Establishing boundary conditions in sewer pipe/soil heat transfer modelling using physics-informed learning

Li, Jiuling; Mohamad, Nur Nabilah Naina; Sharma, Keshab; Yuan, Zhiguo

Published in:
Water Research

Published: 01/10/2023

Document Version:
Final Published version, also known as Publisher’s PDF, Publisher’s Final version or Version of Record

License:
CC BY-NC-ND

Publication record in CityU Scholars:
Go to record

Published version (DOI):
10.1016/j.watres.2023.120441

Publication details:

Citing this paper
Please note that where the full-text provided on CityU Scholars is the Post-print version (also known as Accepted Author Manuscript, Peer-reviewed or Author Final version), it may differ from the Final Published version. When citing, ensure that you check and use the publisher's definitive version for pagination and other details.

General rights
Copyright for the publications made accessible via the CityU Scholars portal is retained by the author(s) and/or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights. Users may not further distribute the material or use it for any profit-making activity or commercial gain.

Publisher permission
Permission for previously published items are in accordance with publisher’s copyright policies sourced from the SHERPA RoMEO database. Links to full text versions (either Published or Post-print) are only available if corresponding publishers allow open access.

Take down policy
Contact lbscholars@cityu.edu.hk if you believe that this document breaches copyright and provide us with details. We will remove access to the work immediately and investigate your claim.
Establishing boundary conditions in sewer pipe/soil heat transfer modelling using physics-informed learning

Jiuling Li a, Nur Nabilah Naina Mohamad a, Keshab Sharma a, Zhiguo Yuan a,b,*

a Australian Centre for Water and Environmental Biotechnology, The University of Queensland, St. Lucia, Brisbane QLD 4072, Australia
b School of Energy and Environment, City University of Hong Kong, Hong Kong SAR, China

ARTICLE INFO

Keywords:
Heat transfer
Boundary conditions
Temperature modelling
Physics-informed model
Digital twin
Sewer system
Pipe-soil heat transfer

ABSTRACT

Modelling heat transfer in sewers and the surrounding soil is important for effective sewer maintenance, and for heat recovery from wastewater. The boundary conditions, including both the thickness of the soil layer to be modelled and the temperature distribution around the boundary of the soil layer, directly determine both the efficiency and accuracy of the models. Yet there is no systematic method to establish these conditions. This study presents a novel and generic approach to establishing efficient boundary conditions for sewer heat transfer modelling. Fourier transform is applied to identify the dominant frequencies of the temperatures of the heat sources/sinks, namely the atmosphere, sewer air and wastewater. A simple data-driven model for determining the thickness of the soil-layer to be included, and three physics-informed models for predicting the temperatures at the soil-layer boundary are then learnt from mechanistic models for sewer heat transfer, taking into consideration the frequency spectra. The methodology achieved high fidelity to the mechanistic models in predicting the soil-layer boundary temperatures and sewer wall temperatures for real-life sewers. This approach offers an easy yet reliable way to obtain efficient boundary conditions that significantly improve both the accuracy and speed of sewer heat transfer modelling.

1. Introduction

Heat transfer between in-sewer fluids, sewer pipes and the surrounding soil are critical processes to be considered in sewer heat transfer modelling. Efficient modelling of sewer heat transfer benefits not only sewer operation but also the energy recovery from sewers. For example, modelling the heat exchange between in-sewer fluid and sewer wall and soil can estimate the potential storage of heat in pipe wall and soil thus supporting optimisation of heat recovery from sewage (Dürrenmatt and Wanner, 2014). Similarly, the prediction of the pipe wall temperature is critical for determining if condensation would occur on the surface of a sewer wall, which along with the hydrogen sulfide concentration in sewer air, determines the sewer corrosion rate (Jiang et al., 2014).

Various studies have attempted to model heat transfer processes in sewer systems, which involves computing heat transfer between the in-sewer fluid (air and water) and the sewer pipe, governed by thermal convection; between and within the sewer pipe and the surrounding soil, governed by thermal conduction (Dürrenmatt and Wanner, 2014). Pipes in saturated soil or below the ground water table, where convective heat transfer dominates, have not been included in these efforts to date. These heat transfer equations are well established based on physical principles (Bergman et al., 2011), and are solved for a field comprising the in-sewer fluid, the pipe walls and the surrounding soil (Zwillinger and Dobrushkin, 1998). To enable accurate and efficient solutions of the governing partial differential equations (PDEs), the thickness of the soil-layer to be included in the heat transfer model (δs), hereafter called the soil-layer for simplicity, and the temperature at the boundary of the soil-layer (Ts,b) need to be properly defined/determined. Dürrenmat and Wanner (2014) concluded that δs, Ts,b, and the soil thermal diffusivity were the most sensitive parameters to temperature modelling. While thermal diffusivity is a well-documented physical property of soil (Arkhangel'skaya and Lukyashchenko, 2018), the soil-layer thickness (δs) and boundary temperature (Ts,b), collectively called boundary conditions in this paper, are often selected empirically and in many cases even arbitrarily.

Table 1 presents an overview of sewer heat transfer modelling in the literature, including the heat transfer processes modelled, model schemes, and boundary conditions. The soil-layer thickness (δs) is treated as either a calibrated parameter (Abdel-Aal et al., 2018, 2014, 2016).
A summary of the sewer heat transfer models.

Table 1
A summary of the sewer heat transfer models.

