \sigma_i\mu_i, \\ \dot{\hat{\theta}}_i =& \frac{\rho_i\varphi_i^T\varphi_is_i^Ts_i}{4b_i^2} - \phi_i\hat{\theta}_i,\end{aligned} \tag{14}$$

where $f_i(X_i) = -F_{fi}(q_i, \dot{q}_i) - G_i(q_i) - M_i(q_i)\ddot{z}_i - V_{mi}(q_i, \dot{q}_i)\dot{z}_i$, and $X_i = \mathrm{col}(q_i, \dot{q}_i, v_i)$.

Denote $\bar{v} = \mathrm{col}(\bar{v}_1, \cdots, \bar{v}_{\mathscr{N}})$, $e = \mathrm{col}(e_1, \cdots, e_{\mathscr{N}})$ and $\mu(t) = \mathrm{diag}\{\mu_1(t), \cdots, \mu_{\mathscr{N}}(t)\}$. Then the following compact form is given,

$$\begin{aligned}\dot{\bar{v}} =& [(\mathbf{I}_{\mathscr{N}} \otimes S) - \mu(t)(\mathcal{H}\otimes\mathbf{I}_{2n})]\bar{v} \\ & - \mu(t)(\mathcal{H}\otimes\mathbf{I}_{2n})e.\end{aligned} \tag{15}$$

Consider the following Lyapunov-like function,

$$V = V_1 + V_2 + V_3 + V_4, \tag{16}$$

where $V_1 = \bar{v}^T(\mathcal{H}\otimes\mathbf{I}_{2n})\bar{v}$, $V_2 = \sum_{i=1}^{\mathscr{N}} \frac{1}{2}s_i^TM_is_i$, $V_3 = \sum_{i=1}^{\mathscr{N}} \frac{1}{4\gamma_i}\tilde{\mu}_i^2$, $V_4 = \sum_{i=1}^{\mathscr{N}} \frac{1}{2\rho_i}\tilde{\theta}_i^2$, $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$, $\tilde{\mu}_i = \mu_i - a_1$, and a_1 is a constant design parameter to be given later.

Define a strictly increasing sequence of the triggering instants $\{t_r : r \in \mathbb{Z}\} = \cup_{i=1}^{\mathscr{N}}\{t_k^i : i \in \mathcal{V},\ k \in \mathbb{Z}\}$ for the whole uncertain EL MAS at which at least one agent is triggered. Then, for $t \in [t_r, t_{r+1})$, taking the time derivative of V_1 along the trajectory of (15) gives

$$\begin{aligned}\dot{V}_1 =& 2\bar{v}^T(\mathcal{H}\otimes\mathbf{I}_{2n})\dot{\bar{v}} \\ =& 2\bar{v}^T[(\mathcal{H}\otimes S) - (\mathcal{H}\mu(t)\mathcal{H}\otimes\mathbf{I}_{2n})]\bar{v} \\ & - 2\bar{v}^T(\mathcal{H}\mu(t)\mathcal{H}\otimes\mathbf{I}_{2n})e \\ \leq& \bar{v}^T\left[\mathcal{H}\otimes\left(S^T + S\right) - (\mathcal{H}\mu(t)\mathcal{H}\otimes\mathbf{I}_{2n})\right]\bar{v} \\ & + e^T(\mathcal{H}\mu(t)\mathcal{H}\otimes\mathbf{I}_{2n})e.\end{aligned} \tag{17}$$

The derivative of V_2 over each time interval $[t_r, t_{r+1})$ is given by

$$\begin{aligned}\dot{V}_2 =& \sum_{i=1}^{\mathscr{N}} s_i^TM_i\dot{s}_i + \sum_{i=1}^{\mathscr{N}} \frac{1}{2}s_i^T\dot{M}_is_i \\ =& \sum_{i=1}^{\mathscr{N}} s_i^T\left[-V_{mi}s_i - l_is_i - \frac{\hat{\theta}_i\varphi_i^T\varphi_is_i}{4b_i^2} + f_i\right] \\ & + \sum_{i=1}^{\mathscr{N}} \frac{1}{2}s_i^T\dot{M}_is_i \\ \leq& - \sum_{i=1}^{\mathscr{N}} l_is_i^Ts_i + \sum_{i=1}^{\mathscr{N}} \|s_i\|\|f_i\| - \sum_{i=1}^{\mathscr{N}} \frac{\hat{\theta}_i\varphi_i^T\varphi_is_i^Ts_i}{4b_i^2}.\end{aligned} \tag{18}$$

In accordance with Lemma 1, one can use fuzzy systems to approximate the unknown term $\chi_i(X_i) = \|f_i(X_i)\|$ with input vector $X_i \in \Omega_{X_i}$, where Ω_{X_i} is a compact set. Then one has

$$\chi_i(X_i) = W_i^{*T}\varphi_i(X_i) + \epsilon_i, \tag{19}$$

$$W_i^* = \arg\min_{W_i}\left[\sup_{X_i\in\Omega_{X_i}} |\chi_i(X_i|W_i) - \chi_i(X_i)|\right], \tag{20}$$

where the optimal parameter is denoted by W_i^*, and its estimate is denoted by W_i, the approximation error is denoted by ϵ_i and satisfies $|\epsilon_i| \leq \epsilon_i^*$ with $\epsilon_i^* > 0$.

Noting that

$$\begin{aligned}\|s_i\|\chi_i \leq& \frac{s_i^Ts_iW_i^{*T}W_i^*\varphi_i^T(X_i)\varphi_i(X_i)}{4b_i^2} + b_i^2 \\ & + \frac{1}{2}s_i^Ts_i + \frac{1}{2}\epsilon_i^2 \\ =& \frac{s_i^Ts_i\theta_i^*\varphi_i^T(X_i)\varphi_i(X_i)}{4b_i^2} + b_i^2 \\ & + \frac{1}{2}s_i^Ts_i + \frac{1}{2}\epsilon_i^2,\end{aligned} \tag{21}$$

where $\theta_i^* = W_i^{*T}W_i^*$, and b_i is a design parameter, then (18) becomes

$$\begin{aligned}\dot{V}_2 \leq & -\sum_{i=1}^{\mathscr{N}} l_i s_i^T s_i - \sum_{i=1}^{\mathscr{N}} \frac{\hat{\theta}_i \varphi_i^T \varphi_i s_i^T s_i}{4b_i^2} \\ & + \sum_{i=1}^{\mathscr{N}} \left(\frac{s_i^T s_i \theta_i^* \varphi_i^T \varphi_i}{4b_i^2} + b_i^2 + \frac{1}{2} s_i^T s_i + \frac{1}{2}\epsilon_i^2 \right) \\ = & -\sum_{i=1}^{\mathscr{N}} l_i s_i^T s_i + \sum_{i=1}^{\mathscr{N}} \frac{\tilde{\theta}_i \varphi_i^T \varphi_i s_i^T s_i}{4b_i^2} \\ & + \sum_{i=1}^{\mathscr{N}} \left(b_i^2 + \frac{1}{2} s_i^T s_i + \frac{1}{2}\epsilon_i^2 \right). \end{aligned} \tag{22}$$

