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Coverage Control for Mobile Sensor Networks with Double-Integrator Dynamics and Unknown Disturbances

Peng Wang, Cheng Song, Lu Liu, *Senior Member, IEEE*

***Abstract*—In this paper, coverage control for mobile sensors with double-integrator dynamics and different maximum velocities on a circle is investigated. A generalized energy function is introduced in the proposed coverage control laws to guarantee order preservation of the sensors despite of the existence of unknown but bounded disturbances. The velocity constraint of each sensor is shown to be always satisfied throughout the coverage task. It is shown that the sensor network can be driven to a neighborhood of the optimal configuration minimizing the coverage cost function and an upper bound on the coverage cost function when time goes to infinity is also provided. Finally, a simulation example is given to illustrate the effectiveness of the proposed coverage control laws.**

***Index Terms*—Coverage control, mobile sensor networks, velocity constraints, unknown but bounded disturbances**

I. Introduction

Cooperative control of mobile sensor networks (MSNs) has drawn much interest in past decades due to their broad range of applications, including environmental monitoring and exploration [1], search and rescue [2], map building [3], and precision agriculture [4, 5]. An MSN is composed of a large number of low-cost sensor nodes with low power consumption. The networked mobile sensors significantly reduce the requirement of the base station while greatly enhancing the robustness and flexibility of the whole sensor network.

One of the fundamental problems in cooperative control of mobile sensor networks is coverage control, whose objective is to drive mobile sensors to desired positions such that the overall sensing performance of the sensor networks is optimized. Various methods have been developed to solve this problem in past few years [6–12]. For example, artificial potential-field-based coverage controller is proposed in [6], and probabilistic-models-based coverage controller is developed in [7–9]. Additionally, many coverage control algorithms have been also developed based on Voronoi partition techniques [10–12].

The coverage control problem for mobile sensor networks in a one-dimensional mission space has received increasing attention in recent years. Under the assumption that sensors may fail with a certain probability in a coverage task, the optimal deployment is achieved in [13] to maximize the expected coverage performance. In order to minimize the shortest response time from a group of sensors to any point on a line with varying roughness, a control scheme using the roughness function is developed in [14] to drive the sensors to the optimal configuration. Then, that work is extended in [15] with a relaxed assumption that each sensor could only access noisy samples of the roughness of the points. By utilizing the decomposition of the Hessian matrix derived from the locational cost function, a sufficient condition for achieving centroidal Voronoi tessellation configurations in a one-dimensional mission space is derived in [16]. Then, the proposed optimal condition is proved to be applicable to several classes of density functions which measures the different relative importance of each point in the space.

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P. Wang and L. Liu are with the Department of Biomedical Engineering, City University of Hong Kong, Hong Kong (e-mail: pwang36-c@my.cityu.edu.hk; luliu45@cityu.edu.hk).

C. Song is with the School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China (e-mail: chengsong@njust.edu.cn).

The coverage control problem on a circle is a typical case of the one-dimensional coverage control problem, which can be applied to many practical scenarios by extending the mission space on a circle to the mission space on a closed curve via a single parameter, such as the curve arc-length [17]. For example, the detection of forest fire is vital for fire fighters to know where the fire is propagating [18]. Thus, mobile sensors need to be deployed along the perimeter of the fire, and each sensor is responsible to surveillance for a perimeter section, which can be finally formulated as the problem of coverage control on a circle. In [19], the mobile sensors need to move around a closed curve periodically for oceanographic sampling, where the coverage control laws can be adopted for the optimal data collection on the surface of the ocean. Moreover, for the target capturing and target enclosing problem [20, 21], the deployment of mobile agents around the target on a circle for better coverage performance will increase the possibility for agents to capture the target or to defend the attackers when the target or attackers reach the circle. In [22], the coverage cost function is defined as the maximum time to reach any point on the circle from the MSN and distributed coverage control laws are designed to minimize the coverage cost function. Then, that work is extended to the case of mobile sensors with limited communication ranges by developing cooperative switching coverage control laws for each sensor in [23].

It is noted that much attention is paid to the coverage control problem for mobile sensors with first-order dynamics in a one-dimensional mission space [14–16, 22–24]. On the one hand, many mobile sensors in real-world applications are of more general dynamics, and the investigation of the coverage control problem for mobile sensors with more general dynamics is thus warranted. First, the double-integrator dynamics contain the acceleration term to take the effect of inertia into consideration, which is important for safety reasons. Second, many mechanical systems governed by the force-position model are of double-integrator dynamics, and the corresponding results can be further extended to more complex systems, such as formation control of unmanned vehicles or even Euler-Lagrange systems [25]. In fact, it is more challenging to study the coverage control problem for mobile sensors with double-integrator dynamics than first-order dynamics on the circle. It is worth noting that the velocity constraints are input saturation constraints for mobile sensors with first-order dynamics in [22–24]. In contrast, for sensors with double-integrator dynamics, the velocity constraints become state saturation constraints. Consequently, the existing approaches dealing with input saturation constraints such as low gain feedback cannot be applied to this case. On the other hand, in real-world applications, mobile sensors are often subject to unmeasurable disturbances due to their environment. These two aspects motivate us to consider the coverage control problem for networked mobile sensors with double-integrator dynamics subject to unknown but bounded (UBB) disturbances in this work.

Several challenges would arise when considering both double-integrator dynamics and UBB disturbances in the coverage control problem. First, how to achieve both the spatial order preservation and maximum velocity constraints for mobile sensors with double-integrator dynamics in the coverage control problem. Though the spatial order preservation is similar to the distance maintaining problem, there are few results considering such a problem with velocity constraints for second-order multiagent systems except [26], where input and velocity constraints, connectivity maintenance, and collision avoidance are investigated simultaneously for second-order multiagent systems. However, the introduction of both a saturation function and a barrier function in control input [26] would significantly increase difficulties for stability analysis of the concerned coverage control problem. Second, how to preserve the spatial order throughout the coverage task in the presence of UBB disturbances. In [27–29], the collision avoidance or obstacle avoidance are guaranteed in the presence of disturbances. However, the methods proposed in those results for distance maintenance in the presence of UBB disturbances are not applicable for the stability analysis of the coverage control problem. Third, how to obtain the upper bound on the coverage cost function as time goes to infinity in the presence of UBB disturbances. On the one hand, sensors will keep moving in their mission space in the presence of UBB disturbances, which implies that the coverage cost function would keep changing. To obtain the upper bound on the coverage cost function in this case, the distance between neighboring sensors needs to be driven into a predictable set as time goes to infinity. On the other hand, the term in the control input designed for spatial order preservation, which can be named as the 'order preservation term' for simplicity, affects the upper bound analysis of the coverage cost function. Thus, the order preservation term also needs to be driven to zero in the presence of UBB disturbances to obtain a tighter upper bound on the coverage cost function.

To overcome these challenges, several important results are obtained for the coverage control problem on a circle. The main contributions of this paper can be summarized as follows. First, compared to previous results [17, 22–24] where the first-order dynamics are considered, this paper develops cooperative coverage control laws for each sensor with double-integrator dynamics, while the order constraints and velocity constraints are considered simultaneously. To handle the difficulties raised by the existence of both input saturation constraints and state saturation constraints, as stated in the previous paragraph, the cooperative coverage control laws are developed by proposing a new generalized energy function which is inspired by [30]. It is worth noting that the reference trajectory is available for all agents in [30], which makes the corresponding convergence analysis more similar to the single-agent system problem. However, in the coverage control problem, mobile sensors need to minimize the coverage cost function by communicating with their neighbors. Thus, the convergence analysis in this work is conducted by taking advantage of the circular communication topology. Second, the UBB disturbances are considered in the coverage control problem and an upper bound on the coverage cost function when time goes to infinity is provided. Contrary to existing results [31–33] in which the UBB disturbances are considered, the objective of this paper is to minimize the coverage cost function that is a nonlinear function with respect to the system states. Then, the bound on the coverage cost function under UBB disturbances cannot be obtained from the closed-loop dynamics. In addition, though [24] and [34] provide the upper bound on the coverage cost function when the measurement errors are investigated, the upper bound on the coverage cost function can be determined by its optimal value and the measurement errors. However, when the UBB disturbances are considered in this work, only the bound on the set into which the system will be driven is available. Thus, an approach to obtain a tighter upper bound on the coverage cost function under the UBB disturbances is provided.

The rest of this paper is organized as follows. In Section II, the problem formulation is given. Section III presents the cooperative coverage control laws. In Section IV, convergence analysis of the coverage control laws is provided. The simulation results are shown in Section V and conclusions are summarized in Section VI.

II. Problem Formulation

Consider a group of n mobile sensors which are constrained to move on a unit circle. The position of an arbitrary point q on the circle is described by the angle measured counterclockwise from the positive direction of the horizontal axis. The remaining part of this work is all based on the angle-measured position on the circular mission field. The position of sensor i is represented by q_i. Let $\mathcal{F}$ denote the set of all points on a unit circle. The physical distance between point $q \in \mathcal{F}$ and sensor i is calculated by $d(q_i, q) = \min\{\bar{d}(q_i, q), 2\pi - \bar{d}(q_i, q)\}$, where $\bar{d}(q_i, q) = (q - q_i) \bmod 2\pi$ is the distance between sensor i and point q in the counterclockwise direction.

Assume that the mobile sensors evolve according to the following double-integrator dynamics,

$$\dot{q}_i = v_i, \dot{v}_i = u_i + \Delta_i(t), \tag{1}$$

where v_i and u_i are the velocity and control input of sensor i, respectively. $\Delta_i(t)$ represents the unmeasurable disturbance due to the environment, whose absolute value is supposed to be bounded by D_i which is known by each sensor *a priori* [35, 36]. That is, $-D_i \leq \Delta_i(t) \leq D_i$, $\forall i \in \mathcal{I}_n = \{1, \cdots, n\}$. In practice, each mobile sensor is often subject to a maximum velocity λ_i. This imposes input constraints on the sensors with first-order dynamics. In contrast, for mobile sensors with double-integrator dynamics considered in this work, it imposes heterogeneous state constraints, that is, $-\lambda_i \leq v_i \leq \lambda_i$, $\forall i \in \mathcal{I}_n$. Leveraging the periodic property of a circle, we can define $\lambda_{n+i} \equiv \lambda_i$, $v_{n+i} \equiv v_i$ and $q_{n+i} \equiv q_i + 2\pi$ for all $i \in \mathcal{I}_n$.