<table>
<thead>
<tr>
<th>Model index</th>
<th>Heat transfer processes</th>
<th>Model scheme</th>
<th>Soil-layer thickness ($\delta_s$)</th>
<th>Temperature at boundary of surrounding soil ($T_{s,b}$)</th>
<th>Remark</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dürrenmatt/TEMPEST model</td>
<td>• Conduction between water and pipe and soil&lt;br&gt;• Conduction between sewer air and pipe and soil&lt;br&gt;• Convection between water and sewer air&lt;br&gt;• Latent heat due to evaporation and condensation&lt;br&gt;• Heat production by biochemical reactions</td>
<td>• Heat conduction between water and pipe surface&lt;br&gt;• 1D PDE (radial) for heat conduction in pipe&lt;br&gt;• 1D PDE (radial) for heat conduction in the surrounding soil</td>
<td>$\delta_s$ is a parameter in the model&lt;br&gt;$\delta_s = 0.1$ m (estimated)</td>
<td>$T_{s,b}$ formed a boundary condition for the 1D radial PDE&lt;br&gt;$T_{s,b} = T_{w,ef}$&lt;br&gt;($T_{w,ef}$ is the undisturbed soil temperature at an infinite depth, either measured or estimated)</td>
<td>The most comprehensive model of sewer wall temperature.</td>
<td>Dürrenmatt and Wanner, 2014</td>
</tr>
<tr>
<td>Elias-Maxil Model</td>
<td>• Conduction between water and pipe and soil&lt;br&gt;• Convection between water and pipe and soil&lt;br&gt;• Latent heat due to evaporation and condensation</td>
<td>• Convection between water and pipe&lt;br&gt;• 2D PDE (radial and tangential) for conduction in pipe&lt;br&gt;• 2D PDE (radial and tangential) for conduction in the surrounding soil</td>
<td>$\delta_s$ is a parameter in the model&lt;br&gt;$\delta_s = 2.61$ m (estimated)</td>
<td>$T_{s,b}$ formed a boundary condition for the 2D radial PDE&lt;br&gt;$T_{s,b} = T_{w,ef}$&lt;br&gt;(Assuming $T_{s,b} \approx T_{w,ef}$ when $\delta_s \approx 2$ m)</td>
<td>The use of 2D PDEs for heat transfer in pipe and soil allows considering tangential variations of pipe and soil temperature, but increases the computational demand.</td>
<td>Elias-Maxil et al. (2017)</td>
</tr>
<tr>
<td>Abdel-Aal Model</td>
<td>• Conduction between water and pipe and soil&lt;br&gt;• Convection between water and sewer air</td>
<td>• Heat transfer rate between water and pipe/solid&lt;br&gt;$T_{w,inf} = T_{s,b}$&lt;br&gt;where $R_0$ is thermal resistivity, a function of $\delta_s$ and thermal conductivity of pipe and surrounding soil</td>
<td>$\delta_s$ is a parameter in the model (via $R$)&lt;br&gt;$\delta_s = 0.4$ m was estimated in Abdel-Aal et al. (2014)</td>
<td>$T_{s,b}$ = undisturbed soil temperature at the depth of the pipe&lt;br&gt;(measured, details not provided)</td>
<td>Sewer air temperature is not predicted, but to be measured.</td>
<td>Abdel-Aal et al. (2014), (2018), (2021)</td>
</tr>
<tr>
<td>Figueroa Model</td>
<td>• Conduction between water and pipe and soil&lt;br&gt;• Convection between water and sewer air&lt;br&gt;• Latent heat due to evaporation and condensation</td>
<td>The same as Abdel-Aal Model, but with the pipe curvature considered. The equation is the steady-state solution of the 1D PDE in the TEMPEST model.</td>
<td>$\delta_s$ is a parameter in the model (via $R$)&lt;br&gt;$\delta_s = 0.1$ m was determined by soil property in the case study.</td>
<td>$T_{s,b}$ = undisturbed soil temperature at the depth of the pipe (measured)</td>
<td>Sewer air temperature is not predicted, but to be measured.</td>
<td>Figueroa et al. (2021), Hadengue et al. (2021)</td>
</tr>
</tbody>
</table>
on the full mechanistic heat transfer models to generate data in support of the boundary condition establishment. A simple, empirical model was then proposed and calibrated, which determines the minimum soil-layer thickness required for modelling based on the soil thermal diffusivity. A set of physics-informed models were further proposed to determine the temperature distribution at the boundary of the soil layer to be modelled. The effectiveness of the developed models was demonstrated through case studies using real-life field data to model the sewer wall temperature.

2. Materials and methods

2.1. Problem formulation and research methodology

Fig. 1 illustrates a cross-section of a sewer pipe buried in soil, filled with either water (e.g. in a pressurised sewer pipe) or air (e.g. in a gravity pipe in a period without wastewater flow). The methodology presented in this paper can be extended to a pipe with both water and air. This aspect is out of the focus of the current study. Dürrenmatt and Wanner (2014) developed a sewer heat transfer model in a cylindrical coordinate, which considers the thermal conduction between the ground, soil, and sewer pipe, as well as thermal convection between the sewer pipe and the in-sewer fluid. Built upon Dürrenmatt and Wanner (2014), this paper has the following intertwined objectives:

1) To establish the thickness ($\delta_s$) of the soil layer to be included in the cylindrical heat transfer model. $\delta_s$ should be as small as possible to reduce the computational demand when running the heat transfer model, yet thick enough so that the impact of the in-sewer fluid temperature on the soil-layer boundary is either negligible or reliably predictable.

2) To determine the temperature distribution at the boundary of the soil layer ($T_{s,b}$), which is impacted by two heat sources/sinks, namely the atmosphere (via the ground surface) and the in-sewer fluid. $T_{s,b}$ thus determined will serve as the boundary conditions for the partial differential equations in the heat transfer model (governing equations of thermal conduction in the soil) (Dürrenmatt and Wanner, 2014). For simplicity of expressions, hereafter, both heat sources and heat sinks are collectively called heat sources, as heat sinks can be regarded as negative heat sources.

Our methodology involves the following steps:

1) We collected atmospheric and in-sewer fluid (air and water) temperature from various places in Australia, and identified their key frequency components via Fourier Transform. The frequency distribution is important as heat fluxes with different frequencies are attenuated to different extents by soil, with higher frequencies more heavily attenuated. Thus, heat fluxes with different frequencies penetrate to different depths in the soil.

2) We implemented the well-established full heat transfer equations (Dürrenmatt and Wanner, 2014) using a Cartesian coordinate system with a soil field of 100 m $\times$ 100 m, in which the pipe is buried at various depths, as a digital twin for heat transfer between atmosphere, soil, and a sewer pipe. This full model was simulated in a wide range of scenarios (Section 3) by varying pipe diameters, pipe burial depth, thermal diffusivity of the soil, and temperature profiles of the atmosphere and in-sewer fluid. For simplicity, we assumed that the ground-surface temperature and the pipe/soil interface temperature are known as boundary conditions for this digital-twin model, to avoid unnecessarily expanding the digital twin to also model heat transfer between atmosphere and ground surface, and between in-sewer fluid, pipe, and soil. This gives convenience without affecting the validity of the data-driven/physics-informed models derived in this work.

3) The data generated with the digital twin were used to develop four data-driven/physics-informed models (Section 3), one for the determination of the optimal thickness of the soil layer (named Model $\delta$), and three for the prediction of the temperature distribution at the boundary of the soil layer (named Model $T_1$, Model $T_2$, and Model $T_3$, respectively, and collectively called Model $T$). The development of Model $\delta$ and Model $T$ will be detailed in Section 3.