Similarly, the derivative of V_3 on each time interval $[t_r, t_{r+1})$ is obtained as

$$\begin{aligned}\dot{V}_3 = & \sum_{i=1}^{\mathscr{N}} \frac{1}{2\gamma_i} \tilde{\mu}_i \left(\gamma_i \hat{\eta}_i^T \hat{\eta}_i - \sigma_i \mu_i \right) \\ = & \sum_{i=1}^{\mathscr{N}} \frac{1}{2} (\mu_i - a_1) \hat{\eta}_i^T \hat{\eta}_i - \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{2\gamma_i} \tilde{\mu}_i \mu_i. \end{aligned} \tag{23}$$

According to the definition of $\hat{\eta}_i$ in (5), one has

$$\begin{aligned}\dot{V}_3 = & \sum_{i=1}^{\mathscr{N}} \frac{1}{2} \mu_i \left[\mathcal{H}_i \bar{v} + \mathcal{H}_i e \right]^T \left[\mathcal{H}_i \bar{v} + \mathcal{H}_i e \right] \\ & - \sum_{i=1}^{\mathscr{N}} \frac{a_1}{2} \hat{\eta}_i^T \hat{\eta}_i - \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{2\gamma_i} \tilde{\mu}_i \mu_i \\ \leq & \bar{v}^T \left(\mathcal{H} \mu(t) \mathcal{H} \otimes \mathbf{I}_{2n} \right) \bar{v} + e^T \left(\mathcal{H} \mu(t) \mathcal{H} \otimes \mathbf{I}_{2n} \right) e \\ & - \frac{a_1}{4} \bar{v}^T \left(\mathcal{H}\mathcal{H} \otimes \mathbf{I}_{2n} \right) \bar{v} + \frac{a_1}{2} e^T \left(\mathcal{H}\mathcal{H} \otimes \mathbf{I}_{2n} \right) e \\ & - \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{2\gamma_i} \tilde{\mu}_i \mu_i, \end{aligned} \tag{24}$$

and

$$\dot{V}_4 = -\sum_{i=1}^{\mathscr{N}} \frac{\tilde{\theta}_i \varphi_i^T \varphi_i s_i^T s_i}{4b_i^2} + \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{\rho_i} \tilde{\theta}_i \hat{\theta}_i. \tag{25}$$

Combining (17), and (22)-(25), for $t \in [t_r, t_{r+1})$, one has

$$\begin{aligned}\dot{V} \leq & \left(2\|\mathcal{H}\|\|S\| - \frac{a_1}{4} \lambda_{\min}(\mathcal{H}\mathcal{H}) \right) \bar{v}^T \bar{v} \\ & + \sum_{i=1}^{\mathscr{N}} \left[4\mu_i(t)(|\mathscr{N}_i| + a_{i0})^2 + \frac{a_1}{2} \|\mathcal{H}\|^2 \right] e_i^T e_i \\ & - \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{2\gamma_i} \tilde{\mu}_i \mu_i - \sum_{i=1}^{\mathscr{N}} l_i s_i^T s_i \\ & + \sum_{i=1}^{\mathscr{N}} \left(b_i^2 + \frac{1}{2} s_i^T s_i + \frac{1}{2}\epsilon_i^2 \right) + \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{\rho_i} \tilde{\theta}_i \hat{\theta}_i. \end{aligned} \tag{26}$$

Since $\mu_i(t) \geq 0$ for $t \geq 0$, one can obtain that

$$\begin{aligned}\dot{V} \leq & \left(2\|\mathcal{H}\|\|S\| - \frac{a_1}{4} \lambda_{\min}(\mathcal{H}\mathcal{H}) \right) \bar{v}^T \bar{v} \\ & + \sum_{i=1}^{\mathscr{N}} (\mu_i(t) + 1) \left[4(\mathscr{N} + 1)^2 + \frac{a_1}{2} \|\mathcal{H}\|^2 \right] e_i^T e_i \\ & - \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{4\gamma_i} \tilde{\mu}_i^2 - \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{2\rho_i} \tilde{\theta}_i^2 - \sum_{i=1}^{\mathscr{N}} (l_i - \frac{1}{2}) s_i^T s_i \\ & + \sum_{i=1}^{\mathscr{N}} \left(b_i^2 + \frac{1}{2}\epsilon_i^2 \right) + \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{4\gamma_i} a_1^2 + \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{2\rho_i} \theta_i^{*2} \\ \leq & -c_1 \bar{v}^T \bar{v} + c_2 \sum_{i=1}^{\mathscr{N}} (\mu_i(t) + 1) e_i^T e_i - \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{4\gamma_i} \tilde{\mu}_i^2 \\ & - \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{2\rho_i} \tilde{\theta}_i^2 - c_3 \sum_{i=1}^{\mathscr{N}} s_i^T s_i + \kappa, \end{aligned} \tag{27}$$

where $a_1 > \frac{4}{\lambda_{\min}(\mathcal{H}\mathcal{H})} (2\|\mathcal{H}\|\|S\| + c_1)$, $c_2 = 4(\mathscr{N} + 1)^2 + \frac{a_1}{2}\|\mathcal{H}\|^2$, $l_i > \frac{1}{2} + c_3$, $c_1 > 0$ and $c_3 > 0$ are two constants, and $\kappa = \sum_{i=1}^{\mathscr{N}} \left(b_i^2 + \frac{1}{2}\epsilon_i^2 \right) + \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{4\gamma_i} a_1^2 + \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{2\rho_i} \theta_i^{*2}$.