In order to simplify the subsequent analysis, the sensors are sorted in the counterclockwise direction based on their initial positions, that is, $0 \leq q_1(0) < \cdots < q_i(0) < q_{i+1}(0) < \cdots < q_n(0) < 2\pi$. Assume that the initial distance between each sensor i and $i+1$ is larger than $\sqrt{2\theta_i}$ and all sensors remain static at the beginning, that is,

$$q_{i+1}(0) - q_i(0) > \sqrt{2\theta_i},\ v_i(0) = 0 \tag{2}$$

holds for all $i \in \mathcal{I}_n$, where θ_i is a positive constant which will be defined later. For practical consideration that the shape of mobile sensors are not negligible, the spatial order of the MSN is said to be preserved if and only if $q_{i+1}(t) - q_i(t) > \sqrt{2\theta_i}$, $\forall i \in \mathcal{I}_n$ always holds, which implies that $\sqrt{2\theta_i}$ is larger than the sum of the radii of the collision regions for sensor i and sensor $i+1$. Assume that each sensor can only communicate with its immediate counterclockwise and clockwise sensors on the circle. Then, the communication topology of the MSN will be fixed if the sensors spatial order is preserved throughout the coverage task.

The coverage cost function is defined as follows,

$$T(q_1, ..., q_n) = \max_{q \in \mathcal{F}} \min_{i \in \mathcal{I}_n} d(q_i, q)/\lambda_i, \tag{3}$$

where $\min_{i \in \mathcal{I}_n} d(q_i, q)/\lambda_i$ is the arrival time for the point q. In other words, the minimum arrival time to q is determined by choosing sensor i from the sensor set $\mathcal{I}_n$ of the MSN. After the arrival time for the point q has been decided, the coverage cost function $T(q_1, ..., q_n)$ is chosen as the largest arrival time with respect to the point q from the set $\mathcal{F}$ on the circle, which implies the largest arrival time from the MSN to the whole mission space [22]. Assuming that the sensing ranges of mobile sensors are negligible with respect to the circumference of a circle, a smaller coverage cost function implies a higher possibility of handling the events taking place on the circle. In this paper, our aim is to design cooperative coverage control laws to drive networked mobile sensors subject to heterogeneous state constraints and unknown but bounded disturbances to a neighborhood of the optimal configuration minimizing $T(q_1, ..., q_n)$ while preserving their spatial order on the circle throughout the coverage task.

Remark 1: This problem can be applied to many practical scenarios including the case in which the sensors are not initially placed on the circle. In this case, the design of the coverage control laws can be divided into the design of circle formation laws and that of coverage control laws on the circle. More details can be found in Remark 6 of [37].

III. Cooperative Coverage Control Laws

In this section, cooperative coverage control laws are proposed for mobile sensors with double integrator dynamics, and a generalized energy function is employed to guarantee order preservation of the sensors.

Before proceeding, a useful lemma is given as follows.

Lemma 1 ([30]): Let the scalar function $h(x, \theta, \gamma)$ be defined as $h(x, \theta, \gamma) = \frac{f(\tau)}{f(\tau)+f(1-\tau)}$ with $\tau = \frac{x-\theta}{\gamma-\theta}$, where $f(\tau) = 0$ if $\tau \leq 0$ and $f(\tau) = e^{-\frac{1}{\tau}}$ if $\tau > 0$, θ and γ are constants such that $0 < \theta < \gamma$, whose ranges will be discussed in Lemma 5 and Lemma 6. Then, the function $h(x, \theta, \gamma)$ is a smooth step function possessing the following properties: 1) $h(x, \theta, \gamma) = 0, \forall x \in (-\infty, \theta]$, 2) $h(x, \theta, \gamma) = 1, \forall x \in [\gamma, \infty)$, 3) $0 < h(x, \theta, \gamma) < 1, \forall x \in (\theta, \gamma)$, 4) $h(x, \theta, \gamma)$ is smooth, 5) $h'(x, \theta, \gamma) > 0, \forall x \in (\theta, \gamma)$, where $h'(x, \theta, \gamma) = \frac{\partial h(x,\theta,\gamma)}{\partial x}$.

Inspired by [30], the generalized distance function between sensor i and sensor j is defined as follows,

$$g_{i,j} = q_{i,j}(\frac{1}{2}q_{i,j} + v_{i,j}),\ i, j \in \mathcal{I}_n, \tag{4}$$

where $q_{i,j} = q_j - q_i$, and $v_{i,j} = v_j - v_i$. Then, one has $g_{i,j} \equiv g_{j,i}$. Using the generalized distance $g_{i,j}$ and the smooth step function $h(x, \theta, \gamma)$ defined in Lemma 1, the following generalized energy function is proposed,

$$P_{i,j} = \begin{cases} 0, & g_{i,j} \geq \gamma_i \\ E_{i,j}(g_{i,j}, \theta_i, \gamma_i) - \frac{\Phi_i}{\sqrt{2\theta_i}}(g_{i,j} - \gamma_i), & \theta_i < g_{i,j} < \gamma_i \end{cases}, \tag{5}$$

where $E_{i,j}(g_{i,j}, \theta_i, \gamma_i) = \frac{1-h(g_{i,j},\theta_i,\gamma_i)}{h(g_{i,j},\theta_i,\gamma_i)}$ with γ_i and θ_i are constants which satisfy the requirement of the parameters of function h in Lemma 1, that is, $0 < \theta_i < \gamma_i$, and $\Phi_i > 0$ is an auxiliary parameter which can be used to tune the upper bound on the coverage cost function in the presence of the UBB disturbances. This will be discussed more in Remark 6. Note that the subscript i always denotes the sensor with smaller counterclockwise distance from the start point between the pair (i, j). Then, it can be verified that the energy function (5) possesses the following properties:

$$\begin{aligned} &1)\ P_{i,j} = 0,\ P'_{i,j} = 0,\ \forall g_{i,j} \in [\gamma_i, +\infty), \\ &2)\ P_{i,j} > 0,\ P'_{i,j} < -\tfrac{\Phi_i}{\sqrt{2\theta_i}},\ \forall g_{i,j} \in (\theta_i, \gamma_i), \\ &3)\ \lim_{g_{i,j}\to\theta_i} P_{i,j} = +\infty,\ \lim_{g_{i,j}\to\theta_i} P'_{i,j} = -\infty, \\ &4)\ P_{i,j} \text{ is continuous}, \forall g_{i,j} \in (\theta_i, +\infty), \end{aligned} \tag{6}$$

where $P'_{i,j} = \partial P_{i,j}/\partial g_{i,j}$. From the definition of $g_{i,j}$ and $P_{i,j}$, one has $P_{i,j} \equiv P_{j,i}$ and $P'_{i,j} \equiv P'_{j,i}$. The first two properties provide a mechanism to activate $P'_{i,j}$ whenever $g_{i,j} \in (\theta_i, \gamma_i)$. When $P'_{i,j}$ is activated, the upper bound of $P'_{i,j}$ is also guaranteed, which will be used to obtain $u_i^{ord} = 0$ as shown in Lemma 6. The property 3) implies that $P'_{i,j}$ will tend to infinity if $g_{i,j}$ approaches θ_i. Moreover, it will be shown in the proof of Lemma 5 that $g_{i,i+1} > \theta_i$, $\forall i \in \mathcal{I}_n$ always holds under the initial condition (2), coverage control laws (7) and parameters chosen according to (15). The property 4) guarantees that $P'_{i,j}$ is always bounded in both (θ_i, γ_i) and $[\gamma_i, +\infty)$, which is necessary to show the monotone decreasing property of the Lyapunov-like function in the convergence analysis.

To guarantee order preservation of the sensors and to satisfy their velocity constraints, the following cooperative coverage control laws are proposed in this paper,

$$u_i = -(v_i - \mu_i \mathrm{sat}(\Omega_i)),\ i \in \mathcal{I}_n, \tag{7}$$

where $\mathrm{sat}(\cdot) : \mathbb{R} \to \mathbb{R}$ is a standard saturation function which is defined by $\mathrm{sat}(x) = \mathrm{sign}(x) \min\{1, |x|\}$, μ_i is a positive constant whose range will be determined in Lemma 2, and

$$\Omega_i = \frac{u_i^{cov} + u_i^{ord}}{\Phi_i}, \tag{8}$$

$$u_i^{cov} = \frac{q_{i,i+1} + v_{i,i+1}}{\lambda_i + \lambda_{i+1}} + \frac{q_{i,i-1} + v_{i,i-1}}{\lambda_i + \lambda_{i-1}}, \tag{9}$$

$$u_i^{ord} = P'_{i,i-1} q_{i,i-1} + P'_{i,i+1} q_{i,i+1}, \tag{10}$$

with $P'_{i,i-1} = \partial P_{i,i-1}/\partial g_{i,i-1}$, $P'_{i,i+1} = \partial P_{i,i+1}/\partial g_{i,i+1}$.