![Fig. 1. Schematic of sewer temperature modelling and the boundary conditions.](image-url)
4) **Model δ and Models T** were then applied to set the boundary for the temperature modelling of a real sewer pipe in a cylindrical coordinate considering a thin soil layer, with the prediction results compared with those obtained with the full digital twin simulating a 100 m × 100 m temperature field using a Cartesian coordinate system.

It is recognised that the fundamental equations adopted in Dürrenmatt and Wanner (2014) are the same as those used for modelling heat exchanging systems (see e.g. (Badescu, 2007; Zeng et al., 2003)). While Dürrenmatt and Wanner (2014) is chosen in this study to illustrate our methodology, the methodology proposed here is potentially applicable to establishing boundary conditions for all these models.

### 2.2. Data collection and Fourier transform

In-sewer fluid temperature data were retrieved from the supervisory control and data acquisition (SCADA) system of Melbourne Water (refer to Section 2.5 for sewer site details). The dataset, collected from three concrete gravity sewers, with diameters of 0.75 m, 1.35 m, and 1.8 m, respectively, along the same sewer line spanning 22.5 km, contains both sewage and headspace air temperature data, with a sampling interval of 15 min for a two-year period from 1/7/2020 to 30/6/2022 (Fig. S1). This means a year-long data series contains 35,040 data points. While we did not have access to the sensor calibration records, all the sensor signals, i.e. temperature of sewer air and sewer water for multiple pipe sections, did not display patterns of typical sensor faults (e.g. drifts, flat signals, instability), suggesting the sensors were well maintained. The raw data were directly used in the analysis reported below without pre-processing.

In addition, historical atmospheric temperature data were obtained from the Australian Bureau of Meteorology (BoM) for all Australian capital cities (distributed in an area of 7.62 million km² with their coordinates shown in Fig. S2) for the period between 1/7/2020 and 30/6/2022 (Fig. S3). These cities cover temperate, Mediterranean, subtropical and tropical climates. The air temperature was measured hourly by a resistance temperature device inside a Stevenson Screen at 1.2 m above the ground (Ashcroft et al., 2022), at the weather station closest to each local airport. While we did not have the specifications of the sensors used nor their calibration records, the data quality can be justifiably assumed as the sensors were maintained and data officially published, by the Australian BoM. Further, our preliminary data analysis did not reveal any anomalies caused by typical sensor faults (e.g. drifts, flat signals, instability), suggesting the sensors were well maintained. The raw data were directly used in the analysis reported below without pre-processing.

Equations of the mechanistic models of heat transfer are elaborated in Supplementary Information. The model was programmed in Matlab and solved using explicit Runge-Kutta (4,5) formula (Shampine and Reichelt, 1997).

2.4. Model fitting and evaluation

Based on the data generated using the digital twin, four empirical equations were proposed to determine the optimal thickness of the soil layer and the temperature distribution at the boundary of the soil layer (Section 3). The model parameters were calibrated through non-linear least square regression using the Trust-Region-Dogleg Algorithm (Conn et al., 2000) in Matlab, which yielded the mean parameter values as well as the 95% confidence intervals. To quantitatively evaluate the model fitness and prediction accuracy, coefficient of determination ($R^2$) and
root mean square error (RMSE) were calculated as described in Supplementary Information.

2.5. A case study demonstrating the method

To demonstrate the methodology of establishing boundary conditions, a real-life sewer system in Melbourne, Australia is selected. The Western Trunk Sewer is located in Werribee, which is 30 km from the Melbourne city centre. It is a 22.5-km long gravity sewer system consisting of sewer pipes with diameters ranging from 0.75 m to 3.5 m. The gravity sewer selected for this case study has a diameter of 1.8 m and a burial depth of 3.4 m (pipe centre to ground). The in-sewer air and wastewater temperature monitored by Melbourne Water and the ground surface temperature for a nearby location, retrieved from BoM, were used for the simulation study. The sewer pipe wall and surrounding soil temperatures were simulated using different model types resulting in a number of scenarios:

1) the full mechanistic model in a Cartesian coordinate system with the 100 m × 100 m field, with full boundary conditions as defined in Section 2.3;
2) the full mechanistic model in a cylindrical coordinate system (Fig. S6) with the optimised soil-layer thickness and boundary temperatures, determined with the developed methodology;
3) the full mechanistic model in a cylindrical coordinate system with a soil-layer thickness of 0.1 m, 0.4 m, and 2.0 m, respectively. The boundary temperature in all these cases was assigned as the undisturbed soil temperature (i.e. average atmospheric temperature).

In all scenarios of the case study, temperatures measured at 10 cm below the ground surface at a location close to the Melbourne airport (see Section 2.2) were assigned as the ground surface temperature. The inner-surface temperature of the sewer pipe was predicted by using the full mechanistic model to simulate thermal convection between sewer wall and in-sewer fluid and thermal conduction between sewer pipe and the surrounding soil.

The model predictions in Scenario 1 are used as a reference, for comparison with results from other scenarios. \( \delta_s = 0.1 \text{ m} \) and \( \delta_s = 2 \text{ m} \), selected for Scenario 3, are about the lower and upper limits of the range of \( \delta_s \) reported in literature. \( \delta_s = 0.4 \text{ m} \) is also included in Scenario 3, to highlight the importance of correctly assigning boundary temperatures through comparison with Scenario 2.

The predicted sewer wall temperature and the computational time in all the scenarios are compared.

Thermal diffusivity of soil used in the case study was calibrated from the soil temperatures measured at different depths (10 cm, 20 cm, 50 cm, and 100 cm). The soil temperature measured at a depth of 10 cm was used as the input. The value of thermal diffusivity was calibrated by minimising the sum of the squares of the difference between the simulated and measured soil temperature at all other depths (Bühlmann and Van De Geer, 2011). The soil thermal diffusivity was found to be 3.35 × 10⁻⁷ m²/s, and the calibration results for soil temperature at depths of 20 cm, 50 cm, and 100 cm are shown in Fig. S4.