Considering the event-triggering function (8), one has

$$\begin{aligned}\dot{V} \leq & -c_1 \sum_{i=1}^{\mathscr{N}} \bar{v}_i^T \bar{v}_i + c_2 m_1 \sum_{i=1}^{\mathscr{N}} \mathrm{e}^{-m_2 t} - \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{4\gamma_i} \tilde{\mu}_i^2 \\ & - \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{2\rho_i} \tilde{\theta}_i^2 - c_3 \sum_{i=1}^{\mathscr{N}} s_i^T s_i + \kappa. \end{aligned} \tag{28}$$

Define

$$\bar{V}(t) = V(t) + \frac{c_2 m_1}{m_2} \sum_{i=1}^{\mathscr{N}} \mathrm{e}^{-m_2 t}. \tag{29}$$

Then one has

$$\begin{aligned}\dot{\bar{V}}(t) \leq & -\left(c_1 \sum_{i=1}^{\mathscr{N}} \bar{v}_i^T \bar{v}_i + \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{4\gamma_i} \tilde{\mu}_i^2 \right. \\ & \left. + \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{2\rho_i} \tilde{\theta}_i^2 + c_3 \sum_{i=1}^{\mathscr{N}} s_i^T s_i \right) + \kappa. \end{aligned} \tag{30}$$

The inequality (30) also holds for $t \in [0, \infty)$. Consequently, $\dot{\bar{V}}(t) < 0$ whenever the trajectory of the closed-loop system is outside the set

$$\begin{aligned}\Gamma = \Bigg\{ & c_1 \sum_{i=1}^{\mathscr{N}} \bar{v}_i^T \bar{v}_i + \sum_{i=1}^{\mathscr{N}} \frac{\sigma_i}{4\gamma_i} \tilde{\mu}_i^2 + \sum_{i=1}^{\mathscr{N}} \frac{\phi_i}{2\rho_i} \tilde{\theta}_i^2 \\ & + c_3 \sum_{i=1}^{\mathscr{N}} s_i^T s_i \leq \kappa \Bigg\}. \end{aligned} \tag{31}$$

Since $0 < V(t) < \bar{V}(t)$, it can be obtained that $\bar{V}(t)$ is bounded. By the definition of $\bar{V}(t)$, $\bar{V}(t)$ is lower bounded by

$$\bar{V}(t) \geq \lambda_1 \sum_{i=1}^{\mathscr{N}} \|\bar{v}_i\|^2 + \frac{d_m}{2} \sum_{i=1}^{\mathscr{N}} \|s_i\|^2 + \sum_{i=1}^{\mathscr{N}} \frac{1}{4\gamma_i} \tilde{\mu}_i^2 + \sum_{i=1}^{\mathscr{N}} \frac{1}{2\rho_i} \tilde{\theta}_i^2. \quad (32)$$

Thus one can conclude that s_i, $\bar{v}_i$, $\tilde{\mu}_i$, and $\tilde{\theta}_i$ are bounded.

Next, from (5) one has

$$\dot{v}_{i1} = v_{i2} - \mu_i \sum_{j\in\mathscr{N}_i} [a_{ij}(\hat{v}_{i1} - \hat{v}_{j1}) + a_{i0}(\hat{v}_{i1} - v_0)]. \quad (33)$$

Substituting (33) into (10) gives

$$\dot{q}_i - \dot{v}_{i1} + \alpha(q_i - v_{i1}) = s_i + \mu_i(t) \sum_{j\in\mathscr{N}_i} [a_{ij}(\hat{v}_{i1} - \hat{v}_{j1}) + a_{i0}(\hat{v}_{i1} - v_0)]. \quad (34)$$

Since the terms on the right hand side of (34) are bounded for $t \geq 0$ and $\alpha > 0$, $q_i - v_{i1}$ is bounded. Since s_i is bounded, it follows from (10) that $\dot{q}_i - v_{i2}$ is bounded. Consequently, the tracking errors

$$\begin{aligned} q_i - v_0 =& (q_i - v_{i1}) + (v_{i1} - v_0), \\ \dot{q}_i - \dot{v}_0 =& (\dot{q}_i - v_{i2}) + (v_{i2} - \dot{v}_0), \end{aligned} \quad (35)$$

are also bounded.

The ultimate bounds of the tracking errors are given as follows. From (31) one has that $\|\bar{v}\| \leq \sqrt{\frac{\kappa}{\varrho}}$, $\|s_i\| \leq \sqrt{\frac{\kappa}{\varrho}}$, $\|\tilde{\theta}_i\| \leq \sqrt{\frac{2\rho_i\kappa}{\varrho}}$, and $\mu_i \leq a_1 + \sqrt{\frac{4\gamma_i\kappa}{\varrho}}$, where $\varrho = \min_{i\in\mathscr{N}}\{c_1, c_3, \sigma_i, \phi_i\}$. Denoting $\chi_i = q_i - v_{i1}$, (34) becomes

$$\begin{aligned} \dot{\chi}_i &\leq -\alpha\chi_i + \|s_i\| + \lambda_{\mathscr{N}}\mu_i(t)\left(\|\bar{v}\| + \|e\|\right) \\ &\leq -\alpha\chi_i + Z_{i1} + Z_{i2}\mathrm{e}^{-m_2 t/2}, \end{aligned} \quad (36)$$

where $Z_{i1} = \sqrt{\frac{\kappa}{\varrho}} + \lambda_{\mathscr{N}}\sqrt{\frac{\kappa}{\varrho}}\left(a_1 + \sqrt{\frac{4\gamma_i\kappa}{\varrho}}\right)$, and $Z_{i2} = \lambda_{\mathscr{N}}\sqrt{\mathscr{N} m_1}\left(a_1 + \sqrt{\frac{4\gamma_i\kappa}{\varrho}}\right)$.