Remark 2: In the control laws (7), the saturation function and the velocity feedback v_i are used to satisfy the maximum velocity constraints, which will be shown in the following lemma. However, the saturation function bounds the term u_i^{ord} which is designed for spatial order preservation in the control input. Thus, under the bounded term u_i^{ord}, even if the distance between two mobile sensors is large, they may still exchange their spatial order after some time considering double-integrator dynamics if their relative velocity is large enough. To preserve the spatial order while maintaining the maximum velocity constraints, the generalized distance $g_{i,j}$ is introduced, which contains not only distances $q_{i,j}$ but also relative velocities $v_{i,j}$ [30]. Thus, the larger relative velocity $|v_{i,j}|$ towards each other leads to the smaller generalized distance $g_{i,j}$. Then, recalling (10) and the property (6) that $P'_{i,j} < -\frac{\Phi_i}{\sqrt{2\theta_i}}$ if $g_{i,j} < \gamma_i$, the larger relative velocity $|v_{i,j}|$ decreasing $|q_{i,j}|$ will activate u_i^{ord} in larger distance compared with other potential functions which contain only the distance information, such as [38]. According to (7)-(10), the decrease of u_i^{ord} leads to smaller control input, which implies that v_i will not increase any more. Thus, one can find from Lemma 5 that sensor i will never chase up sensor $i + 1$ in the counterclockwise direction. This property provides a method to avoid the spatial order exchange under the bounded term u_i^{ord} for mobile sensors with double-integrator dynamics. In this way, both velocity saturation constraints and spatial order preservation can be tackled simultaneously. In addition, the generalized energy function (5) is also designed to drive the term u_i^{ord} to zero as time goes to infinity as shown in Lemma 6, so that the mobile sensors are driven towards their optimal positions only under the term u_i^{cov}, which will be used to obtain a tighter upper bound on the coverage cost function.

To show that the maximum velocity constraints and spatial order preservation will be both tackled under the coverage control laws (7), we establish the following lemma. This lemma is proposed to prove that the velocity constraint $-\lambda_i \leq v_i \leq \lambda_i$ can be satisfied for each sensor throughout the coverage task.

Lemma 2: Given the initial condition (2), if the parameter μ_i satisfies the following inequality

$$\mu_i < \lambda_i - D_i,\ i \in \mathcal{I}_n, \tag{11}$$

the velocity constraints $-\lambda_i \leq v_i \leq \lambda_i$ can be satisfied for all sensors in the network under the coverage control laws (7).

Proof: From the system dynamics (1) and the coverage control laws (7), one can get the close-loop dynamic system $\dot{v}_i = -(v_i - \mu_i \mathrm{sat}(\Omega_i)) + \Delta_i$. Noting the initial condition $v_i(0) = 0$, $\forall i \in \mathcal{I}_n$ in (2), the solution of the given dynamical system is $v_i(t) = \int_0^t (\mu_i \mathrm{sat}(\Omega_i(\tau)) + \Delta_i(\tau)) e^{-(t-\tau)} d\tau$, which suggests that

$$\begin{cases} v_i(t) \leq \int_0^t e^{-(t-\tau)} d\tau \max\{\mu_i \mathrm{sat}(\Omega_i(t)) + \Delta_i(t)\} \\ v_i(t) \geq \int_0^t e^{-(t-\tau)} d\tau \min\{\mu_i \mathrm{sat}(\Omega_i(t)) + \Delta_i(t)\} \end{cases}. \tag{12}$$

Note that $\int_0^t e^{-(t-\tau)} d\tau = 1 - e^{-t}$. From the definition of the saturation function and the bounds of the disturbances $\Delta_i(t)$, it can be obtained that $\max\{\mu_i \mathrm{sat}(\Omega_i(t)) + \Delta_i(t)\} = \mu_i + D_i$ and $\min\{\mu_i \mathrm{sat}(\Omega_i(t)) + \Delta_i(t)\} = -\mu_i - D_i$. Recalling the inequality

(12), one has

$$|v_i(t)| \leq \mu_i + D_i. \tag{13}$$

Thus, if the inequality (11) is satisfied, one has $|v_i(t)| \leq \mu_i + D_i < \lambda_i$ holds for all $t > 0$. ∎

Remark 3: The velocity constraints are necessary for the coverage control problem on a circle. Recall that the coverage cost function is the maximum response time from the MSN to any point on the circle. Once the networked mobile sensors reach their positions where the coverage cost function is minimized, the assigned sensor is supposed to respond to the emergency occurred on the circle at its maximum velocity. Thus, the maximum velocities have been used to define the coverage cost function and they should always hold throughout the coverage task. Moreover, note that the velocity constraints are achieved by a velocity damping term $-v_i$ and the saturation function $\mathrm{sat}(\cdot)$ in the proposed control laws (7). Then, the bounds on the control input can also be determined, which means that a control gain can be applied to adjust the control input amplitude. It follows from inequality (13) and the proposed control laws (7) that, $|u_i| \leq 2\mu_i + D_i, \forall i \in \mathcal{I}_n$. Consequently, under the control law $\bar{u}_i = k_i u_i$ and the maximum control input τ_i, given the condition $0 < k_i \leq \frac{\tau_i}{2\mu_i + D_i}$, one has $|u_i| \leq \tau_i$, $\forall i \in \mathcal{I}_n$. Thus, a low control gain k_i can always be selected for each sensor to guarantee that the control input $|u_i|$ will never violate its maximum value τ_i during the coverage mission.

IV. Convergence Analysis

With the existence of UBB disturbances, the convergence of the closed-loop dynamics of the MSN and the spatial order preservation of the sensors are firstly analyzed by using the Lyapunov stability theory. Then, it will be shown that the MSN can be driven to a neighborhood of the optimal configuration under the proposed coverage control laws and an upper bound on the coverage cost function when time goes to infinity is also provided.

Before proceeding, two useful lemmas are given as follows.

Lemma 3 ([30]): Assume the vector $\boldsymbol{x}(t) \in \mathbb{R}^n$ satisfies the following conditions, $\|\boldsymbol{x}(t_0)\| > a_0$, $\boldsymbol{x}(t)(\dot{\boldsymbol{x}}(t) + \boldsymbol{B}\boldsymbol{x}(t)) > a$, $\forall t \geq t_0 \geq 0$, where $t_0 \geq 0$ is the initial time, $\boldsymbol{B}$ is a symmetric positive-definite matrix, and a_0 and a are strictly positive constants. Then,

$$\|\boldsymbol{x}(t)\| > \min(a_0, \sqrt{\frac{a}{\lambda_M(\boldsymbol{B})}}) \tag{14}$$

for all $t \geq t_0 \geq 0$, where $\lambda_M(\boldsymbol{B})$ is the maximum eigenvalue of the matrix $\boldsymbol{B}$.

Lemma 4 ([22]): The coverage cost function $T(q_1, ..., q_n)$ is minimized if and only if $\frac{\bar{d}(q_i, q_{i+1})}{\lambda_i + \lambda_{i+1}} = \frac{\bar{d}(q_j, q_{j+1})}{\lambda_j + \lambda_{j+1}}$, $\forall i, j \in \mathcal{I}_n$, where $\bar{d}(q_i, q_{i+1})$ is the counterclockwise distance from sensor i to $i+1$. Moreover, $T(q_1, ..., q_n) \leq \max_{i \in \mathcal{I}_n} \bar{d}(q_i, q_{i+1})/(\lambda_i + \lambda_{i+1})$ and the minimum value of the function $T(q_1, ..., q_n)$ is $T^* = \pi / \sum_{i=1}^n \lambda_i$.

In the following, we propose two key lemmas in this work. First, it is shown that the sensors' spatial order on the circle can be always preserved in Lemma 5, where a Lyapunov function is specially chosen for stability analysis of the coverage control problem. Thus, the existence of both the saturation function and energy function terms in the control laws can be handled simultaneously in the stability analysis. Utilizing the proposed control laws (7) and the generalized energy function (5), the term u_i^{cov} will be constructed by differentiating the chosen Lyapunov function. Then, it can be derived that $|u_i^{cov} + u_i^{ord}|$ will be driven into a predictable set $\mathcal{K}$, which will be used to obtain the upper bound on $|u_i^{cov}|$. After obtaining the range of u_i^{cov}, the coverage performance of the proposed control laws (7) can be consequently assessed by deriving the upper bound on the coverage cost function as shown in Theorem 1. Moreover, from the stability analysis in this lemma, the effect of UBB disturbances can be also compensated while maintaining spatial order preservation.

Lemma 5: Under the initial condition (2) and the coverage control laws (7), if the parameters μ_i, θ_i and γ_i are chosen such that

$$\begin{cases} \sum_{i=1}^{n} \sqrt{2\theta_i + (v_i^M + v_{i+1}^M)^2 + 2\epsilon_i} < 2\pi & \text{(15a)} \\ \eta_i + \Sigma_{i=1}^n D_i < \mu_i < \lambda_i - D_i & \text{(15b)} \\ 0 < \theta_i < \gamma_i, & \text{(15c)} \\ v_i^M = \mu_i + D_i, \ \forall i \in \mathcal{I}_n & \text{(15d)} \end{cases}$$

where ϵ_i and η_i are positive constants, one has the following two conclusions:

(1). The networked mobile sensors will be driven into the set $\mathcal{K} = \{(q_i, v_i)| \ |u_i^{cov} + u_i^{ord}| < \Phi_i, \forall i \in \mathcal{I}_n\}$ asymptotically;

(2). The spatial order of the mobile sensors is always preserved throughout the coverage task, that is, the inequalities $q_{i+1}(t) - q_i(t) > \sqrt{2\theta_i}, \forall i \in \mathcal{I}_n$ always hold.

The proof of Lemma 5 can be found in Appendix.

Remark 4: In this work, the distance between each pair of neighboring sensors is proved to be larger than a positive constant. In contrast to the existing works [22–24, 39, 40] where the distance $q_{i+1}(k) - q_i(k)$ is only proved to be larger than zero for all $i \in \mathcal{I}_n$, the result in this paper can be applied to more general scenarios where the shape of sensors is not negligible.

Second, Lemma 6 is established to show that the order preservation term u_i^{ord} can be driven to zero under the proposed control laws (7) if the parameter γ_i is smaller than a certain value. This lemma is critical to obtain a tighter upper bound on the coverage cost function, which will be shown in Theorem 1.

