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Event-Triggered Consensus of Uncertain Euler–Lagrange Multi-Agent Systems over Jointly Connected Digraphs

Yahui Hao, Lu Liu, Senior Member, IEEE

Abstract—In this article, a fully distributed event-triggered protocol is proposed to solve the consensus problem of uncertain Euler–Lagrange (EL) multi-agent systems (MASs) under jointly connected digraphs. First, distributed event-based reference generators are proposed to generate continuously differentiable reference signals via event-based communication under jointly connected digraphs. Unlike some existing works, only the states of agents rather than virtual internal reference variables need to be transmitted among agents. Second, adaptive controllers are exploited based on the reference generators so that each agent can track the reference signals. The uncertain parameters converge to their real values under an initially exciting (IE) assumption. It is proved that the uncertain EL MAS achieves state consensus asymptotically under the proposed event-triggered protocol composed of the reference generators and the adaptive controllers. A unique feature of the proposed event-triggered protocol is its fully distributed property: the protocol does not depend on global information about the jointly connected digraphs. Meanwhile, a minimum inter-event time (MIET) is guaranteed. Finally, two simulations are conducted to show the validity of the proposed protocol.

Index Terms—Adaptive, Consensus, Euler–Lagrange, Event-triggered protocol, Fully distributed, Multi-agent systems, Uncertain, Jointly connected digraphs

I. INTRODUCTION

Over the past two decades, the distributed cooperative control problem of multi-agent systems (MASs) has emerged as a research hotspot in the control community due to its broad applications [1, 2]. Typical topics include consensus, flocking, and formation [3–5], among which consensus is the most basic topic. As a result, many researchers have devoted to studying the consensus problem of MASs with various dynamics [3, 6–8]. Recently, cluster consensus (or multi-consensus) in which different partitions of MASs are enforced to achieve different consensus states has also attracted much attention [9, 10]. It is well known that communication among agents is often necessary to reach consensus in cooperative control of MASs [11]. However, continuous communication is inefficient and even not feasible due to limited communication bandwidth and energy supply of each individual agent. Distributed event-triggered protocols are proposed so that communication between neighboring agents only occurs if a predefined event-triggering condition is satisfied [12]. A key issue in event-triggered protocols is to exclude Zeno behavior, which refers to an infinite number of events occurring within a finite time interval. Two widely adopted approaches to show the exclusion of Zeno behavior involve utilizing the contradiction argument [13–16] and providing a positive minimum inter-event time (MIET) [17–19].

The topologies of networked MASs significantly affect the design of event-triggered protocols. For connected undirected topologies, the symmetry property of the Laplacian matrix is utilized as a key in the design [14, 16–21]. Since directed topologies usually have asymmetric Laplacian matrices, the graph symmetrization technique has been developed [22, 23]. Due to power constraints, multipath fading, or external attacks, time-varying topologies may appear in applications [24]. Therefore, some researchers have investigated event-triggered protocols of MASs under time-varying topologies. For example, jointly connected undirected topologies are investigated in [25, 26]. Inspired by [27], an event-triggered protocol to handle jointly connected directed topologies is developed in [28].

Only linear MASs are investigated in the above-mentioned literature [13–26]. Cooperative control of nonlinear MASs has broader applications, such as the on-orbit autonomous assembly of a flexible spacecraft team [29]. Some research on the consensus problem of MASs with nonlinear dynamics has been conducted in [7, 30–33]. Specifically, the consensus problem of Euler–Lagrange (EL) MASs under fixed topologies is solved via distributed control protocols using continuous communication in [7, 31], with the research then being extended to jointly connected topologies in [30, 32]. More recently, event-triggered communication mechanisms are also adopted for cooperative control of EL MASs [34, 35]. A quantized sampled-data control method is developed to achieve robust consensus of a class of nonlinear MASs in [33]. However, the topologies are restricted to be fixed in [33, 34]. The proposed event-triggered protocols in [34, 35] rely on global information about the communication topologies, and no positive MIET is provided.

Motivated by the above observations, a valuable and open research topic is to design a novel fully distributed event-triggered protocol with a positive MIET to solve the consensus problem of uncertain EL MASs under jointly connected digraphs. To achieve this objective, the following three main challenges need to be overcome:

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First, we intend to propose an event-triggered protocol where the transmitted data are the states of the EL agents rather than virtual variables so that both communication and state measurement can be adopted in the proposed protocol. In contrast to [31, 34], the coupling of uncertain EL system dynamics and the switching network poses challenges in the control protocol design. Second, both the design and analysis of the event-triggered protocol are challenging since the jointly connected digraph may be disconnected at any time. Unlike [7, 31], it is difficult to apply the Lyapunov method in our work. Third, it is challenging to ensure a positive MIET for the event-triggered protocol without using any global information about the communication topology.

Compared with existing results, the unique features of this work are threefold:

First, a novel fully distributed event-triggered protocol, which only requires to transmit the states of the EL MAS over the network, is proposed to solve the consensus problem. Unlike [31, 34], where virtual internal variables can only be transmitted via communication, the states of an agent can be either transmitted to its neighbors via communication or measured by its neighbors. This flexibility enables the protocol to be adapted to a broader range of application scenarios.

Second, inspired by [36], a new adaptive control method is developed for uncertain EL MASs to track continuous signals. Contrary to [30, 31, 37], the proposed method not only ensures that each agent maintains a positive MIET but also enables the uncertain parameters of the EL MAS to converge to their true values.

Third, unlike the Lyapunov method used in [34, 35, 38, 39], by adopting several technical lemmas and integral inequalities, the challenge in consensus analysis of the closed-loop system under jointly connected digraphs and the proposed event-triggered protocol is overcome.

The structure of the remainder of this paper is as follows. In Section II, the preliminary and problem description of this work are presented. The main results are shown in Section III. Next, numerical examples are provided to verify the proposed event-triggered protocol. Finally, the conclusions are summarized in Section IV.

Notations: $\mathbb{Z}$, $\mathbb{Z}_{\geq 0}$ and $\mathbb{Z}_{> 0}$ represent integers, positive integers and nonnegative integers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ are the sets of real vectors with dimension $n$ and real matrices with dimension $m \times n$, respectively. $X^T \in \mathbb{R}^{m \times n}$ represents the transpose of matrix $X \in \mathbb{R}^{m \times n}$. The stack vector composed of $x_i \in \mathbb{R}^{n_i}$ is denoted by \( \text{col}(x_1, \ldots, x_N) = (x_1^T, \ldots, x_N^T)^T \).

Define vec($X$) = col($x_1, \ldots, x_m$) as the vector from matrix $X \in \mathbb{R}^{m \times n}$, where $x_i \in \mathbb{R}^n$ is the $i$th column of matrix $X$. $\| \cdot \|$ denotes the Euclidean norm for vectors or induced 2-norm for matrices. diag{$r_1, \ldots, r_N$} $\in \mathbb{R}^{N \times N}$ represents a diagonal matrix whose diagonal elements are $r_i \in \mathbb{R}$, $i = 1, \ldots, N$. For a matrix $A \in \mathbb{R}^{m \times n}$, $\lambda_A$ and $\lambda_A^*$ represent its maximum and minimum eigenvalue, respectively. The symbol $I_n$ represents the identity matrix with dimension $n$. $0_n$ and $1_n$ are the column vectors with dimension $n$ whose elements all equal 0 and 1, respectively. The Kronecker product is denoted by $\otimes$. $C_1[0, +\infty)$ represents the collection of continuously differentiable functions defined on $[0, +\infty)$. Denote $[a] = \min\{m \in \mathbb{Z}|m \geq a, a \in \mathbb{R}\}$ as the ceiling function. The abbreviations of this article is listed in TABLE I.