According to the comparison lemma [49], one has

$$\chi_i(t) \leq \mathrm{e}^{-\alpha t}\chi_i(0) + \frac{Z_{i1}}{\alpha}(1 - \mathrm{e}^{-\alpha t}) + \Xi_i(t), \quad (37)$$

where

$$\Xi_i(t) = \begin{cases} \frac{Z_{i2}}{\alpha - m_2/2}\left(\mathrm{e}^{-m_2 t/2} - \mathrm{e}^{-\alpha t}\right), & 2\alpha \neq m_2 \\ t Z_{i2}\mathrm{e}^{-\alpha t}, & 2\alpha = m_2 \end{cases}.$$

Thus $\|q_i - v_{i1}\| \leq \frac{Z_{i1}}{\alpha}$ when $t \to \infty$. From (10), $\|\dot{q}_i - v_{i2}\| \leq Z_{i1} + \sqrt{\frac{\kappa}{\varrho}}$.

Finally from (35), the ultimate bounds satisfy

$$\|q_i - v_0\| \leq \|q_i - v_{i1}\| + \|v_{i1} - v_0\| \leq \frac{Z_{i1}}{\alpha} + \sqrt{\frac{\kappa}{\varrho}}, \quad (38)$$

$$\|\dot{q}_i - \dot{v}_0\| \leq \|\dot{q}_i - v_{i2}\| + \|v_{i2} - \dot{v}_0\| \leq Z_{i1} + 2\sqrt{\frac{\kappa}{\varrho}}. \quad (39)$$

The proof of Theorem 1 is thus completed. ■

Remark 6: The ultimate bounds of the consensus tracking errors are dependent on the agent dynamics, the communication graph, as well as the design parameters b_i, σ_i, γ_i, ϕ_i, ρ_i, and α, $i \in 1, \cdots, \mathscr{N}$. Theoretically, by increasing ϱ or decreasing κ, the error bound can be reduced. For a given multiagent system and a given fixed graph, the values of ϱ and κ can be adjusted by choosing proper design parameters, for example, if one chooses a larger σ_i, the values of ϱ and κ will also become larger. In addition, it is worth noting that the choice of triggering parameters m_1 and m_2 will not affect the ultimate bounds of the consensus tracking errors.

Remark 7: Different from the most recent work [44], the distributed event-triggered adaptive fuzzy control strategy proposed in this work has the following distinctive features. First, the distributed compensator proposed in this work contains an adaptive gain, such that no global information is required. Second, there is no extra assumption on S, such that the leader system can include more kinds of signals, for example, the ramp functions with arbitrary slopes. Third, by using the open-loop estimate technique, the relative state estimates are used to construct the distributed compensator. With the proposed control protocol, communication among neighboring agents is aperiodic intermittent, and thus can be significantly reduced.

C. Exclusion of Zeno Behavior

In this subsection, we proof that the developed event-triggered control protocol is free from Zeno behavior, i.e., there exists a minimal inter-event time (MIET) $\tau_{miet} > 0$ such that $t_{k+1}^i - t_k^i \geq \tau_{miet}$. The result is described in Theorem 2.

Theorem 2: Suppose Assumptions 1-2 hold. Consider the graph $\bar{\mathcal{G}}$, the multiple uncertain EL systems (1), and the leader system (2). With the proposed distributed ETM (7) and the distributed adaptive fuzzy control law (11), Zeno behavior is excluded for all agents in the MAS (1).

Proof: The proof will be established by argument of contradiction. Suppose Zeno behavior happens, which means that there exists a finite T_d such that $\lim_{k\to+\infty} t_k^i = T_d$. Then from the property of limit, there exists a positive integer N_d such that $t_k^i \in [T_d - \epsilon_l, T_d]$, $\forall k \geq N_d$, where the constant $\epsilon_l > 0$ is set as $\epsilon_l = \frac{1}{2(1+2\|S\|)} \ln\left(1 + \frac{m_1(1+2\|S\|)\mathrm{e}^{-m_2 T_d}}{2\mu_i^3(T_d)\|\mathcal{H}\|^2\left(\frac{\bar{V}(0)}{\lambda_1} + m_1\mathscr{N}\right)}\right)$.

For $t \in \left[t_k^i, t_{k+1}^i\right)$, the derivative of $e_i^T(t)e_i(t)$ is calculated by

$$\begin{aligned} &\frac{d}{dt}e_i^T(t)e_i(t) \\ &\leq (2\|S\| + 1)\, e_i^T(t)e_i(t) + \mu_i^2(t)\hat{\eta}_i^T\hat{\eta}_i \\ &\leq (2\|S\| + 1)\, e_i^T(t)e_i(t) \\ &\quad + 2\mu_i^2(t)\|\mathcal{H}\|^2\left(\frac{\bar{V}(0)}{\lambda_1} + m_1\mathscr{N}\right). \end{aligned} \quad (40)$$

Since $e_i(t_k^i) = 0$, one can calculate that, for $t \in \left[t_k^i, t_{k+1}^i\right)$, $\mu_i(t)e_i^T(t)e_i(t)$ is upper bounded by

$$\begin{aligned} &\mu_i(t)e_i^T(t)e_i(t) \\ \leq& 2c_4\mu_i^3(t)\left(\frac{\bar{V}(0)}{\lambda_1} + m_1\mathscr{N}\right)\left(\mathrm{e}^{(2\|S\|+1)(t-t_k^i)} - 1\right), \end{aligned} \quad (41)$$

where $c_4 = \frac{\|\mathcal{H}\|^2}{2\|S\|+1}$.

Since $\mu(t)$ is monotonically nondecreasing, it can be derived from (7) and (41) that the MIET τ_d is lower bounded by

$$\tau_{miet} \geq \frac{1}{2\|S\|+1} \ln \left(\frac{m_1 e^{-m_2 T_d}}{2c_4 \mu_i^3(T_d) \left(\frac{\bar{V}(0)}{\lambda_1} + m_1 \mathscr{N} \right)} + 1 \right) = 2\epsilon_l. \quad (42)$$

From (42) one can obtain that $t_{k+1}^i \geq t_k^i + \tau_{miet} > T_d + \epsilon_l$, which contradicts the hypothesis. Therefore, the developed event-triggered control protocol is free from Zeno behavior. The proof is thus completed. ■