Lemma 6: Under the coverage control laws (7), if the parameters μ_i, θ_i and γ_i are chosen such that (15) and

$$\gamma_i \leq \frac{1}{2}\{[\pi(\lambda_i + \lambda_{i+1}) / \sum_{l=1}^{n} \lambda_l]^2 - (v_i^M + v_{i+1}^M)^2\} \tag{16}$$

hold, and if

$$|u_i^{cov} + u_i^{ord}| < \Phi_i, \ \forall i \in \mathcal{I}_n, \tag{17}$$

one has

$$u_i^{ord} = 0, \ \forall i \in \mathcal{I}_n. \tag{18}$$

Proof: It is firstly shown that there must not exist any pair of neighboring sensors i and $i+1$, such that $q_{i,i+1} + v_{i,i+1} < \sqrt{2\gamma_i + v_{i,i+1}^2}$ by a contradiction. Recalling $\sum_{i=1}^{n}[\pi(\lambda_i + \lambda_{i+1})/\sum_{l=1}^{n} \lambda_l] = 2\pi$ and equation (33), one has $\sum_{i=1}^{n}(q_{i,i+1} + v_{i,i+1}) = \sum_{i=1}^{n}[\pi(\lambda_i + \lambda_{i+1})/\sum_{l=1}^{n} \lambda_l]$. Under this constraint, the value of $q_{i,i+1} + v_{i,i+1}$ for each pair of neighboring sensors can be only classified into the following two cases:

1) Each pair of neighboring sensors satisfies $q_{i,i+1} + v_{i,i+1} = \pi(\lambda_i + \lambda_{i+1})/\sum_{l=1}^{n} \lambda_l$, $\forall i \in \mathcal{I}_n$;

2) Let $\Omega_1 = \{i \in \mathcal{I}_n | q_{i,i+1} + v_{i,i+1} \geq \pi(\lambda_i + \lambda_{i+1})/\sum_{l=1}^{n} \lambda_l\}$ and $\Omega_2 = \{i \in \mathcal{I}_n | q_{i,i+1} + v_{i,i+1} < \pi(\lambda_i + \lambda_{i+1})/\sum_{l=1}^{n} \lambda_l\}$. The two sets Ω_1 and Ω_2 are both non-empty.

Assume that there exists a pair of neighboring sensors i and $i+1$, such that

$$q_{i,i+1} + v_{i,i+1} < \sqrt{2\gamma_i + v_{i,i+1}^2}. \tag{19}$$

It follows from inequality (13) and (15d) that $v_{i,i+1}^2 \leq (|v_i| + |v_{i+1}|)^2 \leq (\mu_i + D_i + \mu_{i+1} + D_{i+1})^2 = (v_i^M + v_{i+1}^M)^2$. Recalling (16), one has $\gamma_i \leq \frac{1}{2}\{[\pi(\lambda_i + \lambda_{i+1})/\sum_{l=1}^{n} \lambda_l]^2 - v_{i,i+1}^2\}$ which is equivalent to $\sqrt{2\gamma_i + v_{i,i+1}^2} \leq \pi(\lambda_i + \lambda_{i+1})/\sum_{l=1}^{n} \lambda_l$, $\forall i \in \mathcal{I}_n$. Then, one has $q_{i,i+1} + v_{i,i+1} < \pi(\lambda_i + \lambda_{i+1})/\sum_{l=1}^{n} \lambda_l$, which implies that case 1) is eliminated under this assumption. Accordingly, case 2) arises, in other words, all the sensors in the network will be only classified into Ω_1 and Ω_2, and both Ω_1 and Ω_2 are non-empty. Thus, there exists at least one pair of neighboring sensors, denoted as $j-1$ and j, such that one of them belongs to set Ω_1 while the other belongs to set Ω_2.

Without loss of generality, assume $j-1 \in \Omega_1$ and $j \in \Omega_2$. Recalling (16), (19) and the definitions of Ω_1 and Ω_2, one has

$$\begin{cases} q_{j-1,j} + v_{j-1,j} \geq \pi(\lambda_j + \lambda_{j-1})/\sum_{l=1}^{n} \lambda_l \geq \sqrt{2\gamma_{j-1} + v_{j,j-1}^2} \\ q_{j,j+1} + v_{j,j+1} < \sqrt{2\gamma_j + v_{j,j+1}^2} \leq \pi(\lambda_j + \lambda_{j+1})/\sum_{l=1}^{n} \lambda_l \end{cases}. \tag{20}$$

Then, the following inequality can be obtained,

$$u_j^{cov} = \frac{q_{j,j+1} + v_{j,j+1}}{\lambda_j + \lambda_{j+1}} - \frac{q_{j-1,j} + v_{j-1,j}}{\lambda_j + \lambda_{j-1}} < 0. \tag{21}$$

Next, the upper bound of u_j^{ord} will be derived. From the inequalities (20), one has $(1/2)(q_{j,j+1} + v_{j,j+1})^2 = (1/2)q_{j,j+1}^2 + q_{j,j+1}v_{j,j+1} + (1/2)v_{j,j+1}^2 < \gamma_j + (1/2)v_{j,j+1}^2$ and $(1/2)(q_{j,j-1} + v_{j,j-1})^2 = (1/2)q_{j,j-1}^2 + q_{j,j-1}v_{j,j-1} + (1/2)v_{j,j-1}^2 \geq \gamma_{j-1} + (1/2)v_{j,j-1}^2$. Furthermore, it can be obtained that $g_{j,j+1} < \gamma_j$ and $g_{j,j-1} \geq \gamma_{j-1}$. From the property (6) of the generalized energy function (5), one has $P'_{j,j-1} = 0$ and $P'_{j,j+1} < -\frac{\Phi_j}{\sqrt{2\theta_j}}$. Moreover, according to the condition (15) and Lemma 5, the spatial order of the sensor network is preserved, which means that $q_{j,j+1} > \sqrt{2\theta_j}$. It then follows from equation (10) that $u_j^{ord} = P'_{j,j+1}q_{j,j+1} < -\Phi_j$. Thus, recalling inequality (21), one has $u_j^{cov} + u_j^{ord} < -\Phi_j$, which contradicts the condition (17). Via the argument of contradiction, one has that

$$q_{i,i+1} + v_{i,i+1} \geq \sqrt{2\gamma_i + v_{i,i+1}^2}, \ \forall i \in \mathcal{I}_n. \tag{22}$$

Then, it follows from (22) that $g_{i,i+1} = \frac{1}{2}q_{i,i+1}^2 + q_{i,i+1}v_{i,i+1} \geq \gamma_i$ and thereby $u_i^{ord} = 0$, $\forall i \in \mathcal{I}_n$ according to the equation (10) and property (6). Thus, equation (18) can be finally obtained. ■

Now we are ready to present the main results, that is, the MSN under the control laws (7) will be driven to the neighborhood of their optimal positions.

Theorem 1: If the parameters μ_i, θ_i and γ_i are chosen such that (15) and (16) hold, under the proposed coverage control laws (7), networked mobile sensors will be driven to a neighborhood of the optimal configuration minimizing the coverage cost function $T(q_1, ..., q_n)$ with order preservation such that

$$0 \leq T(q_1, ..., q_n) - T^* \leq \max_{i \in \mathcal{I}_n}\{Q_i\} - \frac{\pi}{\Sigma_{i=1}^n \lambda_i}, \tag{23}$$

where

$$Q_i = \frac{G_i + \Delta d_i^-(\lambda_{i-1} + \lambda_i) + \Delta d_i^+(\lambda_{i+1} + \lambda_{i+2})}{\lambda_{i-1} + 2\lambda_i + 2\lambda_{i+1} + \lambda_{i+2}}, \tag{24a}$$

$$\Delta d_i^- = \Phi_i + (v_i^M + \max\{v_{i-1}^M, v_{i+1}^M\})(\frac{1}{\lambda_i + \lambda_{i+1}} + \frac{1}{\lambda_i + \lambda_{i-1}}), \tag{24b}$$

$$\Delta d_i^+ = \Phi_i + (v_{i+1}^M + \max\{v_i^M, v_{i+2}^M\})(\frac{1}{\lambda_{i+1} + \lambda_{i+2}} + \frac{1}{\lambda_{i+1} + \lambda_i}), \tag{24c}$$

$$G_i = 2\pi - \Sigma_{j \in \mathcal{I}_n, j \neq \{i-1,i,i+1\}}[\sqrt{2\gamma_j} - (v_j^M + v_{j+1}^M)]. \tag{24d}$$

Proof: In the first conclusion of Lemma 5, it has been shown that the MSN will be driven to the set $\mathcal{K} = \{(q_i, v_i) | \ |u_i^{cov} + u_i^{ord}| < \Phi_i, \forall i \in \mathcal{I}_n\}$ asymptotically. Then, it follows from Lemma 6 that the equation (18) holds when time goes to infinity. Therefore, one has $|u_i^{cov}| < \Phi_i$, $\forall i \in \mathcal{I}_n$ as time goes to infinity, which implies that $-\Phi_i - (\frac{v_{i,i+1}}{\lambda_i + \lambda_{i+1}} + \frac{v_{i,i-1}}{\lambda_i + \lambda_{i-1}}) < \frac{q_{i,i+1}}{\lambda_i + \lambda_{i+1}} + \frac{q_{i,i-1}}{\lambda_i + \lambda_{i-1}} < \Phi_i - (\frac{v_{i,i+1}}{\lambda_i + \lambda_{i+1}} + \frac{v_{i,i-1}}{\lambda_i + \lambda_{i-1}})$. It follows from inequality (13) and (15d) that $|v_{i,i+1}| \leq |v_i| + |v_{i+1}| \leq \mu_i + D_i + \mu_{i+1} + D_{i+1} = v_i^M + v_{i+1}^M$. Consequently,

$$|\frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}} - \frac{q_i - q_{i-1}}{\lambda_i + \lambda_{i-1}}| < \Phi_i + (v_i^M + \max\{v_{i-1}^M, v_{i+1}^M\}) (\frac{1}{\lambda_i + \lambda_{i+1}} + \frac{1}{\lambda_i + \lambda_{i-1}}), \tag{25}$$

which sets a constraint on the difference of $\frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}}$ among each pair of neighboring sensors.