### TABLE I: ABBREVIATION TABLE

<table>
<thead>
<tr>
<th>Full name</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler–Lagrange</td>
<td>EL</td>
</tr>
<tr>
<td>Event-triggering mechanism</td>
<td>ETM</td>
</tr>
<tr>
<td>Initially exciting</td>
<td>IE</td>
</tr>
<tr>
<td>Minimum inter-event time</td>
<td>MIET</td>
</tr>
<tr>
<td>Multi-agent system</td>
<td>MAS</td>
</tr>
</tbody>
</table>

## II. GRAPH THEORY AND PROBLEM FORMULATION

### A. Graph Theory

A directed topology is described by $G=(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, N\}$ represents the set of $N$ agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the edge set among agents. Denote $a_{ij}=1$ if $(i,j)$ $\in$ $\mathcal{E}$, which means agent $i$ is able to transmit data to agent $j$. Then, agent $j$ is an out-neighbor of agent $i$. Otherwise, denote $a_{ij}=0$. An edge sequence in the form of \{$(i,m_1),(m_2,m_3),\ldots,(m_l,j)$\} is called a path from agent $i$ to agent $j$. A graph contains a spanning tree if there exist paths from an agent $i$ to all the other agents, and agent $i$ is thus called the root of the spanning tree. The adjacency matrix of $G$ is denoted by $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ with $a_{ii} = 0$. By conventional, define the Laplacian matrix as $L = [l_{ij}] \in \mathbb{R}^{N \times N}$ with $l_{ij} = -a_{ij}$, for $i \neq j$ and $l_{ii} = \sum_{j=1}^{N} a_{ij}$. In this work, the switching directed topology is described by $G_\sigma(t) = (\mathcal{V}, \mathcal{E}_\sigma(t))$ with a piecewise constant switching signal $\sigma(t):[0, \infty) \rightarrow \mathcal{P} = \{1, 2, \ldots, p\}$. It is assumed that the interval between two consecutive switching instants $t_m$ and $t_{m+1}$ is lower bounded by $\tau$, that is, $t_{m+1} - t_m \geq \tau > 0$.

**Definition 1:** (Jointly Connected Digraph [40]) A switching directed topology is said to be a jointly connected digraph if there exists a positive constant $\bar{\tau} > 0$ such that the union of the subgraphs over $[t, t+\tau], \forall t \geq 0$, that is, $\cup_{[t,t+\bar{\tau}]} G_\sigma(t)$, contains a spanning tree.

### B. Problem Description

The networked fully actuated EL system considered in this work is described by the following dynamics:

$$
M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + G_i(q_i) = \tau_i, \quad i = 1, \ldots, N, \quad (1)
$$

where $q_i \in \mathbb{R}^n$, $\dot{q}_i \in \mathbb{R}^n$, $\ddot{q}_i \in \mathbb{R}^n$ and $\tau_i \in \mathbb{R}^n$ represent the generalized position, velocity, acceleration and the control force of agent $i$. $M_i(q_i) \in \mathbb{R}^{n \times n}$ and $C_i(q_i, q_i) \in \mathbb{R}^{n \times n}$ are the inertia matrix and the Coriolis and centrifugal forces matrix of agent $i$. $G_i(q_i) \in \mathbb{R}^n$ represents the vector of gravitational force.

The EL systems (1) has the following three properties:

1. The matrix $M_i(q_i)$ is positive definite and bounded, that is, there exist two positive constants $0 < m_i < \bar{m}_i$ such that $m_i I_n \leq M_i(q_i) \leq \bar{m}_i I_n, \forall q_i$. The vector $G_i(q_i)$ is also bounded.
2. $M_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is antisymmetric.
3. Linearity in the parameters:

$$
M_i(q_i) x + C_i(q_i, \dot{q}_i) y + G_i(q_i) = Y_i(q_i, \dot{q}_i, x, y) \theta_i, \quad (2)
$$
where \( x, y \in \mathbb{R}^n \), \( Y_i(q_i, \dot{q}_i, x, y) \in \mathbb{R}^{nxn} \) is the known regressor, and \( \theta_i \in \mathbb{R}^{nxn} \) represents agent \( i \)'s constant parameter. For ease of presentation, denote \( \dot{Y}_i(t) \triangleq \dot{Y}_i(q_i, \dot{q}_i, x, y)|_{x=q_i, y=\dot{q}_i} \). Then it follows from (1) and (2) that
\[
\dot{Y}_i(t)\theta_i = \tau_i(t). \tag{3}
\]

For more details about the properties of EL dynamics, one can refer to [41]. The parameter \( \theta_i \) is not precisely known for the concerned uncertain EL systems. Moreover, each agent is allowed to have nonidentical system matrices \( M_i(q_i) \in \mathbb{R}^{nxn} \), \( C_i(q_i, \dot{q}_i) \in \mathbb{R}^{nxn} \) and \( G_i(q_i) \in \mathbb{R}^n \), that is, heterogeneous EL agents with the same state dimension can be handled in this work.

The consensus problem considered in this work is presented as follows.

**Definition 2**: For the uncertain EL MAS (1) with a jointly connected digraph defined in Definition 1, design a feasible fully distributed event-triggered protocol such that the following control objective can be achieved,
\[
\begin{align*}
\lim_{t \to \infty} q_i(t) &= q_\infty \quad \forall i \in \mathcal{V}, \\
\lim_{t \to \infty} \|\dot{q}_i(t)\| &= 0 \quad \forall i \in \mathcal{V},
\end{align*}
\]  
where \( q_\infty \in \mathbb{R}^n \) represents the final common constant state of the MAS.

The following assumption on the topology is necessary to achieve state consensus shown in Definition 2.

**Assumption 1**: The topology \( \mathcal{G}_{\sigma(t)} \) is a jointly connected digraph.

To deal with the uncertainty of the EL systems, another assumption on the initially exciting (IE) condition is needed and given in Section III.B because its presentation depends on the proposed adaptive controller in the next section.

**Remark 1**: The consensus problem has broad applications in engineering, such as in the on-orbit assembly of a spacecraft team [29]. In [32, 34], the consensus objective is described as \( \lim_{t \to \infty} \|q_i(t) - q_j(t)\| = 0 \) and \( \lim_{t \to \infty} \|\dot{q}_i(t)\| = 0 \), \( \forall i, j \in \mathcal{V} \), which does not imply all agents converge to a common constant state. For example, \( \lim_{t \to \infty} q_i(t) = f(t) \times 1_n \) with \( f(t) = \ln(t+1) \). In this work, we aim to enforce all agents converging to a common constant state \( q_\infty \in \mathbb{R}^n \) asymptotically. What make our work unique and challenging are: 1) we intend to design a fully distributed event-triggered protocol without using any global information about the network; and 2) the topologies are extended to jointly connected, directed topologies.

### III. MAIN RESULTS

In this section, fully distributed event-triggered reference generators are designed to generate continuous differentiable reference signals under jointly connected digraphs. Then, composite adaptive controllers are proposed for the uncertain EL MAS to track the reference signals. Finally, the closed-loop system is analyzed.