IV. An Example

The effectiveness of the proposed control protocol will be demonstrated in this section with its distinct properties. As a popular example of practical Euler-Lagrange systems, the simulation study on a group of two-link robotic arms is conducted, whose dynamic model is taken from [47],

$$\begin{bmatrix} M_i^{11} & M_i^{12} \\ M_i^{21} & M_i^{22} \end{bmatrix} \ddot{q}_i + \begin{bmatrix} V_{mi}^{11} & V_{mi}^{12} \\ V_{mi}^{21} & V_{mi}^{22} \end{bmatrix} \dot{q}_i + \begin{bmatrix} F_{fi}^1 \\ F_{fi}^2 \end{bmatrix} + \begin{bmatrix} G_i^1 \\ G_i^2 \end{bmatrix} = \begin{bmatrix} \tau_{i1} \\ \tau_{i2} \end{bmatrix}, \; i = \{1,2,3,4\} \quad (43)$$

where $q_i = \text{col}(q_{i1}, q_{i2})$, $\tau_i = \text{col}(\tau_{i1}, \tau_{i2})$, and

$$\begin{aligned} M_i^{11} =& \varpi_1(i) + \varpi_2(i) + 2\varpi_3(i)\cos(q_{i2}), \\ M_i^{12} =& M_i^{21} = \varpi_2(i) + \varpi_3(i)\cos(q_{i2}), \\ M_i^{22} =& \varpi_2(i), \\ V_{mi}^{11} =& -\varpi_3(i)\dot{q}_{i2}\sin(q_{i2}), \\ V_{mi}^{12} =& -\varpi_3(i)(\dot{q}_{i1} + \dot{q}_{i2})\sin(q_{i2}), \\ V_{mi}^{21} =& \varpi_3(i)\dot{q}_{i1}\sin(q_{i2}), \\ V_{mi}^{22} =& 0, \\ F_{fi}^1 =& i\cos(0.5t), \\ F_{fi}^2 =& i\sin(0.5t), \\ G_i^1 =& \varpi_4(i)g\cos(q_{i1}) + \varpi_5(i)g\cos(q_{i1} + q_{i2}), \\ G_i^2 =& \varpi_5(i)g\cos(q_{i1} + q_{i2}). \end{aligned}$$

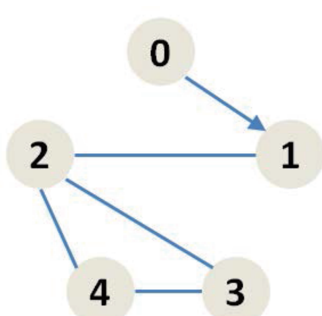

Fig. 1. Communication topology.

Fig. 1 shows the communication topology $\bar{\mathcal{G}}$ of the MAS including one leader and four followers. For the convenience of simulation, the parameters of 4 robot systems are taken respectively as $\varpi_1 = \{0.6, 0.8, 0.9, 1.1\}$, $\varpi_2 = \{1.1, 1.2, 1.3, 1.4\}$, $\varpi_3 = \{0.1, 0.1, 0.2, 0.3\}$, $\varpi_4 = \{0.6, 0.9, 1.1, 0.7\}$, $\varpi_5 = \{0.3, 0.5, 0.6, 0.7\}$, $g = 9.8$N/kg. The control law of form (11) is designed with $\alpha = 10$, $l_i = 5$, $b_i^2 = 0.1$, $\rho_i = 2$, $\phi_i = 0.1$, $\gamma_i = 0.9$, $\sigma_i = 0.001$, $m_1 = 5$, and $m_2 = 0.5$.

The fuzzy membership functions are set as

$$\mu_{\mathcal{F}_j^l}(X_{ij}) = \exp\left[-\frac{(X_{ij} - 3 + l)^2}{16} \right], \quad i \in \mathscr{N}, \; l \in \mathscr{L}, \; j = 1, \cdots, g \quad (44)$$

with the fuzzy basis functions

$$\varphi_{il}(X_i) = \frac{\prod_{j=1}^{g} \mu_{\mathcal{F}_j^l}(X_{ij})}{\sum_{l=1}^{r} \left[\prod_{j=1}^{g} \mu_{\mathcal{F}_j^l}(X_{ij}) \right]}.$$

The initial conditions are randomly chosen for the simulations. Some results are given here with the following initial conditions, $q_1(0) = \text{col}(4,8)$, $q_2(0) = \text{col}(8,9)$, $q_3(0) = \text{col}(5,2)$, $q_4(0) = \text{col}(2,4)$, $\dot{q}_1(0) = \text{col}(1,3)$, $\dot{q}_2(0) = \text{col}(4,2)$, $\dot{q}_3(0) = \text{col}(8,2)$, $\dot{q}_4(0) = \text{col}(8,2)$, $v_1(0) = \text{col}(8,9,5,2)$, $v_2(0) = \text{col}(6,7,1,3)$, $v_3(0) = \text{col}(4,6,4,2)$, $v_4(0) = \text{col}(7,3,6,2)$, $x_0(0) = \text{col}(0.5, 0.5, 1/3, 1/3)$, $\mu_i(0) = 2$, and $\theta_i(0) = 1.5$.

A. Simulation Results of the Proposed Control Scheme

Case 1: It is assumed that the leader system is described by (2) with $F_1 = F_2 = \mathbf{0}_{2\times 2}$, $v_0 = \text{col}(v_{01}, v_{02})$. Applying the proposed event-triggered control scheme, the tracking performance of q_i and $\dot{q}_i$, $i = 1,2,3,4$, is shown in Figs. 2-3. The compensator state errors and the corresponding adaptive parameters are depicted in Figs. 4-5 and Figs. 6-7, respectively.

Case 2: It is assumed that the leader system is described by (2) with

$$F_1 = \begin{bmatrix} -(2\pi)^2 & 0 \\ 0 & -(2\pi)^2 \end{bmatrix}, \; F_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (45)$$

that is,

$$v_0 = [\cos(2\pi t + \pi/3) \;\; \sin(2\pi t + \pi/6)]^T. \quad (46)$$

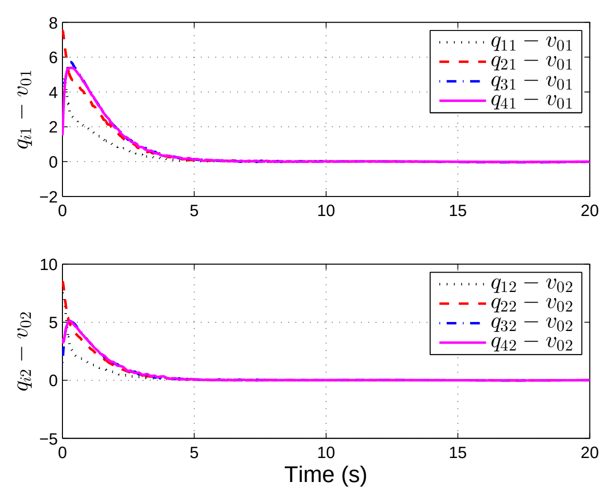

Fig. 2. Consensus tracking error $q_i - v_0$ for Case 1.