3. Results

3.1. Fourier analysis of source temperatures

Fourier analysis was applied to all atmospheric and in-sewer air and water temperature data series to identify their dominant frequencies. The two-year time series in each case was evenly divided into two consecutive one-year-long datasets (indicated as Year 1 and Year 2), with each year analysed individually. The amplitude corresponding to each frequency (yearly, \( \frac{1}{2} \)-yearly, \( \frac{1}{3} \)-yearly, ..., \( \frac{1}{12} \)-yearly), along with the average temperature (called the direct current or DC in the Fourier Transform), for each year-long data series was obtained from the Fourier Transform (Eq. (1)). As examples, the frequency spectra of the atmospheric temperature in Brisbane (Fig. 2(A)), and the in-sewer air (Fig. 2(B)) and water temperature (Fig. 2(C)) of the gravity sewer RIS003 are used to illustrate the variation characteristics of the source temperatures. All other results are presented in Fig. S7 and Fig. S8. Details of the DC and amplitudes for both years are summarised in Table S1. For all datasets, the amplitudes for most frequencies were negligible (<<5% of the DC value), except for the yearly and daily (i.e. \( \frac{1}{1} \)-yearly) and, in some cases, \( \frac{1}{2} \)-yearly frequencies. Therefore, for readability, only the DC value and the amplitudes for seven frequencies, namely annually (i.e. yearly), semiannually (i.e. \( \frac{1}{2} \)-yearly), quarterly (i.e. \( \frac{1}{4} \)-yearly), monthly (i.e. \( \frac{1}{12} \)-yearly), fortnightly (i.e. \( \frac{1}{24} \)-yearly), weekly (i.e. \( \frac{1}{7} \)-yearly), and daily (i.e. \( \frac{1}{365} \)-yearly), are shown in the figures and table.

The temperature profiles all exhibit the highest amplitudes in annual and daily variations (Fig. 2, Fig. S7 and Fig. S8). All temperature profiles in Fig. 2 start from 1st January. The annual variation is generally larger than the daily variation, with the atmospheric temperature in Darwin being the only exception. Due to its tropical savanna climate, Darwin only has two seasons, a wet season and a dry season, of approximately equal duration (McKnight and Hess, 2000). Therefore, the atmospheric temperature in Darwin has a much smaller annual variation than the daily variation (Fig. S7). The semiannual amplitude is generally the third highest but significantly lower than the annual or diurnal amplitudes. In comparison to atmospheric temperature, the daily variation amplitudes of in-sewer air/water temperatures are much smaller. Air temperature in better vented sewers will likely present higher daily variation as the ventilation carries fresh air from the atmosphere, with significant daily variations, into sewers.

From the frequency spectra obtained, it can be concluded that the dominant frequencies in most cases annually and daily, and in some cases, semiannually and daily. Fig. 2(D) to (F) compare the real temperature of Year 1 and the temperature reconstructed using low-frequency features (annual) for the same period with Eq. (1), for atmospheric, in-sewer air and in-sewer water temperature, respectively, which shows that the long-term variations in atmospheric, and in-sewer fluid temperatures are well captured by the annual frequencies.

In addition, Fig. 2(D) to (F) also depict the temperature reconstructed using the DC and annual features of Year 2, which resembles the temperature patterns of Year 1, with differences less than 0.80°C. Moreover, from Fig. 2(A) to (C) and Table S1, it can be observed that the key features used to reconstruct the temperature profiles are close between the consecutive years, with differences ranging from 0.03°C to 0.27°C for DC, and from 0.05°C to 0.5°C for annual amplitude. These findings indicate that all heat sources relevant for sewer temperature modelling have similar, reproducible annual patterns.

3.2. Model \( \delta \) - Determination of soil-layer thickness

It is well known that attenuation of heat fluxes depends on the frequency, with fluxes with higher frequencies traverse shorter distances (Hillel, 2003). Considering that the dominant low and high frequencies of in-sewer air or water are annually (in some cases semiannually as well) and daily, respectively, we propose that the soil-layer thickness is selected such that the heat flux from the in-sewer fluid to the soil layer with a daily frequency is (almost) fully attenuated, e.g. the induced temperature variation at the boundary of the soil layer is less than 1% of the amplitude of the daily variation in the in-sewer fluid temperature. In this way, the impact of the daily temperature variation of the in-sewer fluid, which is difficult to predict, on the soil-layer boundary can be neglected, while the impact of the annual temperature variation of the in-sewer fluid can be reliably estimated considering that the annual variation of the source temperature is reproducible and hence predictable (Fig. 2). The soil-layer thickness thus selected is the least allowing the boundary temperatures to be reliably established.

The 2D heat transfer model (Cartesian coordinate system, 100 m ×
100 m field) was simulated to produce temperature distribution data in the soil, leading to a data-driven model to determine the optimal soil-layer thickness, which is expected to depend on the soil thermal diffusivity ($\alpha$) and sewer pipe radius ($R$). To build a generic model to determine the soil-layer thickness, we investigated temperature distribution in soil surrounding sewer pipes with a radius of 0.2 m, 0.5 m, 1 m, and 2 m, respectively, each with the soil thermal diffusivity varying in a broad range from $2 \times 10^{-7}$ m$^2$/s to $8 \times 10^{-7}$ m$^2$/s with an increment of $1 \times 10^{-7}$ m$^2$/s (Standard), resulting in 28 scenarios. Given the purpose of this simulation study, the pipe was buried at a depth of 50 m to avoid the temperature field around the pipe being influenced by the variations in the atmospheric temperature. The in-sewer fluid temperature was assumed to be $T_f(t) = T_{\text{atm,ave}} + A_{365}\cos(365 \times 2\pi t)$ (i.e. with the frequency being ‘daily’), with $T_{\text{atm,ave}}$ being the average atmospheric temperature, which is also the undisturbed soil temperature, and $A_{365}$ the amplitude corresponding to the ‘daily’ frequency. $T_{\text{atm,ave}}$ and $A_{365}$ were arbitrarily chosen to be 20°C and 5°C, respectively, which do not influence Model $\delta$ to be developed here.

Table 2 summarises the optimal boundary thickness for all 28 scenarios. The results indicate that an effective soil-layer thickness can be as thin as 0.3 m, while 0.6 m is sufficient for all scenarios. Moreover, it can be further observed that the soil-layer thickness increases dramatically with soil thermal diffusivity, but remains relatively independent of pipe radius, with the thickness increasing slightly with $R$. Hence, soil thermal diffusivity is selected as the sole variable for determining the soil-layer thickness. To ensure a conservative thickness, the dataset with $R = 2$ m, after additional simulations with more $\alpha$ values ($2 \times 10^{-7}$ m$^2$/s to $8 \times 10^{-7}$ m$^2$/s with an increment of $0.1 \times 10^{-7}$ m$^2$/s), was employed to develop the model. The data in Fig. 3 displayed features of a power function with respect to $\alpha$. We attempted to fit the data with a power function with a power of 0.5:

$$\delta_s = k \sqrt{\alpha}$$

where $\delta_s$ is soil-layer thickness (m) and $\alpha$ is soil thermal diffusivity (m$^2$/s). Through calibrating parameter $k$ (681.9 with a 95% confidence interval of [680.8, 683.0]), we obtained an excellent fit between the model and the data ($R^2 = 0.9987$). Eq. (2) is named Model $\delta$. 