Next, the upper bound on $\frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}}$ will be derived for each sensor i and sensor $i+1$ via the following three steps.

Step 1: The upper bound on $\frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}}$ can be converted to $\max_{q_i \in \mathcal{F}, i \in \mathcal{I}_n} \min\{U_1, U_2, U_3\}$.

It follows from (25) that $\frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}}$ will be upper bounded by $U_1 = \frac{q_i - q_{i-1}}{\lambda_{i-1} + \lambda_i} + \Delta d_i^-$, where Δd_i^- is given by (24b). Similarly, it will also be upper bounded by $U_2 = \frac{q_{i+2} - q_{i+1}}{\lambda_{i+1} + \lambda_{i+2}} + \Delta d_i^+$, where Δd_i^+ is given by (24c). Thus, a tighter upper bound on $\frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}}$ can be represented by $\min\{U_1, U_2, U_3\}$, where $U_3 = \frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}}$. However, $\min\{U_1, U_2, U_3\}$ will be time-varying depending on the evolution of q_{i-1}, q_i, q_{i+1} and q_{i+2}. Thus, for each sensor i and sensor $i+1$, the tighter upper bound on $\frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}}$ is equivalent to $\max_{q_i \in \mathcal{F}, i \in \mathcal{I}_n} \min\{U_1, U_2, U_3\}$.

Step 2: Under the condition that $q_{i+2} - q_{i-1}$ is fixed, obtain the value of $\max_{q_i, q_{i+1} \in [q_{i-1}, q_{i+2}]} \min\{U_1, U_2, U_3\}$.

Suppose that the difference $q_{i+2} - q_{i-1}$ is fixed, which can be expressed by

$$\sum_{j=i-1}^{i+1} (q_{j+1} - q_j) = C_i, \tag{26}$$

where $0 < C_i < 2\pi$. Note that U_1, U_2, U_3 are all linear functions of $q_i - q_{i-1}$, $q_{i+2} - q_{i+1}$ and $q_{i+1} - q_i$, respectively. The increase of U_1 for a given mobile sensor network can be only caused by the increase of $q_i - q_{i-1}$, and U_2, U_3 also have similar properties. Thus, the increase of any element in the set $\{U_1, U_2, U_3\}$ causes the decrease of one of the remaining elements in that set under the constraint (26). Then, the value of $\max_{q_i, q_{i+1} \in [q_{i-1}, q_{i+2}]} \min\{U_1, U_2, U_3\}$ with the constraint (26) can be obtained under the following condition,

$$U_1 = U_2 = U_3. \tag{27}$$

In other words, any other solution deviating from the equilibrium point (27) will decrease the value of $\min\{U_1, U_2, U_3\}$ under the constraint (26). Thus, by combining the definitions of U_1, U_2, U_3 and the conditions (26) and (27), one has

$$\max_{q_i, q_{i+1} \in [q_{i-1}, q_{i+2}]} \min\{U_1, U_2, U_3\} = \frac{C_i + \Delta d_i^-(\lambda_{i-1} + \lambda_i) + \Delta d_i^+(\lambda_{i+1} + \lambda_{i+2})}{\lambda_{i-1} + 2\lambda_i + 2\lambda_{i+1} + \lambda_{i+2}}, \tag{28}$$

if $q_{i+2} - q_{i-1}$ is fixed.

Step 3: The value of $\max_{q_i \in \mathcal{F}, i \in \mathcal{I}_n} \min\{U_1, U_2, U_3\}$ is finally obtained by finding the upper bound on C_i.

Due to $\min\{U_1, U_2, U_3\}$ is only related to the positions of sensors $i-1$, i, $i+1$ and $i+2$, one has the following equations, $\max_{q_i \in \mathcal{F}, i \in \mathcal{I}_n} \min\{U_1, U_2, U_3\} = \max_{\{q_{i-1}, q_i, q_{i+1}, q_{i+2}\} \in \mathcal{F}} \min\{U_1, U_2, U_3\} = \max_{\{q_{i-1}, q_{i+2}\} \in \mathcal{F}} \max_{q_i, q_{i+1} \in [q_{i-1}, q_{i+2}]} \min\{U_1, U_2, U_3\} = \max_{C_i \in (0, 2\pi)} \frac{C_i + \Delta d_i^-(\lambda_{i-1} + \lambda_i) + \Delta d_i^+(\lambda_{i+1} + \lambda_{i+2})}{\lambda_{i-1} + 2\lambda_i + 2\lambda_{i+1} + \lambda_{i+2}}$, where the last equation can be obtained from (26) and (28). It follows from the inequality (22) that $q_{i+1} - q_i \geq \sqrt{2\gamma_i + v_{i,i+1}^2} - v_{i,i+1} \geq \sqrt{2\gamma_i} - v_{i,i+1} \geq \sqrt{2\gamma_i} - (v_i^M + v_{i+1}^M)$. Noting the definition (26) and the fact that $\Sigma_{i=1}^n (q_{i+1} - q_i) = 2\pi$, the upper bound on C_i is $2\pi - \Sigma_{j \in \mathcal{I}_n, j \neq \{i-1,i,i+1\}}[\sqrt{2\gamma_j} - (v_j^M + v_{j+1}^M)]$, which is shown by (24d). Thus, for each sensor i and sensor $i+1$, $\max_{q_i \in \mathcal{F}, i \in \mathcal{I}_n} \min\{U_1, U_2, U_3\}$ is given by equation (24a) which is a tighter upper bound on $\frac{q_{i+1} - q_i}{\lambda_i + \lambda_{i+1}}$ as what Step 1 shows.

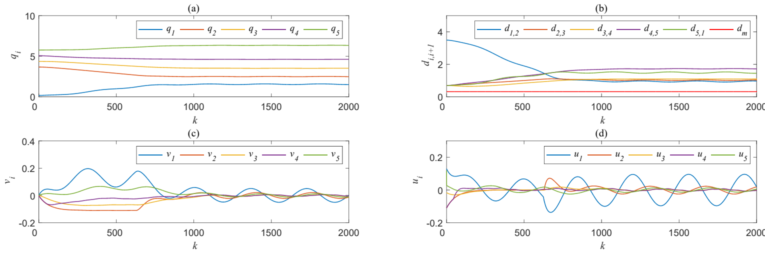

Fig. 1. Simulation results of the coverage control laws (7). (a) The sensors' positions; (b) The sensors' relative distances; (c) The sensors' velocities; (d) The sensors' control inputs.

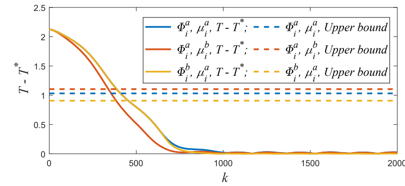

Fig. 2. The function $T - T^*$ for different parameters combination of Φ_i and μ_i.

It follows from Lemma 4 that $T(q_1, ..., q_n) \leq \max_{i\in\mathcal{I}_n} \bar{d}(q_i, q_{i+1})/(\lambda_i + \lambda_{i+1})$ and $T^* = \pi/\sum_{i=1}^{n}\lambda_i$. Note that $\bar{d}(q_i, q_{i+1}) = (q_{i+1} - q_i) \mod 2\pi = q_{i+1} - q_i$ due to the order preservation of the mobile sensors. Then, an upper bound on $T(q_1, ..., q_n) - T^*$ can be finally given by the inequality (23). ■

Remark 5: In order to preserve the spatial order for the sensor network, the term $P'_{i,j}$ is introduced in the proposed coverage control laws (7) to provide a repulsion force when the generalized distance function $g_{i,j}$ is less than a threshold. In this work, γ_i is designed to be small enough such that $P'_{i,j} = 0$ and thus $u_i^{ord} = 0$ when the mobile sensor network converges to a neighborhood of the optimal configuration. Then, the sensors can be driven only under the term u_i^{cov} in the neighborhood of the optimal configuration. Moreover, the condition $u_i^{ord} = 0$ is also helpful to reduce the range of $|u_i^{cov}|$ given the inequality (17). Then, the estimation of the upper bound on the coverage cost function with spatial order preservation in the presence of UBB disturbances can be finally tackled.

Remark 6: It can be observed from equations (23)-(24d) that the upper bound on the coverage cost function $T(q_1, ..., q_n)$ critically depends on D_i which is the bound of UBB disturbances, the maximum velocity λ_i, the parameter μ_i and the parameter Φ_i. For a given MSN, λ_i and D_i are fixed. Then, recalling equations (7) and (8), one can find that the parameter μ_i and the parameter Φ_i can be tuned to determine the upper bound on the coverage cost function $T(q_1, ..., q_n)$. It follows from (15d) that smaller μ_i leads to smaller v_i^M which is the maximum possible velocity of sensor i throughout the coverage task. Then, according to (23)-(24d), the decrease of v_i^M will reduce the upper bound on the coverage cost function, which implies that the estimation of the coverage cost function can be more accurate by choosing a smaller parameter μ_i. However, the parameter μ_i cannot be designed to be too small due to (15b), which suggests that μ_i should be large enough to compensate for the effect of disturbances. Moreover, from the proof of Theorem 1, the MSN will be driven to the set $\mathcal{K} = \{(q_i, v_i)|\ |u_i^{cov} + u_i^{ord}| < \Phi_i, \forall i \in \mathcal{I}_n\}$, and the upper bound on the coverage cost function is then obtained based on the bound Φ_i on the set $\mathcal{K}$. Similarly, it can be obtained from equations (23)-(24d) that the upper bound on the coverage cost function can also be reduced by choosing a smaller parameter Φ_i. Though the parameter Φ_i needs only to be larger than 0, the reduction of Φ_i makes $\text{sat}(\Omega_i)$ in (7) approaches $\text{sign}(u_i^{cov} + u_i^{ord})$ which will induce the discontinuity of the control input.