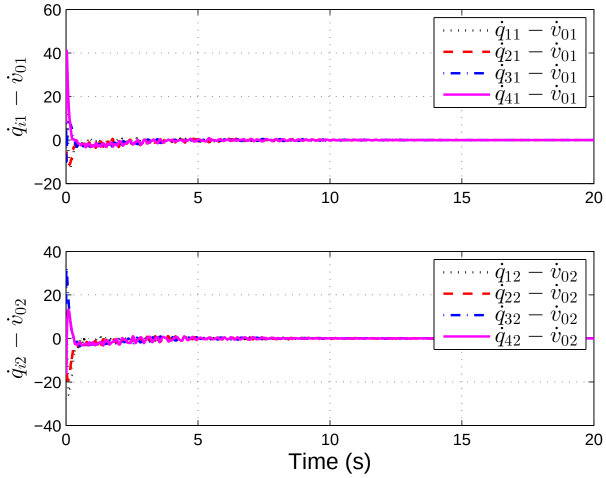

Fig. 3. Consensus tracking error $\dot{q}_i - \dot{v}_0$ for Case 1.

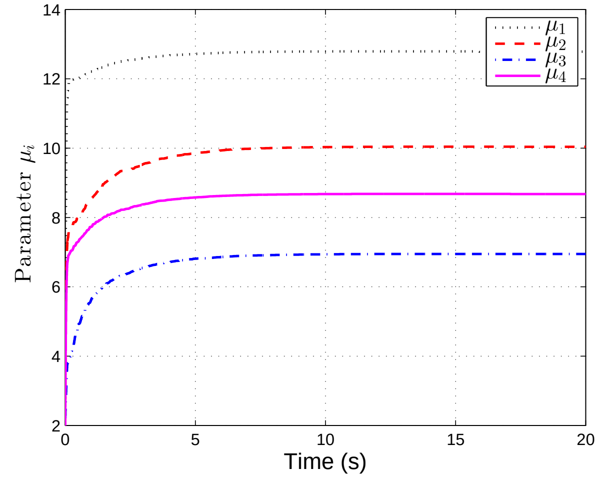

Fig. 6. Adaptive coupling gains for Case 1.

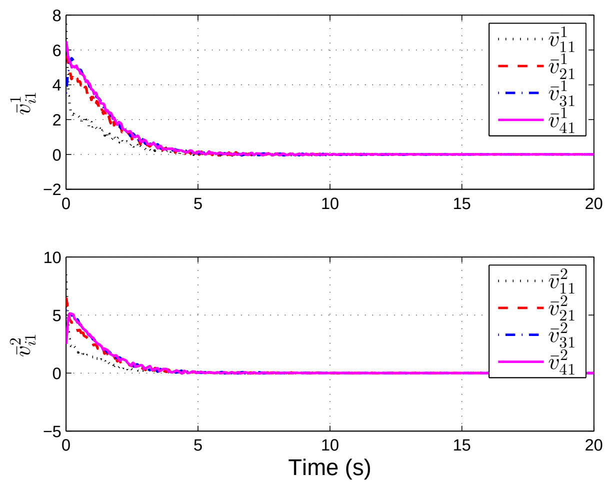

Fig. 4. State errors of the compensators $\bar{v}_{i1} = \mathrm{col}(\bar{v}_{i1}^1, \bar{v}_{i1}^2)$ for Case 1.

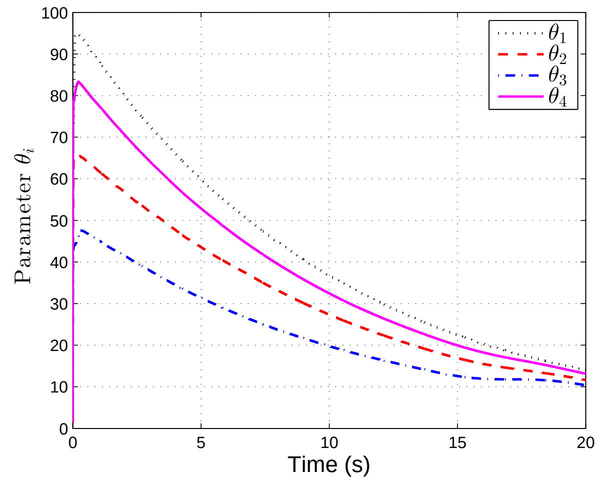

Fig. 7. Adaptive parameters for Case 1.

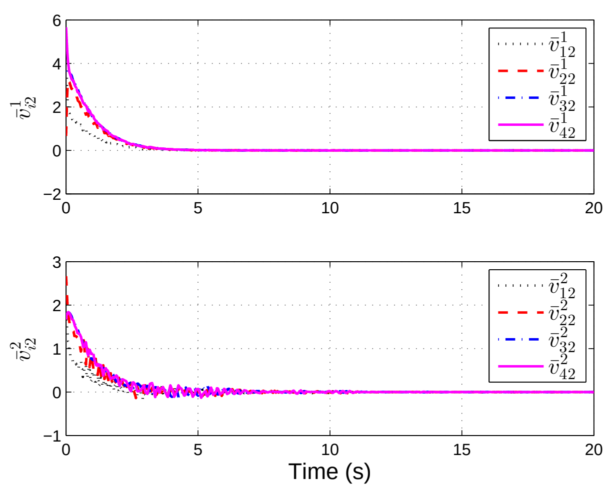

Fig. 5. State errors of the compensators $\bar{v}_{i2} = \mathrm{col}(\bar{v}_{i2}^1, \bar{v}_{i2}^2)$ for Case 1.