### Table 2

Soil-layer thickness ($\delta_s$) of diurnal oscillation of in-sewer fluid temperature.

<table>
<thead>
<tr>
<th>Soil thermal diffusivity (m$^2$/s)</th>
<th>Soil-layer thickness (m)</th>
<th>$R = 0.2$ m</th>
<th>$R = 0.5$ m</th>
<th>$R = 1$ m</th>
<th>$R = 2$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 10^{-7}$</td>
<td>0.29</td>
<td>0.30</td>
<td>0.31</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>$3 \times 10^{-7}$</td>
<td>0.36</td>
<td>0.37</td>
<td>0.38</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>$4 \times 10^{-7}$</td>
<td>0.41</td>
<td>0.42</td>
<td>0.43</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>$5 \times 10^{-7}$</td>
<td>0.47</td>
<td>0.47</td>
<td>0.48</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>$6 \times 10^{-7}$</td>
<td>0.50</td>
<td>0.51</td>
<td>0.52</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>$7 \times 10^{-7}$</td>
<td>0.54</td>
<td>0.55</td>
<td>0.56</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>$8 \times 10^{-7}$</td>
<td>0.57</td>
<td>0.59</td>
<td>0.60</td>
<td>0.61</td>
<td></td>
</tr>
</tbody>
</table>

Note: $R$ is sewer pipe radius.
3.3. Boundary soil temperature determination

With \( \delta \) determined in Section 3.2, the aim of this section is to develop equations to estimate the temperature at the boundary of the soil layer \( (T_{s,b}) \):

1) \( \delta \) is designed such that the daily variation of the in-sewer fluid temperature is fully attenuated at the boundary. However, \( T_{s,b} \) will be influenced by the low-frequency variations, particularly the annual variation, and in some cases semiannual variation as well (Section 3.1). We define this component as \( \Delta T_{s,b,f} \).

2) Further, \( T_{s,b} \) will also be influenced by the heat source from the ground. Considering a sewer pipe is typically buried at least 1.5 m below the ground (District, 2012; Utility, 2018), it will be the low-frequency components (annual and semiannual), rather than the high-frequency component (i.e. daily, referring to Section 3.1), of the above-ground heat flux that will influence \( T_{s,b} \). We define this component as \( \Delta T_{s,b,g} \).

3) In addition, the DC component of the in-sewer fluid temperature and the atmosphere temperature will likely be different, which will cause additional heat transfer influencing \( T_{s,b} \). We define this component as \( T_{s,b,ave} \).

Thermal conduction is a linear process, governed by linear heat transfer equations (Supplementary Information). Consequently, the principle of superposition applies, and hence:

\[
T_{s,b,\text{tot}} = \Delta T_{s,b,f} + \Delta T_{s,b,g} + T_{s,b,\text{ave}} \quad (\text{Model} \ T)
\]

In the remainder of Section 3.3, three physics-informed data-driven models will be developed to estimate \( \Delta T_{s,b,f} \), \( \Delta T_{s,b,g} \) and \( T_{s,b,\text{ave}} \), respectively, utilising data generated by the digital twin implementing the 2D mechanistic heat transfer model.

3.3.1. Model T1 for estimating \( \Delta T_{s,b,f} \)

Assuming the in-sewer fluid temperature varies according to \( A \cos(\omega t) \), where \( \omega \) is the angular frequency (\( = 2\pi \) or \( 4\pi \) for annual or semiannual variations, respectively), and \( A \) is the variation magnitude, the aim of this section is to derive an empirical model to predict the induced temperature variation in the soil surrounding the pipe.

Imagine a case where the pipe radius \( R \) is large approaching infinity, the temperature variation in the soil with a distance of \( \delta \) from the pipe surface \( (\Delta T_\delta) \) has been well established in literature (see e.g. Hillel, 2003), which is:

\[
\Delta T_\delta(\delta,t) = e^{-\sqrt{2}\delta/\alpha} A \cos \left( \omega t - \delta \sqrt{\frac{\omega}{2\alpha}} \right)
\]

where \( e^{-\sqrt{2}\delta/\alpha} \) describes the attenuation of the variation altitude, \( \delta \sqrt{\frac{\omega}{2\alpha}} \) represents the phase lag (Hillel, 2003) (see also Supplementary Information). For a smaller \( R \), we introduce a new term \( f(\delta, R) \) to describe the curvature effect on the attenuation:

\[
\Delta T_\delta(\delta,t) = f(\delta, R) e^{-\sqrt{2}\delta/\alpha} A \cos \left( \omega t - \delta \sqrt{\frac{\omega}{2\alpha}} \right)
\]

To find \( f(\delta, R) \), the 2D mechanistic model (digital twin) was used to generate soil temperature data for scenarios with pipe radii of 0.2, 0.5, 1, and 2 m, and soil thermal diffusivities ranging from \( 2 \times 10^{-7} \) to \( 8 \times 10^{-7} \) \( \text{m}^2/\text{s} \) (with an increment of \( 10^{-7} \) \( \text{m}^2/\text{s} \)). The simulation settings were the same as those used for the development of \( \delta \), but with \( \omega = 2\pi \) or \( 4\pi \). The peak soil temperatures at \( \delta = 0.2, 0.3, 0.4, 0.5, \) and 0.6 m at quasi-steady state were extracted to find the form of \( f(\delta, R) \) and calibrate the parameters involved. All data used for model calibration is provided in Table S2. The process yielded the following \( f(\delta, R) \):

\[
f(\delta, R) = \left( \frac{\pi R}{\delta} \right)^\nu \]

where \( \nu \) and \( \xi \) are estimated to be 0.6495 and 1.320, with 95\% confidence intervals of [0.630, 0.669] and [1.279, 1.361], respectively.

Eq. (4) (see Table S3).