V. Simulations

In this section, a simulation example is presented to demonstrate the effectiveness of the proposed coverage control laws. A group of five sensors is considered, where the maximum velocity λ_i of each sensor is given by $[0.5, 0.7, 0.6, 0.8, 1.4]$. The initial positions of sensors are chosen as $[0.1745, 3.6652, 4.3633, 5.0615, 5.7596]$. The disturbance functions are given by $\Delta_i(t) = (-1)^i D_i \sin(ti\pi/180)$, where the upper bounds D_i's are chosen from $[0.15, 0.001, 0.002, 0.003, 0.001]$. For the generalized energy function, the parameters are selected as $\Phi_i = [1.2, 0.3, 0.9, 0.3, 0.6]$, $\theta_i = [0.0493, 0.0415, 0.0343, 0.0278, 0.0219]$, and $\gamma_i = [0.2176, 0.1974, 0.1781, 0.1599, 0.1511]$, where the parameter γ_i satisfies (16). In the condition (15) in Theorem 1, η_i and ϵ_i are chosen as $[0.003, 0.001, 0.002, 0.001, 0.002]$ and $[0.001, 0.002, 0.001, 0.002, 0.003]$, respectively. Moreover, the parameters μ_i are selected as $[0.18, 0.16, 0.17, 0.16, 0.17]$. Under the coverage control laws (7), the simulation results are shown in Figs. 1-2. Fig. 1 (a) shows time evolution of the sensors' positions. Fig. 1 (b) shows that the distance between each pair of neighboring sensors is always larger than $d_m = \max_{i\in\mathcal{I}_5}\{\sqrt{2\theta_i}\}$ which is represented by the red line. Fig. 1 (c) shows each sensor's moving velocity. It can be found that under the proposed control laws (7), the velocity constraints $|v_i| \leq \lambda_i, \forall i \in \mathcal{I}_5$ are satisfied throughout the coverage task. The sensors control inputs are shown in Fig. 1 (d). Finally, Fig. 2 shows time evolution of the function $T - T^*$ in different design parameters, where $\Phi_i^a = \Phi_i$ and $\mu_i^a = \mu_i$ denote the previous settings, $\Phi_i^b = 0.3\Phi_i^a$, $\mu_i^b = 1.3\mu_i^a$. The upper bound on $T - T^*$ given by Theorem 1 is denoted by the dashed line in corresponding colors. It can be seen that the coverage cost functions under different combinations of those parameters are all driven to a neighborhood of its minimum as time goes to infinity. Moreover, it can be observed from this figure that smaller μ_i and smaller Φ_i lead to a smaller upper bound of the coverage cost function, which coincides with Remark 6. The UBB disturbances keep affecting the system (1), which caused persistent fluctuations as shown in Fig. 1 and Fig. 2.

VI. Conclusion

The coverage control problem for MSNs with double-integrator dynamics and UBB disturbances have been studied in this paper. Cooperative coverage control laws with state saturation constraints have been designed for each sensor via the generalized energy function. It is shown that the MSN can be driven to a neighborhood of the optimal configuration under the proposed control laws and an upper bound on the limit of the coverage cost function is also derived. In the future, it is of practical significance to extend the current work to coverage control for mobile sensors with limited communication ranges.

Appendix
Proof of Lemma 5

Consider the Lyapunov-like function candidate $V(t) = V_{cov} + V_{ord}$, where

$$V_{cov} = \frac{1}{2}\sum_{i=1}^{n}\frac{(q_{i,i+1} + v_{i,i+1})^2}{\lambda_i + \lambda_{i+1}}, \tag{29}$$

and

$$V_{ord} = \sum_{i=1}^{n} P_{i,i+1}. \tag{30}$$

Note that $V(t)$ is a continuous function but it is not continuously differentiable at γ_i from the property (6). However, the energy function $P_{i,j}$ is smooth when $g_{i,j}$ is not at γ_i, which implies that $V(t)$ is a piecewise continuously differentiable function along the mobile sensors' state trajectories. Thus, denote $\Delta_j - \Delta_i$ by $\Delta_{i,j}$, and calculate the upper right-hand time derivative of V_{cov}. Then, we can get $D^+V_{cov} = \sum_{i=1}^{n} \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}}(v_{i,i+1}+u_{i,i+1}+\Delta_{i,i+1}) = \sum_{i=1}^{n} \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}} v_{i+1} - \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}} v_i + \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}}(u_{i+1} + \Delta_{i+1}) - \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}}(u_i+\Delta_i)$. Denote the first term and the third term of D^+V_{cov} as $D^+V_{cov}^1$ and $D^+V_{cov}^3$, respectively. Then, one has $D^+V_{cov}^1 = \sum_{i=1}^{n} \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}} v_{i+1} = \sum_{i=1}^{n-1} \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}} v_{i+1} + \frac{q_{n,n+1}+v_{n,n+1}}{\lambda_n+\lambda_{n+1}} v_{n+1}$. Recalling $\lambda_{n+i} \equiv \lambda_i$, $v_{n+i} \equiv v_i$ and $q_{n+i} \equiv q_i + 2\pi$, one has $D^+V_{cov}^1 = \sum_{i=2}^{n} \frac{q_{i-1,i}+v_{i-1,i}}{\lambda_i+\lambda_{i-1}} v_i + \frac{q_{0,1}+v_{0,1}}{\lambda_0+\lambda_1} v_1 = \sum_{i=1}^{n} \frac{q_{i-1,i}+v_{i-1,i}}{\lambda_i+\lambda_{i-1}} v_i = -\sum_{i=1}^{n} \frac{q_{i,i-1}+v_{i,i-1}}{\lambda_i+\lambda_{i-1}} v_i$. Similarly, $D^+V_{cov}^3$ can be calculated as $D^+V_{cov}^3 = \sum_{i=1}^{n} \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}}(u_{i+1}+\Delta_{i+1}) = -\sum_{i=1}^{n} \frac{q_{i,i-1}+v_{i,i-1}}{\lambda_i+\lambda_{i-1}}(u_i+\Delta_i)$. Consequently, $D^+V_{cov} = \sum_{i=1}^{n} -\frac{q_{i,i-1}+v_{i,i-1}}{\lambda_i+\lambda_{i-1}} v_i - \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}} v_i - \frac{q_{i,i-1}+v_{i,i-1}}{\lambda_i+\lambda_{i-1}}(u_i + \Delta_i) - \frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}}(u_i + \Delta_i) = -\sum_{i=1}^{n}(\frac{q_{i,i+1}+v_{i,i+1}}{\lambda_i+\lambda_{i+1}} + \frac{q_{i,i-1}+v_{i,i-1}}{\lambda_i+\lambda_{i-1}})(v_i+u_i+\Delta_i)$.

Moreover, from the proposed energy function (5), one has $P'_{i,j}(g_{i,j})|_{g_{i,j}\to\gamma_i^+} = 0$ and $P'_{i,j}(g_{i,j})|_{g_{i,j}\to\gamma_i^-} = -\frac{\Phi_i}{\sqrt{2\theta_i}}$, which implies that $P'_{i,j}$ is a continuous function with a a jump discontinuity at γ_i. Noting that $g_{i,j}$ may approach γ_i from both two sides, one has $D^+P_{i,j} = P'_{i,j}\dot{g}_{i,j}$. Then, calculating the upper right-hand time derivative of V_{ord} along the mobile sensors' state trajectories, we can obtain $D^+V_{ord} = \sum_{i=1}^{n} P'_{i,i+1}\dot{g}_{i,i+1} = \sum_{i=1}^{n} P'_{i,i+1}[v_{i,i+1}(\frac{1}{2}q_{i,i+1} + v_{i,i+1}) + q_{i,i+1}(\frac{1}{2}v_{i,i+1} + u_{i,i+1} + \Delta_{i,i+1})] = \sum_{i=1}^{n} P'_{i,i+1}q_{i,i+1}(v_{i,i+1}+u_{i,i+1}+\Delta_{i,i+1}) + \sum_{i=1}^{n} P'_{i,i+1}v_{i,i+1}^2 = \sum_{i=1}^{n} P'_{i,i+1}q_{i,i+1}v_{i+1} - \sum_{i=1}^{n} P'_{i,i+1}q_{i,i+1}v_i + \sum_{i=1}^{n} P'_{i,i+1}q_{i,i+1}(u_{i+1} + \Delta_{i+1}) - \sum_{i=1}^{n} P'_{i,i+1}q_{i,i+1}(u_i + \Delta_i) + \sum_{i=1}^{n} P'_{i,i+1}v_{i,i+1}^2$. Denote the first term and the third term of D^+V_{ord} as $D^+V_{ord}^1$ and $D^+V_{ord}^3$, respectively. Following the similar operations as $D^+V_{cov}^1$ and $D^+V_{cov}^3$, one has $D^+V_{ord}^1 = -\sum_{i=1}^{n} P'_{i,i-1}q_{i,i-1}v_i$ and $D^+V_{ord}^3 = -\sum_{i=1}^{n} P'_{i,i-1}q_{i,i-1}(u_i+\Delta_i)$. Recall that $P'_{i,i+1} \leq 0$, $\forall g_{i,j} \in [\theta_i, \infty)$. Then, one has $D^+V_{ord} = -\sum_{i=1}^{n} P'_{i,i-1}q_{i,i-1}v_i - \sum_{i=1}^{n} P'_{i,i+1}q_{i,i+1}v_i - \sum_{i=1}^{n} P'_{i,i-1}q_{i,i-1}(u_i + \Delta_i) - \sum_{i=1}^{n} P'_{i,i+1}q_{i,i+1}(u_i + \Delta_i) + \sum_{i=1}^{n} P'_{i,i+1}v_{i,i+1}^2 = -\sum_{i=1}^{n}(P'_{i,i-1}q_{i,i-1}+P'_{i,i+1}q_{i,i+1})(v_i+u_i+\Delta_i)+\sum_{i=1}^{n} P'_{i,i+1}v_{i,i+1}^2 \leq -\sum_{i=1}^{n}(P'_{i,i-1}q_{i,i-1} + P'_{i,i+1}q_{i,i+1})(v_i + u_i + \Delta_i)$. In consequence, under the cooperative coverage control laws (7), the upper right-hand time derivative of V satisfies

$$
\begin{aligned}
D^+V =& D^+V_{cov} + D^+V_{ord} \leq -\sum_{i=1}^{n}(u_i^{cov} + u_i^{ord})(v_i + u_i + \Delta_i) \\
=& -\sum_{i=1}^{n}(u_i^{cov} + u_i^{ord})(\mu_i \mathrm{sat}(\frac{u_i^{cov} + u_i^{ord}}{\Phi_i}) + \Delta_i). \qquad (31)
\end{aligned}
$$