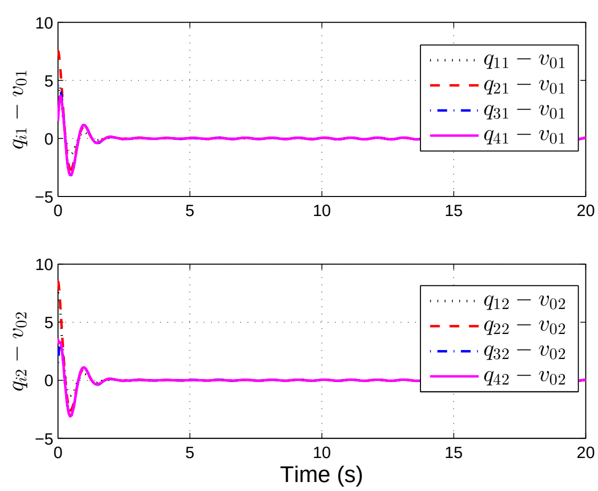

Fig. 8. Consensus tracking error $q_i - v_0$ for Case 2.

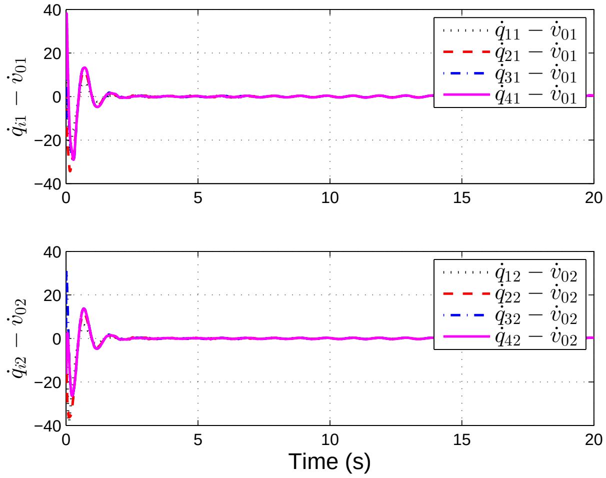

Fig. 9. Consensus tracking error $\dot{q}_i - \dot{v}_0$ for Case 2.

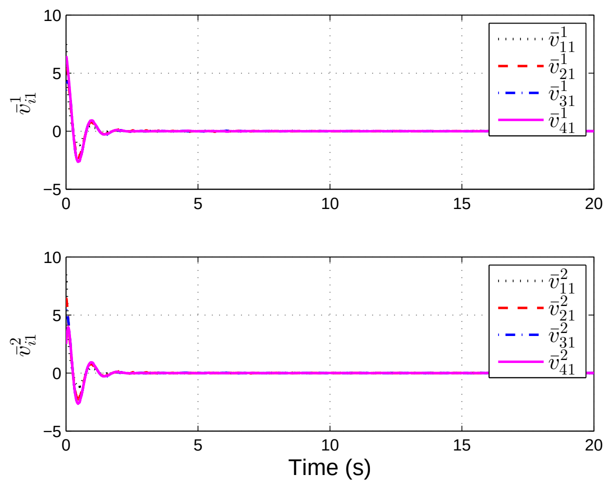

Fig. 10. State errors of the compensators $\bar{v}_{i1} = \text{col}(\bar{v}_{i1}^1, \bar{v}_{i1}^2)$ for Case 2.

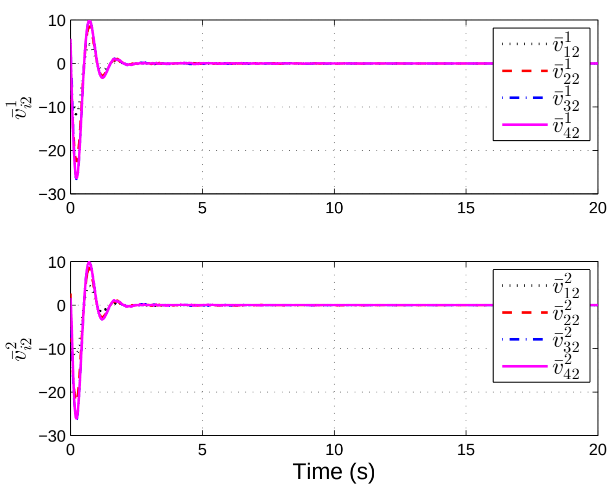

Fig. 11. State errors of the compensators $\bar{v}_{i2} = \text{col}(\bar{v}_{i2}^1, \bar{v}_{i2}^2)$ for Case 2.

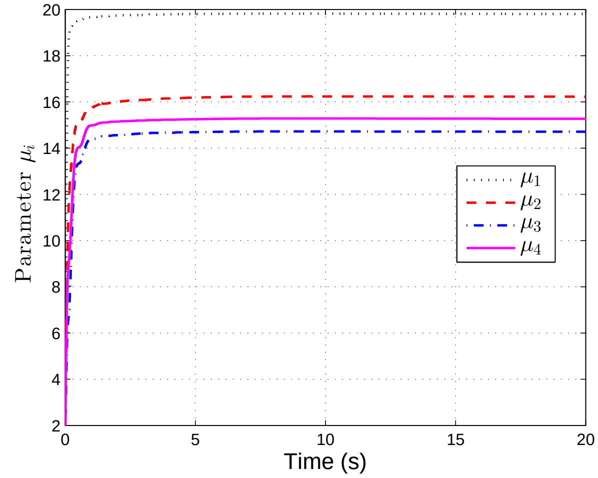

Fig. 12. Adaptive coupling gains for Case 2.

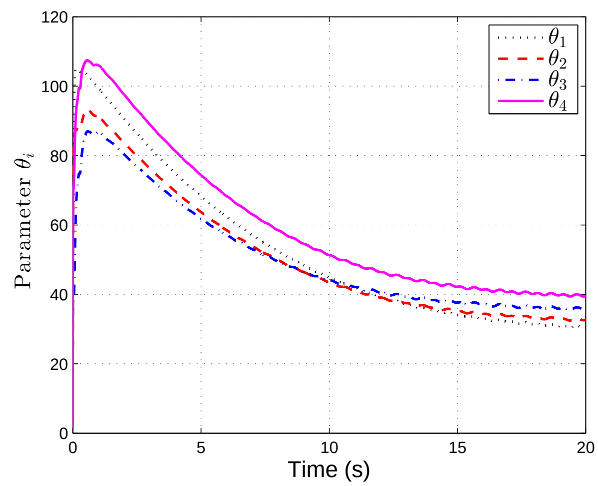

Fig. 13. Adaptive parameters for Case 2.