3.4. Model T2 for estimating \( \Delta T_{s,b,g} \)

The second heat source impacting soil temperature is the heat from the ground. The aim of this section is to derive an empirical model to predict the temperature variation in the soil surrounding the pipe, induced by the ground temperature variation of \( A_0 \cos(\omega t, \phi) \) at any location \( (\delta, \phi) \) on the vertical cross-section (Fig. 1). The depth of location \( (\delta, \phi) \) from the ground is therefore \( d = \sqrt{d_b^2 + (R + \delta + \phi \cos \phi)^2} \), where \( d_b \) is the depth of the pipe centre. The temperature variation at location \( (\delta, \phi) \), induced by the ground temperature variation of \( A_0 \cos(\omega t) \), is denoted as \( \Delta T_{s,g}(\delta, \phi, t) \).

If there were no sewer pipe buried in soil, \( \Delta T_{s,g}(\delta, \phi, t) \) at any location \( (\delta, \phi) \) could be calculated as (Hillel, 2003):

\[
\Delta T_{s,g}(\delta, \phi, t) = e^{-\sqrt{2}_s \delta \cos \phi} A_0 \cos \left( \omega t - (d \cos \phi) \sqrt{\frac{\omega}{2\alpha}} \right)
\]

(6)

As specific cases, at locations of the pipe-soil interface \( (\delta = 0) \), \( \Delta T_{s,g}(0, \phi, t) \) in the absence of the sewer pipe is calculated as:

\[
\Delta T_{s,g}(0, \phi, t) = e^{-\sqrt{2}_s \phi} A_0 \cos \left( \omega t - (d \cos \phi) \sqrt{\frac{\omega}{2\alpha}} \right)
\]

(7)

In the presence of the sewer pipe, a second heat source (in-sewer fluid) is added, the effect of which is considered separately via the principle of superposition (see Section 3.3.1). Thus, the temperature at the pipe-soil interface for the development of Model T2 can be considered 0, rather than \( \Delta T_{s,g}(0, \phi, t) \) calculated with Eq. (7). This is equivalent to adding another (virtual) heat source at the pipe-soil interface with a
temperature distribution of \( \Delta T_{s,b}(0, \varphi, t) \), Eq. (6) needs to be corrected to consider the effect of this virtual heat source (via Eq. (5)) and becomes:

\[
\Delta T_{s,e}(\delta, \varphi, t) = e^{-(d_p - R + 5\cos(\alpha t))} \sqrt{\frac{\omega}{2}} A\cos(\omega t - d_p - (R + \delta)\cos(\varphi)) + e^{-(d_p - R + 5\cos(\alpha t))} \sqrt{\frac{\omega}{2}} A\cos(\omega t - 2d_p - (2R + \delta)\cos(\varphi)) \sqrt{\frac{\omega}{2n}}
\]

Eq. (8) is named Model T2. Specifically, the component of the boundary temperature \( \Delta T_{s,b}(\varphi, t) = \Delta T_{s,b}(\delta, \varphi, t) \). Fig. 5 compares the performance of Model T2 with the full model in predicting boundary soil temperature corresponding to a ground surface temperature \( T_f(t) = 20 + 5\cos(\omega t) \), with \( \omega = 2\pi \). Model T2 achieved high fidelity with \( R^2 \) values of 0.9985 to 0.9998 for each comparison, with the detailed \( R^2 \) and RMSE shown in Fig. 5.

3.5. Model T3 for estimating \( T_{s,b,ave} \)

Model T1 and Model T2 focus on the low-frequency oscillation terms of the heat sources. In this section, the focus is on the constant parts of the two heat sources (i.e., the average temperatures of the two sources, or the DC parts of the Fourier transforms), which lead to temperature gradients in the soil if there is a difference between the two terms. Here we assume the average ground surface temperature is \( T_{s,ave} \), and the average pipe surface temperature is \( T_{f,ave} \). The soil temperature at \((\delta, \varphi)\) caused by these constant heat sources is \( T_{s,ave}(\delta, \varphi) \).

Let’s imagine that we move the pipe (radius \( R \)) and the surrounding soil of a thickness of \( \zeta \cdot R \) to above the ground with the outer surface of the soil directly exposed to the atmosphere. The steady-state temperature distribution of the soil surrounding the pipe is well established, and can be described as (Dürennatt and Wanner, 2014):

\[
T_{s,ave}(\delta, \varphi) = T_{f,ave} + (T_{g,ave} - T_{f,ave}) \ln\left(\frac{\rho + \varphi}{\rho}\right)
\]

(9)

where \( \zeta \) is the radial distance (m) from the pipe centre to the outer soil surface.

If we place the cylindrical soil/pipe block back into the soil, Eq. (9) is no longer valid as a continuum of soil forms and the clear soil/air boundary disappears. We hypothesise a virtual boundary may still exist with an unknown \( \zeta \) for each \((\delta, \varphi)\). \( \zeta(\delta, \varphi) \) could be estimated using the following equation by rearranging Eq. (9):

\[
\zeta(\delta, \varphi) = R e^{\frac{T_{s,ave}(\delta, \varphi)}{\rho - T_{f,ave}}} \ln\left(\frac{R + \delta}{R}\right)
\]

(10)

if the temperature field \( T_{s,ave}(\delta, \varphi) \) is known.

We then used the digital twin to generate \( T_{s,ave} \) for a range of conditions by systematically varying the pipe burial depth \( d_p = 2.0 \text{ m}, 2.5 \text{ m}, 3.0 \text{ m}, 3.5 \text{ m}, 4.0 \text{ m} \) and the pipe radius \( R = 0.2 \text{ m}, 0.5 \text{ m}, 1.0 \text{ m}, 1.5 \text{ m}, 2.0 \text{ m} \) with the implausible cases such as \( R = 2.0 \text{ m} \) and \( d_p = 2.0 \text{ m} \) excluded (see Table S4 for the simulated cases). \( \alpha = 4 \times 10^{-7} \text{ m}^2/\text{s} \) was
used, and not varied because it does not affect the steady-state temperature field. \( T_{\text{ave}} \) and \( T_{\text{ave}} \) were arbitrarily assumed to be 20°C and 25°C, respectively, which do not affect the result. Each scenario was simulated on the field of 100 m × 100 m for 100 years to steady state. \( T_{\text{ave}}(\delta \rho) \) in the region of \( \delta \in [0.2 m, 0.6 m], \varphi \in [0°,180°] \) were then used to estimate \( \varphi \) using Eq. (10). This region was used because the aim is to obtain a value of \( \varphi \), which will be used in the empirical equation to estimate \( T_{\text{ave}}(\delta, \varphi) \) in this region, recalling that \( \delta_i \) is in the region \( [0.2 m, 0.6 m] \) (Fig. 3). Fig. 6(A) depicts the distribution of \( \varphi(\delta, \varphi) \) for sewer pipe buried at 3 m with different \( R \), with full results presented in Fig. S4. \( \varphi(\delta, \varphi) \) is found to be dependant on \( d_p, R \), and \( \varphi \), but largely independent of \( \delta \) in the region of \([0.2 m, 0.6 m]\).