Then, it will be shown that the mobile sensor network will be driven to the set $\mathcal{K} = \{(q_i, v_i)|\ |u_i^{cov} + u_i^{ord}| < \Phi_i, \forall i \in \mathcal{I}_n\}$ when time goes to infinity by showing that $D^+V < 0$ as long as the sensor network does not belong to the set $\mathcal{K}$. If the networked mobile sensors belong to the set $\mathcal{D} = \{(q_i, v_i)|\ |u_i^{cov} + u_i^{ord}| \geq \Phi_i, \forall i \in \mathcal{I}_n\}$, one has $D^+V \leq -\sum_{i=1}^{n}[\mu_i|u_i^{cov} + u_i^{ord}| + (u_i^{cov} + u_i^{ord})\Delta_i] \leq -\sum_{i=1}^{n}(\mu_i|u_i^{cov}+u_i^{ord}|-|u_i^{cov}+u_i^{ord}|D_i) = -\sum_{i=1}^{n}|u_i^{cov}+u_i^{ord}|(\mu_i - D_i) \leq -\sum_{i=1}^{n}\Phi_i(\mu_i - D_i)$. The last inequality is established due to the fact $\mu_i - D_i > 0$ according to (15b). Noting that the Lyapunov-like function V is lower bounded by 0 and D^+V is negative definite, it can be obtained that the sensor network will be finally driven out of the set $\mathcal{D}$.

If the sensor network belongs to the set $\mathcal{J}/\mathcal{K}$, where $\mathcal{J} = \{(q_i, v_i)|\ |u_i^{cov}+u_i^{ord}| < \Phi_i, \text{for some } i \in \mathcal{I}_n\}$, assume that there is at least one sensor j which satisfies $|u_j^{cov}+u_j^{ord}| \geq \Phi_j$, and the other sensors satisfy $|u_i^{cov} + u_i^{ord}| < \Phi_i$, where $i, j \in \mathcal{I}_n, i \neq j$. Under this assumption, recalling the expressions (31), one can obtain that $D^+V \leq -(u_j^{cov}+u_j^{ord})(\mu_j \mathrm{sat}(\frac{u_j^{cov}+u_j^{ord}}{\Phi_j})+\Delta_j)+\sum_{i\in\mathcal{I}_n, i\neq j}(u_i^{cov}+u_i^{ord})(\mu_i \mathrm{sat}(\frac{u_i^{cov}+u_i^{ord}}{\Phi_i}) + \Delta_i) = -\mu_j|u_j^{cov} + u_j^{ord}| - (u_j^{cov} + u_j^{ord})\Delta_j + \sum_{i\in\mathcal{I}_n, i\neq j}[-\frac{\mu_i}{\Phi_i}(u_i^{cov} + u_i^{ord})^2 - (u_i^{cov} + u_i^{ord})\Delta_i] \leq -\mu_j|u_j^{cov}+u_j^{ord}| - (u_j^{cov}+u_j^{ord})\Delta_j - \sum_{i\in\mathcal{I}_n, i\neq j}(u_i^{cov}+u_i^{ord})\Delta_i \leq -\mu_j|u_j^{cov}+u_j^{ord}| + |u_j^{cov}+u_j^{ord}|D_j + \sum_{i\in\mathcal{I}_n, i\neq j}|u_i^{cov}+u_i^{ord}|D_i < -\mu_j|u_j^{cov} + u_j^{ord}| + |u_j^{cov} + u_j^{ord}|\sum_{i=1}^{n} D_i = -|u_j^{cov} + u_j^{ord}|(\mu_j - \sum_{i=1}^{n} D_i) < -\Phi_j(\mu_j - \sum_{i=1}^{n} D_i) < -\Phi_j\eta_j$. Thus, if there exists one sensor j which is out of the set $\mathcal{K}$, one has $D^+V < -\Phi_j\eta_j$, which is still negative definite. Recalling that $V > 0$ always holds, networked mobile sensors will be finally driven out of the set $\mathcal{J}/\mathcal{K}$. Therefore, the sensor network will be only driven to the set $\mathcal{K}$ as time goes to infinity, which implies that the first conclusion of this lemma is proved.

Next, the minimum distance between neighboring sensors will be analyzed for all the cases mentioned above. In the case that D^+V is guaranteed to be strictly less than 0, the sensor network can be in the set $\mathcal{D}$ or $\mathcal{J}/\mathcal{K}$. From the initial condition (2), the initial value of the Lyapunov-like function V is upper bounded by a certain positive number. It then follows from $V > 0$ and $D^+V < 0$ that V is lower and upper bounded by 0 and its initial value, respectively. Recalling $V = V_{cov} + V_{ord}$ and equation (29), one has $V_{cov} \geq 0$ and therefore V_{ord} is upper bounded. Then, it follows from equation (30) and $P_{i,i+1} \geq 0$ in property (6) that each $P_{i,i+1}$ is upper bounded. Noting $\lim_{g_{i,j}\to\theta_i} P_{i,j} = +\infty$ in property (6), one has $g_{i,i+1}$ will not tend to θ_i for each sensor i in the network throughout the coverage mission. Recalling the initial condition (2), it can be derived that $g_{i,i+1}(0) > \theta_i$. From the above results, one then has $g_{i,i+1} > \theta_i$, $\forall i \in \mathcal{I}_n$ throughout the coverage mission. Then, it follows from Lemma 3 that $|q_{i,i+1}(t)| > \sqrt{2\theta_i}$, which suggests that

$$
q_{i+1}(t) - q_i(t) > \sqrt{2\theta_i},\ t > 0,\ \forall i \in \mathcal{I}_n \qquad (32)
$$

are satisfied when $D^+V < 0$.

As for the case that the sensor network belongs to the set $\mathcal{K}$, due to $D^+V < 0$ is no longer guaranteed, only the boundedness of $|u_i^{cov} + u_i^{ord}|$ and (15) will be utilized to analyze the minimum distance between neighboring sensors. Noting (15a) and the fact

$$
\sum_{i=1}^{n}(q_{i,i+1} + v_{i,i+1}) = 2\pi, \qquad (33)
$$

there must exist a sensor $j \in \mathcal{I}_n$ such that $q_{j,j+1} + v_{j,j+1} > \sqrt{2\theta_j + (v_j^M + v_{j+1}^M)^2 + 2\epsilon_j}$. This inequality implies that there exists at least one pair of sensors such that $\frac{1}{2}(q_{j,j+1} + v_{j,j+1})^2 >$

$\theta_j + \frac{1}{2}(v_j^M + v_{j+1}^M)^2 + \epsilon_j$. It follows from inequality (13) and (15d) that $|v_{j,j+1}| \leq |v_j| + |v_{j+1}| \leq \mu_j + D_j + \mu_{j+1} + D_{j+1} = v_j^M + v_{j+1}^M$. Thus, there exists at least a sensor j satisfying $g_{j,j+1} > \theta_j + \epsilon_j$. Since ϵ_j is a positive constant, according to property (6), there always exists one pair of sensor j and sensor $j+1$ such that $P'_{j,j+1}$ is bounded.

Then, it will be shown that $g_{i,i+1} > \theta_i$, $\forall i \in \mathcal{I}_n$ when the sensor network belongs to the set $\mathcal{K}$ by a contradiction. Assume that $g_{m,m+1}$ first becomes θ_m at time t_1, that is, $g_{m,m+1}(t_1) = \theta_m$ and $g_{l,l+1}(t_1) > \theta_l$, $\forall l \neq m$, $l \in \mathcal{I}_n$, which implies that $g_{i,i+1}(t) \neq 0$, $t_0 \leq t \leq t_1$, $\forall i \in \mathcal{I}_n$, where t_0 is the earliest moment when the sensor network enters into the set $\mathcal{K}$. Then, it follows from equation (4) that $q_{i,i+1}(t) \neq 0$. Recalling the initial condition (2), one has $q_{i,i+1}(t) > 0$, $t_0 \leq t \leq t_1$, $\forall i \in \mathcal{I}_n$. Consequently, $q_1(t) \leq \cdots \leq q_i(t) \leq q_{i+1}(t) \leq \cdots \leq q_n(t) \leq q_1(t) + 2\pi$ holds for all $t \in [t_0, t_1]$. Thus, one has $0 < q_{i,i+1}(t) < 2\pi$, $t_0 \leq t \leq t_1$, $\forall i \in \mathcal{I}_n$. Noting that $|v_{m,m+1}| \leq v_m^M + v_{m+1}^M$, $|u_m^{cov}| = |\frac{q_{m,m+1}+v_{m,m+1}}{\lambda_m+\lambda_{m+1}} + \frac{q_{m,m-1}+v_{m,m-1}}{\lambda_m+\lambda_{m-1}}|$ is upper bounded when $t_0 \leq t \leq t_1$. Moreover, it has been shown in the previous paragraph that there exists a pair of sensors such that $P'_{j,j+1}$ is bounded. Then, recalling the assumption that $g_{m,m+1}(t_1) = \theta_m$, noting the property of the continuous system, it can be obtained from property (6) that there exists a pair of sensors such that $P'_{m,m+1}(t)$ is unbounded, where $t_0 \leq t < t_1$. Noting $P'_{i,j} \equiv P'_{j,i}$, one can find at least one group of sensors such that one element in $\{P'_{m,m-1}(t), P'_{m,m+1}(t)\}$ is bounded while the other is unbounded. Therefore, it can be derived that $|u_m^{ord}| = |P'_{m,m-1}q_{m,m-1} + P'_{m,m+1}q_{m,m+1}| = \infty$ and thus $|u_m^{cov} + u_m^{ord}| = \infty$ during $t_0 \leq t < t_1$, which contradicts the definition of the set $\mathcal{K}$. Therefore, $g_{i,i+1} > \theta_i$, $\forall i \in \mathcal{I}_n$ holds when the sensor network belongs to the set $\mathcal{K}$. Following a similar procedure as $D^+V < 0$, it can be shown that the equation (32) always holds when the sensor network is in the set $\mathcal{K}$. ■

References

[1] M. Dunbabin and L. Marques, "Robots for environmental monitoring: Significant advancements and applications," *IEEE Robotics & Automation Magazine*, vol. 19, no. 1, pp. 24–39, 2012.