#### A. Fully Distributed Event-Triggered Reference Generator

To generate reference signals \( v_i(t) \in C^1[0, +\infty) \) for each agent \( i \) under jointly connected topologies and event-based communication among agents, we design the following fully distributed reference generators:
\[
\begin{align*}
\dot{q}_i(t) &= (A_i \otimes I_n) q_i(t) \nonumber \\
&\quad + (B_i \otimes I_n) \left( \sum_{j=1}^{N} a_{ij} (t) \tilde{q}_j(t) + o_i(t) \right), \tag{5}
\end{align*}
\]
where \( \eta_i = \text{col}(\eta_{i1}, \eta_{i2}, \ldots, \eta_{im}) \) represents agent \( i \)'s reference generator state with \( \eta_{il} \in \mathbb{R}^n, l = 1, \ldots, m \). \( \gamma_i \) is a positive constant. \( o_i(t) = (o_{i1}(t), \ldots, o_{im}(t))^T \) represent exponentially vanishing exciting signals adopted to handle the uncertainty of EL dynamics in the sequel, which can be designed as follows:
\[
\dot{\bar{q}}_i(t) = a_{il}^t e^{-\gamma_l t} \sum_{p=1}^{M_i} \sin \left( w_{lp}^i t + \varphi_{lp}^i \right), \quad l = 1, \ldots, n, \tag{6}
\]
with \( M_i \in \mathbb{Z}_{>0}, a_{il}^t, \varphi_{lp}^i \in \mathbb{R} \) and \( w_{lp}^i, \varphi_{lp}^i > 0 \). The matrices \( A_i \) and \( B_i \) are designed as follows:
\[
A_i = \begin{bmatrix}
-\alpha_{i1} & \alpha_{i1} & 0 & \cdots & 0 \\
0 & -\alpha_{i2} & \alpha_{i2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & -\alpha_{i(m-1)} & \alpha_{i(m-1)} \\
0 & 0 & \cdots & 0 & -\alpha_{im} \sum_{j=1}^{N} a_{ij}(t)
\end{bmatrix},
\]
\[
B_i = [0, \ldots, 0, \alpha_{im}]^T,
\]
with positive constants \( \alpha_{il} > 0, l = 1, \ldots, m \). \( \bar{q}_i(t), \forall i \in \mathcal{V} \) are defined as follows:
\[
\bar{q}_i(t) = q_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), \tag{7}
\]
where \( t_k^i, k \in \mathbb{Z}_{>0} \) represents the \( k \)th event-triggering instant of agent \( i \). The following event-triggering mechanism (ETM) is proposed to determine the event-triggering time sequence \( \{t_k^i\} \):
\[
t_k^{i+1} = \begin{cases}
> t_k^i \left| q_i(t_k^i) - \bar{q}_i(t) \right| \geq \frac{a}{(t+b)^\mu} \quad \forall i \in \mathcal{V},
\end{cases}
\]  
with positive constants \( a > 0, b \geq 1 \) and \( \mu > 1 \).

Given that only states \( q_i(t_k^i), \forall k \in \mathbb{Z}_{>0} \) of agent \( i \) are transmitted over the network, we can use either communication or measurement for data transmission depending on the application scenarios. If communication is adopted, the communication mechanism is described from the prospective of agent \( i \) as follows.

**Remark 2**: In the measurement mechanism, agent \( i \) needs to send a signal, such as an optical or acoustic signal, so that its out-neighboring agent \( j \) can be informed to measure its neighbor \( i \)'s state in the
measurement mechanism, which can be modelled as packet losses. Since packet losses can be regarded as time-varying topologies, if there exists a maximum allowable number of successive packet losses for each edge \((i, j)\), such that the equivalent topology is still a jointly connected digraph, the proposed control protocol in this work can still achieve the consensus objective. In addition, the measurement mechanism may also lead to delays, which has great impact on the protocol design because an agent cannot obtain real-time information from its neighboring agents at the triggering instants. As a result, addition errors would be induced, and it will affect the MAS achieving consensus. To the best of our knowledge, the event-triggered protocol that can deal with data transmission delays for the concerned problem in context is still an open and valuable research topic.

For ease of presentation, we will focus our analysis on the controlled MAS with the communication mechanism. To facilitate the analysis of the closed-loop system composed of the MAS (1) and the proposed reference generators (5) under jointly connected topologies, we need to introduce the following lemmas.

**Lemma 1** ([32]): Under Assumption 1, if \(A_{\sigma(t)}\) and \(D_{\sigma(t)}\) represent the adjacency matrix and in-degree matrix of \(G_{\sigma(t)}(V, \tilde{E}_{\sigma(t)})\), respectively, then the switching graph \(\tilde{G}_{\sigma(t)}(V, \tilde{E}_{\sigma(t)})\) corresponding to the following Laplacian matrix \(\tilde{L}_{\sigma(t)}\) is also a jointly connected digraph,

\[
\tilde{L}_{\sigma(t)} = 
\begin{bmatrix}
\Lambda_1 & -\Lambda_1 & 0 & \cdots & 0 & 0 \\
0 & \Lambda_2 & -\Lambda_2 & \cdots & 0 & 0 \\
& \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & -\Lambda_{m-1} & 0 & 0 \\
0 & 0 & \cdots & 0 & -\Lambda_mD_{\sigma(t)} & -\Lambda_mA_{\sigma(t)} \\
0 & 0 & \cdots & 0 & 0 & \Gamma \\
\end{bmatrix},
\]

where \(\Lambda_l = \text{diag}(\alpha_{2l}, \ldots, \alpha_{Nl})\), \(l = 1, \ldots, m\) and \(\Gamma = \text{diag}(\gamma_1, \ldots, \gamma_N)\).

**Lemma 2**: Under Assumption 1, the MAS described by the following dynamics,

\[
\dot{x}(t) = -\left(\tilde{L}_{\sigma(t)} \otimes I_n\right)x(t),
\]

where \(x(t) = \text{col} (x_1(t), \ldots, x_{N \times (m+1)}(t))\) with \(x_i(t) \in \mathbb{R}^n\), \(i = 1, \ldots, N \times (m + 1)\) being the state of the \(i\)-th agent, achieves state consensus exponentially. Let \(\Delta_x(t) = x(t) - 1_{N \times (m+1)} \otimes x_1(t)\). One has

\[
\dot{\Delta}_x(t) = -\left(\tilde{L}'_{\sigma(t)} \otimes I_n\right)\Delta_x(t),
\]

where the elements of \(\tilde{L}'_{\sigma(t)}\) are defined as \(\tilde{L}'_{\sigma(t)ij} = \tilde{L}_{\sigma(t)ij} - \tilde{L}_{\sigma(t)ji}\), \(i, j = 1, \ldots, N \times (m + 1)\). Then, the state transition matrix \(\Phi(t, \tau)\), \(t \geq \tau\) of the time-varying system (10) satisfies the following inequality:

\[
\|\Phi(t, \tau)\| \leq \beta_1 e^{-\lambda_1(t-\tau)}, \; t \geq \tau \geq 0,
\]

where \(\beta_1, \lambda_1\) are positive constants.

**Proof**: The proof can be completed by the methods given in [27, 28] or [42] with the condition that the time-varying communication topology \(\tilde{G}_{\sigma(t)}\) is jointly connected by Lemma 1.

**Remark 3**: In this work, the threshold function in (8) should meet three requirements. 1) To achieve consensus asymptotically, the threshold function, i.e., the upper bound of the error between the broadcast and real-time states, should be designed to converge to 0 as \(t \to +\infty\). 2) The proposed threshold can ensure a positive MIET for each agent. 3) The parameters in the threshold should be designed independent of any global information about the switching topology, because we intend to develop a fully distributed event-triggered protocol. In existing works such as [18], the time-dependent threshold function \(ae^{-at}\) can satisfy the first and second requirements. Unfortunately, its parameter \(a\) needs to be designed less than \(\lambda_1\) in (11) to guarantee a strict positive MIET. Motivated by these observations, a decay power function \(\frac{a^t}{(t+1)^\mu}\) is selected as the threshold, which satisfies all these three requirements.