The results manifest that, for a pair of \( d_p \) and \( R \), the distribution of \( \varphi(\delta, \varphi) \) in the region \( \delta \in [0.2 m, 0.6 m], \varphi \in [0°,180°] \) form a circle with a radius of \( R \) (m) and a centre with a depth of \( R - \zeta_u \) (m) from the ground, where \( \zeta_u \) denotes the vertical distance starting from the circle top to ground, as indicated in Fig. 6(B). Hence, in a Cartesian coordinate system with the sewer pipe centre at \((0, -d_p)\), the coordinates of \( \zeta(x, y) \) follow \( x^2 + (y + (R - \zeta_u))^2 = R^2 \) (the green dashed lines in Fig. 6(A) and 6(B)).

From the simulation data, the values of \( R \) and \( \zeta_u \) for each pair of \( d_p \) and \( R \) were calculated, and then correlated with \( d_p \) and \( R \) using the nonlinear least square method. The corresponding circles are depicted by the green dashed line in Fig. 6(A) and (B) with \( R^2 \) of the fitness shown in Fig. 6(A). Analysis of the data generated by the digital twin revealed a linear relationship between \( R \) and \( \zeta_u \) and \( d_p \) and \( R \) (Eq. (11)), but the relationship between \( \zeta_u \) and \( d_p \) and \( R \) was nonlinear (Eq. (12)):

\[
R_i = \lambda d_p + \eta R
\]  
(11)

\[
\ln \frac{\zeta_u}{R} = \mu \left(1 - e^{-\gamma \left(x-x_i\right)}\right)/\xi
\]  
(12)

where \( \lambda \) and \( \eta \) are calibrated to be 2.279 and 1.455, with 95% confidence intervals of [2.163, 2.394] and [1.158, 1.752], respectively, \( \mu \) and \( \gamma \) are estimated to be 2.820 and 0.309, with 95% confidence intervals of [2.726, 2.914] and [0.289, 0.329], respectively.

The calibration results of \( R \) and \( \zeta_u \) are depicted in Fig. 6(C) and (D), which achieved \( R^2 \) of 0.9762 and 0.9964, respectively, demonstrating that the designed functions are capable of describing \( \zeta_u \) in any direction given a buried pipe. Details of the simulation conditions, calibration data, and calibration results are summarised in Table S4. Accordingly, \( \zeta_u \) can be obtained by solving the Euclidean distance from the sewer pipe centre to the circle of \( \zeta_u \) in the radial direction of \( \varphi \) based on the geometrical relation, i.e., \( \zeta_u = \sqrt{x^2 + (y + (R - \zeta_u))^2} \). The process of solving \( \zeta_u \) is provided in Supplementary Information. Finally, feeding \( \zeta_u \) into Eq. (9) yields the soil temperature contributed by the difference between the averages of the heat sources. The set of Eq. (9) to (12) is named Model T3. Specifically, the component of the boundary temperature \( T_{\text{b}} \) is in the region \([0.2 m, 0.6 m]\).

3.6. Application - Establishment of boundary conditions and prediction of sewer wall temperature for a real-life sewer

Case studies were performed to verify the proposed methodology, and the accuracy of the data-driven models proposed. Gravity sewers in Melbourne Western Trunk Sewer system (presented in Section 2.5), and historical temperature data of air and wastewater in these sewers, as well as ground-surface temperature, were used. One representative example, conducted on an empty real-life sewer with an inner diameter of 1.8 m, is shown here. This pipe has the lowest burial depth of 3.4 m amongst the studied pipes and thus can best demonstrate the ability of our methodology. One-year field data of the soil surface temperature (10 cm deep) and in-sewer air temperature are shown in Fig. 7. Based on the Fourier analysis results, the annual frequency was applied for the in-sewer fluid temperature, and annual and semiannual frequencies were applied for the ground surface temperature. The frequency characteristics of the source temperature, along with all information for simulation are listed in Table 3. Thermal convection was simulated for the interface between the in-sewer fluid and sewer pipe inner surface, while conduction was simulated for pipe wall, interface between pipe outer surface and soil, and soil. All simulations were run for two years, and the results of the second year were collected.

First, the simulation was performed in a Cartesian coordinate system of 100 m × 100 m (grid=0.1 m) using the full model with full boundary conditions (shown in Section 2.3). The simulated soil temperature and
sewer wall temperature were used as benchmark to evaluate the performance of the physics-informed models. Next, the physics-informed model \( \delta \) was employed to find the optimal boundary thickness \( 0.4 \) m, hence the computational region was reduced to a circular area with a radius of \( 1.4 \) m, i.e. pipe radius + wall thickness + boundary thickness. Accordingly, the soil-layer boundary temperature was obtained by physics-informed model \( T \). A summary of the detailed procedures of applying the developed methodology to establish boundary conditions is provided in Supplementary Information. Then, the circular area was discretised with an angular increment of 15° and a radial increment of 0.1 m. Pipe wall temperature was modelled in the cylindrical coordinate system with the optimised boundary conditions.

Fig. 8A compares the boundary soil temperature at five locations in radial directions of 0°, 45°, 90°, 135°, and 180° on the boundary (locations indicated by Fig. 5F), calculated with the physics-informed models and the full mechanistic model. The developed physics-informed models achieved \( R^2 \) of 0.9460 to 0.9784 and RMSE of 0.13°C to 0.39°C, demonstrating high precision in predicting the soil-layer boundary temperature. Fig. 8B compares the pipe wall temperature modelled with the full 2D model and the cylindrical model with optimised boundary conditions established with our methodology. It reveals that the physics-informed models achieved high fidelities to the full model, with the lowest \( R^2 \) of 0.9854, the highest RMSE of 0.17°C, and the largest absolute error of 0.20°C.