Choosing the same initial conditions and detailed parameters as in Case 1, simulation results are recorded in Figs. 8-13. Among them, Figs. 8-9 and Figs. 10-11 depict the tracking errors and the compensator state errors, respectively. Figs. 12-13 show the evolution of adaptive parameters. It can be seen that the LFCP is also solved in Case 2 with satisfactory tracking performance.

B. Comparisons

Comparisons are made in this subsection to verify the obtained theoretical results.

First, we compare the performance of the proposed event-triggered control scheme with that of the periodic sampling control approach with the same average sampling period. For Case 1 and Case 2 under the proposed event-triggered control strategy, the average inter-event time in the first 20 seconds are $t_{ave1} = 0.1016\text{s}$ and $t_{ave2} = 0.0904\text{s}$, respectively. In contrast, however, with the same average sampling period, the consensus tracking cannot be achieved via periodic sampling control.

Reducing the sampling period of periodic sampling control, consensus tracking is achieved when the sampling period is reduced to $t_{ave1} = 0.04$s for Case 1 and $t_{ave2} = 0.02$s for Case 2, respectively. In addition, for the two cases considered, Figs. 14-15 illustrate the evolution of consensus tracking errors $\|q - q_0\|$ and $\|\dot{q} - \dot{q}_0\|$ via periodic sampling control strategy and event-triggered control strategy, respectively, where

$$\|q - q_0\| = \sqrt{\sum_{i=1}^{4} \|q_i - v_0\|^2},$$

and

$$\|\dot{q} - \dot{q}_0\| = \sqrt{\sum_{i=1}^{4} \|\dot{q}_i - \dot{v}_0\|^2},$$

with $q = \text{col}(q_1, q_2, q_3, q_4)$, and $q_0 = \text{col}(v_0, v_0, v_0, v_0)$. The superiority of the proposed evert-triggered control strategy is thus demonstrated.

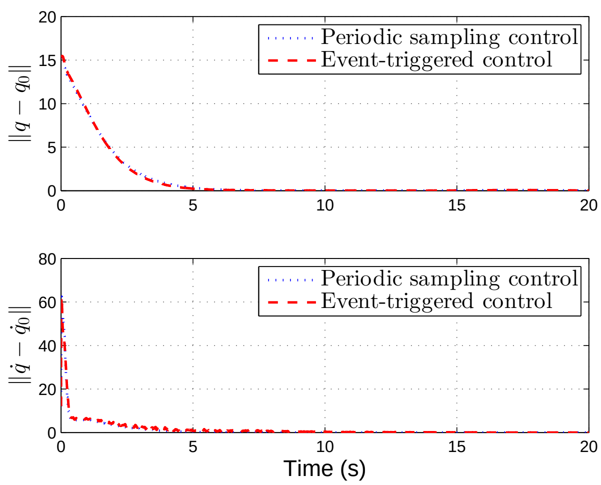

Fig. 14. Evolution of the consensus tracking errors for Case 1 via event-triggered control strategy ($t_{ave1} = 0.1016$s) and periodic sampling control strategy ($t_{ave1} = 0.04$s), respectively.

TABLE I
TRACKING PERFORMANCE WITH DIFFERENT VALUES OF ϕ_i

Design parameter	Time	Ultimate bound of $\|q - q_0\|$	
		Case 1	Case 2
$\phi_i = 0.1$	0-20s	0.08	0.18
$\phi_i = 1.0$	0-20s	0.18	0.46
$\phi_i = 5.0$	0-20s	0.30	0.76

Next, to illustrate how the selection of different design parameters, for example, b_i, σ_i, γ_i, ϕ_i, ρ_i, and α will affect the consensus tracking performance under the proposed control strategy, we take ϕ_i as an example, and test the ultimate bound of the consensus tracking error $\|q - q_0\|$ with different values of ϕ_i. The comparison result is given in Table I, from which one can observe that the smaller ϕ_i leads to the better tracking performance. For other parameters, similar comparisons can also be made, which are omitted here.

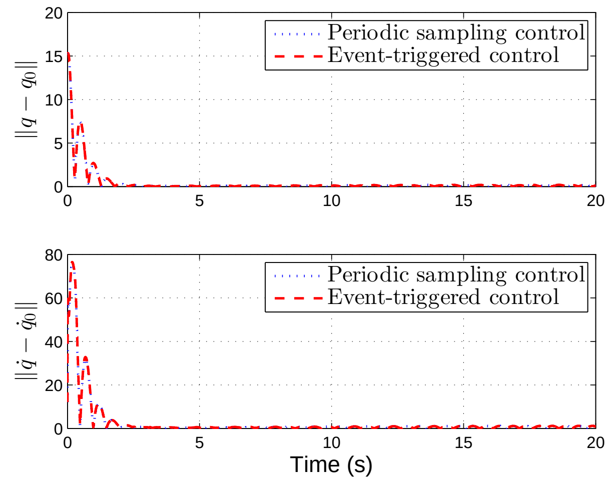

Fig. 15. Evolution of the consensus tracking errors for Case 2 via event-triggered control strategy ($t_{ave2} = 0.0904$s) and periodic sampling control strategy ($t_{ave2} = 0.02$s), respectively.

V. CONCLUSIONS

In this paper, we have investigated the LFCP of multiple uncertain EL systems subject to unknown agent dynamics with a dynamic leader system. FLSs have been employed to approximate the unknown nonlinear functions. Based on a novel event-triggered transmission scheme, a distributed ETM and a distributed adaptive fuzzy controller have been developed. The proposed control strategy requires no global information and needs only intermittent communication among agents, which makes it more practical and more energy efficient. It is shown that the consensus tracking errors of the closed-loop system converge to small sets around the origin which are tunable by appropriate design parameters.

One possible future research could be the LFCP of high-order uncertain nonlinear MASs via distributed event-triggered adaptive fuzzy control.

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