**Remark 4**: With the proposed reference generators (5), each agent \(i\) is able to convert the received event-based data, \(q_j(t_k^i)\), to the continuously differentiable reference signal \(v_{\sigma(i)}(t)\). The reference generators extend the dimension of the original MAS, and Lemma 1 is introduced to deal with the extended MAS. Lemma 2 shows that the MAS described by (9) achieves exponential consensus under jointly connected digraphs. The result (11) in Lemma 2 will play a vital role in the analysis of the closed-loop system.

**B. Composite Adaptive Controller**

Composite adaptive controllers will be proposed in this subsection to control the uncertain EL agents. First, the following

---

**TABLE II: THE COMMUNICATION MECHANISM**

<table>
<thead>
<tr>
<th>WHILE (t &gt; 0) DO</th>
</tr>
</thead>
<tbody>
<tr>
<td>IF the triggering condition in (8) is satisfied THEN</td>
</tr>
<tr>
<td>Update (q_i(t_k^i)) with (t_k^i = t) in (7); Reset (e_i(t) = 0);</td>
</tr>
<tr>
<td>Broadcast (q_i(t_k^i)) to agent (j) with (a_{ji}(t) = 1).</td>
</tr>
<tr>
<td>END IF</td>
</tr>
<tr>
<td>IF there is a new out-neighbor (j) THEN</td>
</tr>
<tr>
<td>Broadcast (q_i(t_k^i)) with (t_k^i = \max{t_k^i</td>
</tr>
<tr>
<td>END IF</td>
</tr>
<tr>
<td>IF agent (i) receive new data from agent (j) THEN</td>
</tr>
<tr>
<td>Update (q_j(t_k^j)) for (5).</td>
</tr>
<tr>
<td>END IF</td>
</tr>
<tr>
<td>END WHILE</td>
</tr>
</tbody>
</table>

**TABLE III: THE MEASUREMENT MECHANISM**

<table>
<thead>
<tr>
<th>WHILE (t &gt; 0) DO</th>
</tr>
</thead>
<tbody>
<tr>
<td>IF the triggering condition in (8) is satisfied THEN</td>
</tr>
<tr>
<td>Update (q_i(t_k^i)) with (t_k^i = t) in (7); Reset (e_i(t) = 0);</td>
</tr>
<tr>
<td>Send a signal that can be observed by agent (j) with (a_{ji}(t) = 1).</td>
</tr>
<tr>
<td>END IF</td>
</tr>
<tr>
<td>IF there is a new out-neighbor (j) THEN</td>
</tr>
<tr>
<td>Update (q_i(t_k^i)) with (t_k^i = t) in (7); Reset (e_i(t) = 0);</td>
</tr>
<tr>
<td>Send a signal that can be observed by agent (j) with (a_{ji}(t) = 1).</td>
</tr>
<tr>
<td>END IF</td>
</tr>
<tr>
<td>IF agent (i) observe a signal from agent (j) THEN</td>
</tr>
<tr>
<td>Measure and update agent (j)’s state (q_j(t_k^j)) with (t_k^j = t) for (5).</td>
</tr>
<tr>
<td>END IF</td>
</tr>
<tr>
<td>END WHILE</td>
</tr>
</tbody>
</table>
two filters are introduced:

\[
\begin{align*}
\dot{Y}_{F_i}(t) &= -k_i Y_{F_i}(t) + \tilde{Y}_i(t), \quad Y_{F_i}(0) = 0 \\
\dot{T}_{F_i}(t) &= -k_i T_{F_i}(t) + \tau_i(t), \quad T_{F_i}(0) = 0,
\end{align*}
\]

(12)

where \( \dot{Y}_{F_i}(t) \in \mathbb{R}^{n \times r_i}, \quad \tau_{F_i}(t) \in \mathbb{R}^n, \) \( Y_{F_i}(t) \in \mathbb{R}^{r_i \times r_i}, \) and \( k_i \) and \( \tau_i \) are positive constants, and \( \dot{z}_{21} = (\vec{v}^T \vec{y}_{F_i} Y_{F_i}, \vec{y}_{F_i}^T \tau_{F_i}), \quad \dot{z}_{22} = (\vec{v}^T \tilde{Y}_{F_i}, \vec{y}_{F_i}^T \tau_{F_i}). \) The design of (13) can ensure \( \dot{Y}_{F_i}(t) \) and \( T_{F_i}(t) \) are bounded. Denote \( \epsilon_i = \dot{Y}_{F_i}(t) - \tau_{F_i}(t), \) then it follows from (3) and (12) that

\[
\dot{\epsilon}_i = -k_i \epsilon_i, \quad \epsilon_i(0) = 0,
\]

which yields \( \epsilon_i = 0. \) Thus one has

\[
\dot{Y}_{F_i}(t) = \tau_{F_i}(t).
\]

(15)

Similarly, according to (13) and (15), one can derive

\[
\dot{Y}_{F_i}(t) = \tau_{F_i}(t).
\]

(16)

Based on the filters (12) and (13), the following composite adaptive controllers are designed for each agent \( i \in \mathcal{V} \) to track \( v_{ri}(t) = C^1[0, +\infty) \) as follows:

\[
\begin{align*}
\tau_i(t) &= -k_i \delta_i(t) + Y_i(q_i, q_i, \dot{v}_{ri}, v_{ri}) \hat{\theta}_i(t), \\
\dot{\theta}_i(t) &= \nu_{i1}(t) + \nu_{i2}(t) + \nu_{i3}(t),
\end{align*}
\]

(17a, 17b)

where \( k_i \) and \( \delta_i(t) = q_i(t) - v_{ri}(t), \hat{\theta}_i(t) \) is the estimate of \( \theta_i, \) and the functions \( \nu_{i1}, \nu_{i2} \) and \( \nu_{i3} \) are defined as follows:

\[
\begin{align*}
\nu_{i1}(t) &= -\Pi_i Y_i^T(q_i, q_i, \dot{v}_{ri}, v_{ri}) \delta_i(t) \\
\nu_{i2}(t) &= -\Pi_i Y_i^T_t(t) \left( Y_i(t) \dot{\theta}_i(t) - T_{F_i}(t) \right) \\
\nu_{i3}(t) &= -\Pi_i \left( Y_i(t) \dot{\theta}_i(t) - T_{F_i}(t) \right)
\end{align*}
\]

(18)

with \( \Pi_i \) being a positive definite matrix, \( Y_{F_i}(t) \) and \( T_{F_i}(t), \) \( \dot{Y}_{F_i}(t) \) and \( \tau_{F_i}(t) \) are the filter states of the filters (12) and (13), respectively.

To ensure both \( \dot{\theta}_i(t) - \theta_i \) and \( \delta_i(t) \) tend to 0 exponentially as \( t \to +\infty, \) we need the following IE assumption [36].