[2] G. Kantor, S. Singh, R. Peterson, D. Rus, A. Das, V. Kumar, G. Pereira, and J. Spletzer, *Distributed Search and Rescue with Robot and Sensor Teams*. Heidelberg, Germany: Springer, 2006.

[3] N. Atanasov, J. Le Ny, K. Daniilidis, and G. J. Pappas, "Decentralized active information acquisition: Theory and application to multi-robot SLAM," in *Proceedings of the IEEE International Conference on Robotics and Automation*, Seattle, USA, 2015, pp. 4775–4782.

[4] A. Barrientos, J. Colorado, J. d. Cerro, A. Martinez, C. Rossi, D. Sanz, and J. Valente, "Aerial remote sensing in agriculture: A practical approach to area coverage and path planning for fleets of mini aerial robots," *Journal of Field Robotics*, vol. 28, no. 5, pp. 667–689, 2011.

[5] W. Kazmi, M. Bisgaard, F. Garcia-Ruiz, K. D. Hansen, and A. la Cour-Harbo, "Adaptive surveying and early treatment of crops with a team of autonomous vehicles," in *Proceedings of the 5th European Conference on Mobile Robots*, Örebro, Sweden, 2011, pp. 253–258.

[6] A. Howard, M. J. Matarić, and G. S. Sukhatme, "Mobile sensor network deployment using potential fields: A distributed, scalable solution to the area coverage problem," in *Proceedings of the 6th International Symposium on Distributed Autonomous Robotic Systems*, Fukuoka, Japan, 2002, pp. 299–308.

[7] M. Zhong and C. G. Cassandras, "Distributed coverage control and data collection with mobile sensor networks," *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2445–2455, 2011.

[8] X. Sun, C. G. Cassandras, and X. Meng, "Exploiting submodularity to quantify near-optimality in multi-agent coverage problems," *Automatica*, vol. 100, pp. 349–359, 2019.

[9] C. Sun, S. Welikala, and C. G. Cassandras, "Optimal composition of heterogeneous multi-agent teams for coverage problems with performance bound guarantees," *Automatica*, vol. 117, p. 108961, 2020.

[10] J. Cortés, S. Martínez, T. Karataş, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, 2004.

[11] S. G. Lee, Y. Diaz-Mercado, and M. Egerstedt, "Multirobot control using time-varying density functions," *IEEE Transactions on Robotics*, vol. 31, no. 2, pp. 489–493, 2015.

[12] Y. Stergiopoulos, M. Thanou, and A. Tzes, "Distributed collaborative coverage-control schemes for non-convex domains," *IEEE Transactions on Automatic Control*, vol. 60, no. 9, pp. 2422–2427, 2015.

[13] P. Frasca, F. Garin, B. Gerencsér, and J. M. Hendrickx, "Optimal one-dimensional coverage by unreliable sensors," *SIAM Journal on Control and Optimization*, vol. 53, no. 5, pp. 3120–3140, 2015.

[14] N. E. Leonard and A. Olshevsky, "Nonuniform coverage control on the line," *IEEE Transactions on Automatic Control*, vol. 58, no. 11, pp. 2743–2755, 2013.

[15] P. Davison, N. E. Leonard, A. Olshevsky, and M. Schwemmer, "Nonuniform line coverage from noisy scalar measurements," *IEEE Transactions on Automatic Control*, vol. 60, no. 7, pp. 1975–1980, 2015.

[16] A. Davydov and Y. Diaz-Mercado, "Sparsity structure and optimality of multi-robot coverage control," *IEEE Control Systems Letters*, vol. 4, no. 1, pp. 13–18, 2020.

[17] C. Song, Y. Fan, and S. Xu, "Finite-time coverage control for multiagent systems with unidirectional motion on a closed curve," *IEEE Transactions on Cybernetics*, vol. 51, no. 6, pp. 3071–3078, 2019.

[18] D. W. Casbeer, D. B. Kingston, R. W. Beard, and T. W. McLain, "Cooperative forest fire surveillance using a team of small unmanned air vehicles," *International Journal of Systems Science*, vol. 37, no. 6, pp. 351–360, 2006.

[19] N. E. Leonard, D. A. Paley, F. Lekien, R. Sepulchre, D. M. Fratantoni, and R. E. Davis, "Collective motion, sensor networks, and ocean sampling," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 48–74, 2007.

[20] T.-H. Kim and T. Sugie, "Cooperative control for target-capturing task based on a cyclic pursuit strategy," *Automatica*, vol. 43, no. 8, pp. 1426–1431, 2007.

[21] R. Zheng, Y. Liu, and D. Sun, "Enclosing a target by nonholonomic mobile robots with bearing-only measurements," *Automatica*, vol. 53, pp. 400–407, 2015.

[22] C. Song, L. Liu, G. Feng, and S. Xu, "Coverage control for heterogeneous mobile sensor networks on a circle," *Automatica*, vol. 63, pp. 349–358, 2016.

[23] C. Song and Y. Fan, "Coverage control for mobile sensor networks with limited communication ranges on a circle," *Automatica*, vol. 92, pp. 155–161, 2018.

[24] C. Song, L. Liu, G. Feng, Y. Fan, and S. Xu, "Coverage control for heterogeneous mobile sensor networks with bounded position measurement errors," *Automatica*, vol. 120, p. 109118, 2020.

[25] A. Abdessameud and A. Tayebi, "On consensus algorithms design for double integrator dynamics," *Automatica*, vol. 49, no. 1, pp. 253–260, 2013.

[26] J. Fu, G. Wen, X. Yu, and Z.-G. Wu, "Distributed formation navigation of constrained second-order multiagent systems with collision avoidance and connectivity maintenance," *IEEE Transactions on Cybernetics*, vol. 52, no. 4, pp. 2149–2162, 2022.

[27] P. F. Hokayem, D. M. Stipanović, and M. W. Spong, "Coordination and collision avoidance for lagrangian systems with disturbances," *Applied Mathematics and Computation*, vol. 217, no. 3, pp. 1085–1094, 2010.

[28] S. He, M. Wang, S.-L. Dai, and F. Luo, "Leader–follower formation control of usvs with prescribed performance and collision avoidance," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 1, pp. 572–581, 2019.

[29] A. Filotheou, A. Nikou, and D. V. Dimarogonas, "Robust decentralised navigation of multi-agent systems with collision avoidance and connectivity maintenance using model predictive controllers," *International Journal of Control*, vol. 93, no. 6, pp. 1470–1484, 2020.

[30] K. D. Do, "Bounded assignment formation control of second-order dynamic agents," *IEEE/ASME Transactions on Mechatronics*, vol. 19, no. 2, pp. 477–489, 2013.

[31] D. Bauso, L. Giarré, and R. Pesenti, "Consensus for networks with unknown but bounded disturbances," *SIAM Journal on Control and Optimization*, vol. 48, no. 3, pp. 1756–1770, 2009.

[32] X. Ge, Q.-L. Han, and F. Yang, "Event-based set-membership leader-following consensus of networked multi-agent systems subject to limited communication resources and unknown-but-bounded noise," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 5045–5054, 2016.

[33] T. Meng, Y. Xie, and Z. Lin, "Consensus of second-order multi-agent systems under unknown but bounded measurement noises," *Systems & Control Letters*, vol. 133, p. 104517, 2019.

[34] L. Dou, C. Song, X. Wang, L. Liu, and G. Feng, "Coverage control for heterogeneous mobile sensor networks subject to measurement errors," *IEEE Transactions on Automatic Control*, vol. 63, no. 10, pp. 3479–3486, 2018.

[35] C. Makkar, W. Dixon, W. Sawyer, and G. Hu, "A new continuously differentiable friction model for control systems design," in *Proceedings of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, Monterey, USA, 2005, pp. 600–605.

[36] C. K. Verginis and D. V. Dimarogonas, "Adaptive robot navigation with collision avoidance subject to 2nd-order uncertain dynamics," *Automatica*, vol. 123, p. 109303, 2021.

[37] C. Song, L. Liu, and S. Xu, "Circle formation control of mobile agents with limited interaction range," *IEEE Transactions on Automatic Control*, vol. 64, no. 5, pp. 2115–2121, 2019.

[38] E. J. Rodríguez-Seda, C. Tang, M. W. Spong, and D. M. Stipanović, "Trajectory tracking with collision avoidance for nonholonomic vehicles with acceleration constraints and limited sensing," *The International Journal of Robotics Research*, vol. 33, no. 12, pp. 1569–1592, 2014.

[39] C. Wang, G. Xie, and M. Cao, "Forming circle formations of anonymous mobile agents with order preservation," *IEEE Transactions on Automatic Control*, vol. 58, no. 12, pp. 3248–3254, 2013.

[40] C. Wang and G. Xie, "Limit-cycle-based decoupled design of circle formation control with collision avoidance for anonymous agents in a plane," *IEEE Transactions on Automatic Control*, vol. 62, no. 12, pp. 6560–6567, 2017.