**Assumption 2**: By selecting proper parameters \( a^i_1, M_i, \) \( w^i_{ip}, \) \( \varphi^i_{ip} \) and \( g^i_1 \) in the exciting signals (6), the matrix \( Y_{F_i}(t) \) is IE, that is, there exist positive constants \( T \) and \( \omega_i \) such that

\[
\int_0^T Y_{F_i}^T (s) Y_{F_i}(s) \, ds \geq \omega_i I_{ri}.
\]

(19)

By selecting large enough \( b_i \) in (13), one can ensure \( \dot{Y}_{F_i}(T) = \int_0^T \dot{Y}_{F_i}(t) \dot{Y}_{F_i}(t) \, dt. \) Thus, the inequality (19) is equivalent to \( \dot{Y}_{F_i}(T) \geq \omega_i I_{ri}. \) Moreover, since \( Y_{F_i}^T \dot{Y}_{F_i} \geq 0, \) one can obtain that

\[
\dot{Y}_{F_i}(t) \geq \omega_i I_{ri}, \quad \forall t \geq T.
\]

(20)

**Remark 5**: Compared to traditional adaptive controllers used in [31, 32], the parameter updating law (17b) includes two more terms, \( \nu_{i2} \) and \( \nu_{i3}, \) which are designed based on the filters (12) and (13), respectively. The underlying idea is to incorporate the estimation error \( \theta_i - \hat{\theta}_i \) of the uncertain parameters into the adaptive control law to ensure the convergence of the estimation error \( \theta_i \) towards the true value. Specifically, considering (15) and (16), \( \nu_{i2} \) and \( \nu_{i3} \) can be respectively expressed as \( \nu_{i2} = -\Pi_i Y_{F_i}^T Y_{F_i} \left( \hat{\theta}_i - \theta_i \right) \) and \( \nu_{i3} = -\Pi_i \dot{Y}_{F_i} \left( \hat{\theta}_i - \theta_i \right). \) Thus, the estimation error \( \hat{\theta}_i - \theta_i \) is introduced to the adaptive control law. Furthermore, we design the dynamics of \( \dot{Y}_{F_i} \) in (13) as \( Y_{F_i}(t) Y_{F_i}(t), \) which results in a negative gain matrix of \( \left( \hat{\theta}_i - \theta_i \right), i.e., -\Pi_i \dot{Y}_{F_i} \leq 0, \) \( \forall t \geq T \) under Assumption 2, ensuring the exponential convergence property of the estimation error. To satisfy the inequality (19) in Assumption 2, the exciting signal \( o_i(t) \) designed in (6) needs to contain rich enough different frequencies \( w^i_{ip}. \) The amplitude \( a^i_1 \) and decay rate \( \rho^{i}_1 \) of \( o_i(t) \) would affect the convergence of the uncertain parameters. In general, larger \( a^i_1 \) and smaller \( \rho^{i}_1 \) can induce a smaller parameter \( T \) in (19) and increase the convergence rate of the uncertain parameters. In addition, with the same event-triggered protocol, smaller uncertainty level of the parameter \( \theta_i \) can lead to a faster convergence rate of the consensus error. Thus, it is better to choose the initial value of \( \theta_i \) with smaller error \( \theta_i(0) - \theta_i. \)

Based on the adaptive controllers (17), the following technical lemma can be obtained.

**Lemma 3**: Under Assumption 2, agent \( i \)’s generalized velocity \( \dot{q}_i \) converges to the reference signal \( v_{ri} \) exponentially by the composite adaptive controller of agent \( i \) in (17), and the estimated parameter \( \hat{\theta}_i(t) \) converges to its real value \( \theta_i \) exponentially.

**Proof**: According to (1), (2) and (17), one obtains

\[
\begin{align*}
\dot{\hat{\theta}}_i(t) &= -M_i^{-1} \Delta_i(t) - M_i^{-1} (q_i \dot{q}_i + k_i I_{ni} \dot{\theta}_i(t) \\
&= -M_i^{-1} \Delta_i(t) + \nu_{i1}(t) + \nu_{i2}(t) + \nu_{i3}(t),
\end{align*}
\]

(21)

where \( \nu_{i1}(t) = \dot{Y}_{F_i}(t) - \tau_{F_i}(t). \) Then it follows from (15), (16) and (17b) that

\[
\dot{\hat{\theta}}_i(t) = -\Pi_i Y_{F_i}^T(q_i, q_i, \dot{v}_{ri}, v_{ri}) \delta_i(t) - \Pi_i \left( Y_{F_i}^T Y_{F_i} + \dot{Y}_{F_i} \right) \hat{\theta}_i(t).
\]

(22)

Because the matrix \( M_i(q_i) \) is positive definite, the following Lyapunov function candidate is built:

\[
V_i(t) = \frac{1}{2} \delta_i^T(t) M_i(q_i) \delta_i(t) + \dot{\delta}_i^T(t) \Pi_i^{-1} \dot{\delta}_i(t).
\]

(23)

The derivative of \( V_i(t) \) along the trajectories of (21) and (22) is

\[
\begin{align*}
\dot{V}_i(t) &= \delta_i^T(t) \left( \frac{1}{2} \dot{M}_i(q_i) - C(q_i, q_i) - k_i I_{ni} \right) \delta_i(t) \\
&= -\delta_i^T(t) \left( Y_{F_i}^T Y_{F_i} + \dot{Y}_{F_i} \right) \dot{\theta}_i(t).
\end{align*}
\]

(24)

Recalling the property (P2), one has

\[
\delta_i^T(t) \left( \frac{1}{2} \dot{M}_i(q_i) - C(q_i, q_i) \right) \delta_i(t) = 0.
\]

(25)

Substituting (25) into (24) yields

\[
\dot{V}_i(t) = -k_i \delta_i^T(t) \delta_i(t) - \dot{\delta}_i^T(t) \left( Y_{F_i}^T Y_{F_i} + \dot{Y}_{F_i} \right) \delta_i(t) \leq 0.
\]

(26)
Thus, \( V_i(t) \) is bounded and non-increasing during \([0, T)\). Under Assumption 2, it can be derived from (20) and (26) that
\[
V_i(t) \leq -k_i \theta_i^T \theta_i - \omega_i \dot{\theta}_i^T \dot{\theta}_i \leq -a_{i1} V_i(t), \quad \forall t \geq T,
\]
where \( a_{i1} = \min \{ 2k_i \bar{m}_i^{-1}, 2\omega_i \bar{\Delta}_{Pi} \} \). Along the error system composed of (21) and (22), one thus has
\[
V_i(t) \leq V_i(0) e^{a_{i1}T} e^{-a_{i1}t}, \quad t \in [0, +\infty).
\]

Let \( \theta_i(t) = \text{col} \left( \dot{\theta}_i(t), \ddot{\theta}_i(t) \right) \). It follows from the property (P1) that
\[
\| \theta_i(t) \| \leq \left( \bar{m}_i^{-1} \delta_i^T M_i(q_i) \delta_i + \lambda_i \bar{\Delta}_{Pi} \bar{\Delta}_{Pi}^{-1} \bar{\theta}_i \right)^{1/2} \leq a_{i2} \sqrt{2V_i(t)} \leq \beta_{i2} \| \theta_i(0) \| e^{-\beta_{i1} t},
\]
where \( a_{i2} = \max \left\{ \sqrt{\bar{m}_i^{-1}}, \sqrt{\lambda_i \bar{\Delta}_{Pi}} \right\}, \beta_{i1} = \frac{a_{i1}}{2}, \beta_{i2} = \sqrt{2a_{i2} e^{a_{i1}t}} \) and \( a_{i3} = \max \{ \bar{m}_i^{-1}, \bar{\Delta}_{Pi} \} \).

The proof is thus completed.

**Remark 6:** The terms \( \nu_{i2}(t) \) and \( \nu_{i3}(t) \) in the parameter updating law (17b) lead to the second term in (22) and further result in the negative term, \(-\omega_i \dot{\theta}_i^T \dot{\theta}_i\), in the Lyapunov derivative (27) under Assumption 2. Thus, the exponential convergence property of the closed-loop error system (21)-(22) can be derived, which is key to guarantee a positive MIET for the ETM (8). Compared with traditional adaptive controllers [31, 32], the main advantages of the composite adaptive controller are that it can ensure the uncertain parameters converging to the real values and further lead to the exponential convergence of the closed-loop error system. However, the controller is more complex. In [36], a composite adaptive controller is developed for a single uncertain EL system tracking some predefined continuous trajectories. In this work, the composite adaptive controller is introduced to solve the consensus problem of MAS with switching topologies and intermittent communication. Different from [36], the reference trajectory of each agent is not predefined but depends on the information received from the neighboring agents. To achieve the consensus objective, we elaborately construct the continuous differentiable signal \( v_{ri}(t) \) by agent \( i \)'s neighbors' intermittent states at their event-triggers instant.

### C. Analysis of the Closed-loop System

The control structure diagram of each agent with the proposed fully distributed event-triggered reference generator and the composite adaptive controller is shown in Fig. 1. In the proposed control protocol, the fully distributed event-triggered reference generator (5) is designed to generate the reference signal \( v_{ri}(t) \) using the received data \( q_{ji}(t_{k_{ji}}) \) from its neighbors and its own state \( q_i(t) \). The reference signal \( v_{ri}(t) \) is transmitted to the composite adaptive controller, which contains the controller (17a) and the uncertain parameter updating law (17b)-(18). Then by using the filters (12) and (13), the control input \( \tau_i \) is generated in the composite adaptive controller (17) and adopted to the EL system. Finally, the EL agent will determine when to broadcast a new state to its neighbors according to the ETM (8).

We are ready to present our main theorems.

**Theorem 1:** Under Assumptions 1-2, the uncertain EL MAS (1) achieves consensus (4) by the proposed fully distributed event-triggered protocol composed of the event-triggered reference generators (5) with the ETM (8) and the composite adaptive controllers (17). Meanwhile, the state of the EL MAS is bounded under the event-triggered protocol.

**Proof:** Let \( q = \text{col} (q_1, \ldots, q_N), \) \( q_i = \text{col} (\eta_{11}, \ldots, \eta_{N1}) \), \( l = 1, \ldots, m \) and \( \zeta = \text{col} (\zeta_1, \ldots, \zeta_m, q) \). It can be derived from the reference generators (5) that
\[
\dot{\zeta}(t) = -\left( \tilde{L}_{\sigma(t)} \otimes I_n \right) \zeta(t) + \varepsilon_1(t) + \varepsilon_2(t),
\]
where \( \varepsilon_1 = \text{col}(0_{N \times n \times (m-1)}, 0, \delta), \delta = \text{col}(\delta_1, \ldots, \delta_N), \) \( \varepsilon_2 = \text{col}(0_{N \times n \times (m-1)}, e, 0_{N \times n}) \) and \( e = \text{col}(\bar{q}_1 - q_1, \ldots, \bar{q}_N - q_N) \). Denote \( \Delta_{\zeta} = \zeta - 1_{N \times (m+1)} \otimes \eta_{11} \). It then follows from (29) that
\[
\Delta_{\zeta}(t) = -\left( \tilde{L}_{\sigma(t)} \otimes I_n \right) \Delta_{\zeta}(t) + \varepsilon_1(t) + \varepsilon_2(t),
\]
whose solution is
\[
\Delta_{\zeta}(t) = \Phi(t, 0) \Delta_{\zeta}(0) + \int_0^t \Phi(t, \tau) (\varepsilon_1(\tau) + \varepsilon_2(\tau)) d\tau.
\]

Then, substituting the inequality (11) into (31) yields
\[
\| \Delta_{\zeta}(t) \| \leq \beta_1\| \Delta_{\zeta}(0) \| e^{-\lambda_1 t} + \int_0^t \beta_2 e^{-\lambda_2 (t-\tau)} (\| \varepsilon_1(\tau) \| + \| \varepsilon_2(\tau) \| ) d\tau.
\]

Since \( \| \varepsilon_1(t) \| = \sqrt{\| o \|^2 + \| \bar{o} \|^2} \), it follows from (6) and (28) in Lemma 3 that \( \| \varepsilon_1(t) \| \) converges to 0 exponentially, that is, there exist positive constants \( \beta_2 \) and \( \lambda_2 \) such that
\[
\| \varepsilon_1(t) \| \leq \beta_2 e^{-\lambda_2 t}, \quad \forall t \geq 0.
\]

The ETM (8) yields
\[
\| \varepsilon_2(t) \| \leq \frac{\alpha \sqrt{N}}{(t+b)^\mu}, \quad \forall t \geq 0.
\]

According to (33) and (34), one can verify that there exists a positive constant \( \bar{a} > 0 \) such that
\[
\| \varepsilon_1(t) \| + \| \varepsilon_2(t) \| \leq \frac{\bar{a}}{(t+b)^\mu}, \quad \forall t \geq 0.
\]

Substituting (35) into (32) yields
\[
\| \Delta_{\zeta}(t) \| \leq \beta_1\| \Delta_{\zeta}(0) \| e^{-\lambda_1 t} + \int_0^t e^{-\lambda_1 (t-\tau)} \frac{\beta_1 \bar{a}}{(t+b)^\mu} d\tau.
\]

The right-hand side of (36) is exactly the solution to the following scalar system:
\[
\dot{\phi}(t) = -\lambda_1 \dot{\phi}(t) + \frac{\beta_1 \bar{a}}{(t+b)^\mu}, \quad \phi(0) = \beta_1\| \Delta_{\zeta}(0) \|.
\]

Let \( \phi(t) = (t+b)^\mu \phi(t) \), we will prove \( \phi(t) \) converges to 0 as \( t \to +\infty \) by showing that \( \dot{\phi}(t) \) is bounded. The dynamics of \( \phi(t) \) is derived from (37) as follows:
\[
\dot{\phi}(t) = \left( \frac{\mu}{t+b} - \lambda_1 \right) \phi(t) + \beta_1 \bar{a}.
\]
According to Lemma 4 in the Appendix, there exists a positive constant $d$ such that
\[
\int_{\tau}^{t} \frac{\mu}{s + b} \, ds \leq \frac{\lambda_1}{2} (t - \tau) + d, \quad t \geq \tau \geq 0.
\] (39)

Applying (39) to the solution of (38) yields
\[
\dot{\phi}(t) \leq e^{d(\phi(0) - \phi(t))} e^{\lambda_1 t} + \beta_1 \bar{d} e^{d(t - \tau)} dt
\leq e^{d(\phi(0) + \frac{2\beta_1 \bar{d}}{\lambda_1} \geq c.}
\] (40)

It then follows from $\phi(t) = (t + b)^{-\mu} \dot{\phi}(t)$ and (36) that
\[
\| \Delta_\phi(t) \| \leq \frac{c}{(t + b)^\mu}; \quad \forall t \geq 0.
\]

Recalling the definition of $\Delta_\phi(t)$ in (30), one can verify that
\[
\left\{ \begin{array}{l}
\| q_i(t) - q_j(t) \| \leq \frac{2c}{(t + b)^\mu} \\
\| v_{ri}(t) \| = \gamma_1 \| q_i(t) - q_j(t) \| \leq \frac{2\gamma_1 c}{(t + b)^\mu}, \quad \forall i, j \in \mathcal{V}.
\end{array} \right.
\] (41)

The inequality (28) in Lemma 3 holds under Assumption 2. Considering (41) and $\dot{q}_i(t) = \delta_i(t) + v_{ri}(t)$, one further has
\[
\| \dot{q}_i(t) \| \leq \beta_1 \| v_i(0) \| e^{-\beta_1 t} + \frac{2\gamma_1 c}{(t + b)^\mu}.
\] (42)

Since $\| \dot{q}_i(t) \|$ is absolutely integrable on $[0, +\infty)$, it can be proved that $\lim_{t \to +\infty} \dot{q}_i(t) = q_\infty$, with $q_\infty$ being a constant state by Cauchy’s convergence criteria. The proof of the consensus objective (4) is thus completed.

Next it will be shown $q_i(t)$ is bounded. According to (42), we have
\[
\| q_i(t) \| \leq \| q_i(0) \| + \int_0^t \| \dot{q}_i(t) \| \, dt
\leq \| q_i(0) \| + \frac{\beta_1 \| v_i(0) \|}{\beta_1} + \frac{2\gamma_1 c}{(t + b)^\mu},
\] (43)

which implies $q_i(t)$ is bounded.

Remark 7: It is shown in (43) that the bound of $q_i$ is related to the initial conditions, the event-triggered protocol and the time-varying communication network. Thus, the bound can be explicitly given if all the above information is known. Moreover, the bound is adjustable by setting the initial conditions $q_i(0), \delta_i(0), \theta_i(0)$, and the design of the event-triggered protocol parameters such as $\gamma_1, \mu, a$ and $b$ according to (43). Unfortunately, as pointed out in [32], the final state $q_\infty$ of the MAS is not easily obtained under switching topologies.

Next, we will prove that the proposed event-triggered protocol is feasible by excluding Zeno behavior.

Theorem 2: Under Assumptions 1-2, Zeno behavior does not exhibit for the uncertain EL MAS (1) with the proposed fully distributed event-triggered protocol. The inter-event time is lower bounded by the following positive constant:
\[
\tau_{\text{min}} = \frac{a (\mu - 1)}{\bar{c} \mu + b} + 1 \frac{(\mu - 1) a^\mu}{(t + b)^\mu}.
\] (44)

where $\bar{c}$ is a positive constant.

Proof: Define $e_i(t) = \dot{q}_i(t) - q_i(t)$, then the ETM (8) of agent $i$, $\forall i \in \mathcal{V}$ can be expressed as follows:
\[
t_{k+1} = \left\{ t \geq t_k \left\| e_i(t) \right\| - \frac{a}{(t + b)^\mu} \geq 0 \right\}.
\] (45)

To obtain the inter-event time, the dynamics of $\| e_i(t) \|$ is derived as follows:
\[
\frac{d}{dt} \| e_i(t) \| = \frac{1}{\mu} \frac{d}{dt} \| e_i(t) \| \leq \| e_i(t) \| \leq \| \dot{q}_i(t) \|.
\] (46)

It follows from (42), (46) and some algebraic manipulations that
\[
\frac{d}{dt} \| e_i(t) \| \leq \beta_1 \| v_i(0) \| e^{-\beta_1 t} + \frac{2\gamma_1 c}{(t + b)^\mu} \leq \frac{\bar{c}}{(t + b)^\mu},
\] (47)

where $\bar{c}$ is a positive constant.

The inequality in (45) implies that $\| e_i(t) \|$ increases from 0 to $\frac{a}{(t + b)^\mu}$ during $[t_k, t_{k+1}]$. It thus can be derived from (47) that
\[
\frac{a}{t_{k+1} + b} \leq \int_{t_k}^{t_{k+1}} \frac{\bar{c}}{(t + b)^\mu} dt = \frac{\bar{c}}{\mu - 1} \left( \frac{1}{t_{k+1}^\mu} - \frac{1}{t_k^\mu} \right) t_k.
\] (48)

Denote $\bar{t}_k = t_k + b, k \in \mathbb{Z}_{\geq 0}$. Then, it can be derived from (48) that
\[
\frac{a}{\frac{1}{\bar{t}_k^\mu}} \leq \int_{t_k}^{\bar{t}_k} \frac{\bar{c}}{(t + b)^\mu} dt = \frac{\bar{c}}{\mu - 1} \left( \frac{1}{\bar{t}_{k+1}^\mu} - \frac{1}{\bar{t}_{k+1}^\mu} \right) \bar{t}_k.
\] (49)

Multiplying both sides of (49) by $(\bar{t}_{k+1} - \bar{t}_k)$ yields
\[
\frac{a}{\frac{1}{\bar{t}_k}} \leq \left( \frac{\bar{t}_{k+1} - \bar{t}_k}{\bar{t}_k^\mu} \right) \frac{1}{\bar{t}_{k+1}^\mu} - \frac{1}{\bar{t}_{k+1}^\mu}.
\] (50)

Given that $\bar{t}_{k+1} \geq \bar{t}_k$, it can be derived from (50) that
which completes the proof. 

Applying the difference of two n-th powers to (51), that is,
\[ x^n - y^n = (x - y)\sum_{i=0}^{n-1} x^{n-1-i}y^i \]
with \( x, y \in \mathbb{R} \) and \( n \in \mathbb{Z}_{>0} \), one obtains
\[
t_{k+1} - \bar{t}_k \geq \frac{a(\mu - 1)1}{c} t_k^{\mu - 1} + \frac{a(\mu - 1)}{c} t_k - \bar{t}_k.
\]
Substituting \( \bar{t}_k \) for \( b \) and \( t_{k+1} - t_k = \bar{t}_{k+1} - \bar{t}_k \) into (52) yields
\[
t_{k+1} - t_k \geq \frac{a(\mu - 1)}{c} \sum_{l=0}^{[\mu]-1} (a(\mu - 1) t_k^{\mu - 1} + 1)^{\frac{1}{\mu - 1}},
\]
which completes the proof.

Remark 8: Since the states, \( q_i(t_k) \), \( \forall k \in \mathbb{Z}_{\geq 0}, i \in \mathcal{V} \), are transmitted over the network, the system dynamics and the network are coupling. The design of \( v_{ri}(t) \) is key to achieving consensus under this coupling setting. On the one hand, the reference signals \( v_{ri}(t) \) in the adaptive controllers (17) are generated by the generators (5) with data received from the network. On the other hand, the exponential tracking property of the EL systems under the adaptive controllers (17) is a necessary condition for the consensus of the generators (5). By the co-design of the generators (5) and the adaptive controllers (17) through the reference signals \( v_{ri}(t), \forall i \in \mathcal{V} \), the consensus objective is achieved under the coupling setting.

Remark 9: Contrary to [34, 35], no global information about the switching digraph is utilized in the proposed event-triggered protocol, and the event-triggered protocol is called fully distributed as a result. Before the implementation of the event-triggered protocol, the reference generators (5) with the ETM (8) and the composite adaptive controllers (17) need to be imported into each agent. The parameters including \( a, b \) and \( \mu \) of the event-triggering condition are initial settings of each agent. Though \( a, b, \mu \) are the same for all agents, their determination does not depend on the interaction among agents or any global information about the communication network. Therefore, the event-triggered protocol is still fully distributed. The time-varying topologies considered in this work are only required to be jointly connected digraphs, which contain those topologies considered in [34, 35, 38, 39] as special cases. Given that the jointly connected digraph may be disconnected at any time, the Lyapunov method that used to deal with fixed connected topologies cannot be adopted. To overcome this challenge, a new analysis method based on Lemmas 1–4 is exploited to establish Theorem 1. Another advantage over [14, 15] of this work is that the MIET (44) is provided for each agent under the fully distributed event-triggered protocol.

IV. NUMERICAL EXAMPLES

Example 1: In this example, a numerical simulation is conducted in an open-source, three-dimensional robot simulator Webots [43] to verify the effectiveness of the proposed fully distributed event-triggered protocol.

The MAS is composed of 4 PUMA-560 robots with 6 degrees of freedom. To simplify the simulation, we lock the end-effector and only control the first 3 joints of each robot as in [44], see Fig. 2. The dynamics and parameters of PUMA-560 are presented in details in [44] (equations (33)-(73) and Tables 6-7). In this simulation, for each agent \( i, i = 1, 2, 3, 4 \), we select \( g_1, g_2 \) and \( I_1 + I_m \) as the uncertain parameters and denote \( \bar{\theta}_{i1}, \bar{\theta}_{i2} \) and \( \bar{\theta}_{i3} \) as their estimations, respectively. The initial values of these parameters are randomly chosen as follows:
\[
\begin{align*}
\theta_{i1}(0) &= -40, \quad \bar{\theta}_{i1}(0) = -18, \quad \bar{\theta}_{i3}(0) = 4 \\
\theta_{i2}(0) &= -50, \quad \bar{\theta}_{i2}(0) = -9, \quad \bar{\theta}_{i3}(0) = 6 \\
\theta_{i3}(0) &= -20, \quad \bar{\theta}_{i3}(0) = -14, \quad \bar{\theta}_{i3}(0) = 5 \\
\theta_{i4}(0) &= -30, \quad \bar{\theta}_{i4}(0) = -5, \quad \bar{\theta}_{i3}(0) = 2
\end{align*}
\]
The initial states of the four agents are randomly selected as follows:
\[
\begin{align*}
q_1(0) &= (-1.30, 0.60, -0.90)^T, \quad q_2(0) = (-0.22, 0.40, -0.20)^T, \\
q_3(0) &= (0.80, -1.10, 0.70)^T, \quad q_4(0) = (1.10, -0.50, -0.70)^T.
\end{align*}
\]
As shown in Fig. 3, the switching communication topology includes 3 different graphs \( G_i, i = 1, 2, 3 \). The switching rule \( \sigma(t) \) is given as follows:
\[
\sigma(t) = \begin{cases} 1, & t \in [k\bar{\tau}, (k + 1/3)\bar{\tau}) \\ 2, & t \in [(k + 1/3)\bar{\tau}, (k + 2/3)\bar{\tau}) \\ 3, & t \in [(k + 2/3)\bar{\tau}, (k + 1)\bar{\tau}) \end{cases}.
\]
where \( k \in \mathbb{Z}_{\geq 0} \) and \( \bar{\tau} = 1.5s \). The parameters and the matrices in the proposed protocol composed of (5) and (17) are designed as follows:
\[
\begin{align*}
A_i &= \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \gamma_i = 1.5.
\end{align*}
\]

![Fig. 2: The PUMA-560 robot [43].](image1)

![Fig. 3: The switching directed communication network.](image2)
that the control protocol can enforce all agents approaching comparison simulation, we choose $h$ studied, where the formation variables were defined as $q_i$.

In [37], the time-varying formation control problem was studied for comparison purpose.

Example II: To further show the unique features of the proposed event-triggered protocol, the numerical example in [37] is studied for comparison purpose.

The MAS contains four agents described by the following EL dynamics:

$$m_i \ddot{q}_i + m_i g = \tau_i, \quad i = 1, \ldots, 4,$$

where $q_i \in \mathbb{R}^3$, $m_i$ represents the mass of agent $i$, $g$ is the gravitational acceleration. The values of these parameters are $m_1 = 1$, $m_2 = 2$, $m_3 = 1.5$, $m_4 = 2.5$ and $g = 9.8$. The topology is given as $\mathcal{E} = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$. In [37], the time-varying formation control problem was studied, where the formation variables were defined as $h_i$ for the MAS. Since consensus problem is considered in this comparison simulation, we choose $h_i = 0$, $i = 1, \ldots, 4$ such that the control protocol can enforce all agents approaching a common constant state asymptotically.

The exciting signals (6) are selected as follows for all agents:

$$\begin{align*}
o_1 &= 0.9e^{-0.1t} (\sin (t + \pi/6) - \sin (5t + \pi/4)) \\
o_2 &= 0.7e^{-0.1t} (\sin (3t + \pi) - \sin (8t + \pi/4)) \\
o_3 &= 1.2e^{-0.1t} (\sin (5t + \pi/3) + \sin (1.1t + \pi/5))
\end{align*}$$

The parameters in the ETM (8) are designed as $a = 2$, $b = 1.5$ and $\mu = 1.5$.

With all these initial conditions and the designed parameters, we run the simulation for 60s and record the results. Fig. 4 shows that $\hat{\theta}_{12}$, $\hat{\theta}_{13}$ and $\hat{\theta}_{14}$ converge to their real values $-37.7$, $-8.37$ and $2.63$, respectively. Fig. 5 shows the state trajectories of all agents. Fig. 6 presents the sequences of event-triggering instants, and the event numbers of the four agents are 110, 105, 116 and 101 times, respectively. The minimum-inter event time in the simulation for the MAS is 0.128s, which is larger than the value 0.024s obtained in Theorem 2. Fig. 7 presents 8 scenes at 0s, 1s, 2s, 4s, 8s, 16s, 30s and 60s of the simulation, respectively. It can be seen from Fig. 5–7 that the control objective (4) is achieved. Overall, the effectiveness of the proposed event-triggered protocol is verified by the simulation.

This consensus problem is also solved via the event-triggered protocol proposed in this work. The exciting signals are selected as follows for all agents:

$$\begin{align*}
o_1 &= 10e^{-0.15t} \sin (5t + 2) \\
o_2 &= 11e^{-0.15t} \sin (3t + 5) \\
o_3 &= 1.3e^{-0.15t} \sin (5t + 2)
\end{align*}$$

The parameters in the ETM are designed as $a = 2$, $b = 1$ and $\mu = 1.2$. Other parameters are the same as selected in (54). With the same initial conditions, the simulation is conducted. Fig. 10 and Fig. 11 show the estimations of the uncertain parameters and the agent states, respectively. The MAS achieves consensus asymptotically. The estimate parameters $\hat{\theta}_{11}$, $\hat{\theta}_{21}$, $\hat{\theta}_{31}$, $\hat{\theta}_{41}$ and $\hat{\theta}_{12}$, $\hat{\theta}_{22}$, $\hat{\theta}_{32}$, $\hat{\theta}_{42}$ converge to their real values $1$, $2$, $1.5$, $2.5$ and $9.8$, $19.6$, $14.7$, $24.5$, respectively. TABLE IV shows the number of triggered events for all agents in the simulations. It can be concluded that the event-triggered protocol proposed in this work can effectively reduce the number of triggered events compared with [37].

Example II shows that the proposed event-triggered protocol can solve the consensus problem with the estimations of uncertain parameters converging to their real values and also reduce communication burden of the MAS.

V. CONCLUSION

In this paper, the consensus problem of uncertain EL MASs under jointly connected digraphs has been solved via a novel
MASs, such as MASs with normal form nonlinear dynamics.

**APPENDIX**

**Lemma 4:** (Lemma 9.5 in [45]) For a nonnegative, continuous and bounded function $g(t) : [0, +\infty) \to \mathbb{R}$, there exists a positive constant $d > 0$ for any $\varsigma > 0$ such that

$$
\int_{\tau}^{t} g(s) \, ds \leq \varsigma (t - \tau) + d, \quad t \geq \tau \geq 0,
$$

if $\lim_{t \to +\infty} g(t) = 0$.
Fig. 10: Estimations of the two uncertain parameters in Example II.

Fig. 11: States of the MAS in Example II.

REFERENCES


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