To analyse the importance of boundary conditions in sewer heat...
transfer modelling. Fig. 9 compares the pipe wall temperature profiles predicted in the three cases defined in Scenario 3 (see Section 2.5) with those predicted with the full 2D model. The thin boundary case ($\delta_1 = 0.1$ m) yielded unsatisfactory predictions with RMSE ranging from 2.30°C to 2.47°C, and absolute errors up to 3.21°C (Fig. 9A). All these error indicators are more than an order of magnitude higher than those achieved in Scenario 2 with optimally determined soil-layer thickness and boundary temperatures. With a conservative boundary thickness of $\delta_1 = 2$ m, the model yielded RMSE of 0.24°C to 0.33°C, and absolute errors up to 0.57°C (Fig. 9C). While the computation time was almost one order of magnitude higher compared to our methodology (631 s for the conventional case with $\delta_1 = 2$ m vs. 85 s for our method with $\delta_1 = 0.4$ m), due to the use of a much thicker soil layer, the accuracy was still lower than that of our methodology due to the use of undisturbed earth temperature as the boundary temperature. The importance of using correctly determined boundary temperature(s) was further demonstrated in the case where the undisturbed soil temperature was assigned to the boundary of the soil-layer with the optimal thickness of $\delta_1 = 0.4$ m. The RMSE was in the range of 1.11°C to 1.21°C, and the absolute error was up to 1.21°C (Fig. 9B), both indicators were several times higher than those obtained in Scenario 2. Overall, the simulation results demonstrate the robust and efficient performance of our methodology in establishing boundary conditions for sewer heat transfer modelling.

4. Discussion

4.1. Learning efficient boundary conditions from digital twin

Boundary conditions directly determine the accuracy and efficiency of heat transfer modelling for sewers, but their determination represents a knowledge gap at present. We presented a systematic methodology for generating efficient boundary conditions to enable accurate and efficient heat transfer modelling. The methodology is generic and can be applied to sites with any pipe diameters, pipe burial depths, and soil properties, provided that thermal conduction is the dominant heat transfer mechanism in the soil. The only parameter that needs to be obtained for the site is the soil thermal diffusivity.

Our method relied on a digital twin that implements a well-established mechanistic model for heat transfer, which generates abundant data for studying the thermal field under various conditions including source temperature oscillations, soil physical characteristics, pipe dimension, burial depth, amongst others. With the proposed method, the computational area for modelling heat transfer is defined to a minimum that ensures the temperature distribution at the soil-layer boundary can be reliably predicted. This enhancement is based on the facts that:

1) the source temperatures, i.e. ground surface temperature and temperature of the in-sewer fluid, comprise a set of waves (temperature profiles) with frequencies dominated by 1–2 low frequencies (annually, and in some cases also semiannually), and one high-frequency (daily), which are far-separated;
2) the low-frequency variations and the average values of the above heat source temperatures remain stable across years and are therefore predictable;
3) the higher the wave frequency, the shorter the wave can travel in the soil before being fully attenuated (Bergman et al., 2011).

The above enabled us to choose a boundary thickness that almost completely attenuates (>99%) the effect of daily variations of the in-sewer fluid. A simple model (Model $\delta$) was then developed based on the data to determine the optimal boundary thickness based on the soil thermal diffusivity. A previous study also presented a function for selecting the soil-layer thickness based on the ‘daily’ frequency, in modelling heat transfer between a buried air tunnel and the surrounding soil (Kraft and Kreider, 1996). The authors directly applied a well-established model which was originally developed for calculating the penetration depth from the flat ground surface into the soil, and neglected the impacts of pipe curvature on the conductive heat transfer. Our Model $\delta$ was trained from the simulation data with varied pipe radius and thermal diffusivity, thus fully considered the impacts of pipe curvatures. Model $\delta$ is thus significantly different from that established in the previous study.

The temperature at the selected boundary is affected by the low-frequency variations of both the in-sewer fluid and the atmospheric temperatures, as well as the average temperatures of these two sources. The linearity of the governing equation for thermal conduction enables the use of the superposition principle in determining the boundary temperatures. With this principle, the effect of each of the three factors on the boundary temperature was evaluated independently by focusing on a particular heat source each time. Through learning from the data generated by the digital twin, three physics-informed models (Model T1 to T3) were developed to predict the boundary soil temperature.

As shown in the case study, our approach outperformed both the aggressive ($\delta_1 = 0.1$ m) and conservative ($\delta_1 = 2.0$ m) boundary conditions adopted in literatures (Dürenmatt and Wanner, 2014; Elias-Maxil et al., 2017) in terms of modelling accuracy (Fig. 8 and 9). In fact, the aggressive boundary conditions caused large, unacceptable errors in predictions, and can thus be considered an invalid approach. Our method accelerated the computational speed by over seven times compared to the conservative case, due to the use of a thinner soil-layer. In summary, our approach avoids the use of overly conservative (i.e. too thick), thus computationally intensive, boundary conditions, or overly aggressive (i.e. too thin), thus unreliable, boundary conditions in heat transfer modelling, enabling efficient yet accurate temperature predictions.

4.2. Physics-informed data-driven modelling

Being data-driven models, Models T1-T3 are based on the physical features of sewer pipes and the heat transfer processes. Model T1 is developed based on the analytical solution of one-dimensional plane-
Fig. 8. Real-life case study of temperature modelling: soil boundary temperatures (A) and sewer pipe wall temperatures (B).
surface heat transfer, with a corrected term \( f(\delta, R) \) introduced to account for the impact of a curvature surface on heat transfer. \( f(\delta, R) \) is constructed as an exponential function of \( \delta / (\delta + R) \) (Eq. (5)), derived from a physical analysis of the impact of \( \delta \) and \( R \). We imagined, for a pipe with an infinitesimal \( R \), the in-sewer fluid is a point source and thus the term becomes a constant. On the contrary, if the pipe \( R \) is infinite, the curvature surface approaches a plane surface, and \( f(\delta, R) \) becomes 1, enabling Model T1 to return to the analytical solution of plane-surface heat transfer. Similarly, Model T2 is based on the analytical solution of one-dimensional plane-surface heat transfer and Model T1. The analytical solution describes the temperature field caused by the ground surface temperature variation, in the absence of a sewer pipe. The pipe modified the field, falsifying the analytical solution. However, based on our physical knowledge, the impact of the pipe can be described by introducing an imaginary heat source at the pipe surface “reflecting” the heat flux. The latter can be described by Model T1. The integration of the analytical solution and Model T1 thus yields Model T2, predicting the impact of the ground surface temperature variation on the soil temperature field in the presence of a sewer pipe that altered the field. Model T3 is developed from a steady-state heat transfer solution. However, the critical variable involved in Model T3, the so-called undisturbed soil thickness \( \zeta \), is impossible to measure in practice. Informed by the reality that \( \zeta \) only depends on the pipe radius and burial depth, the spatial distribution of \( \zeta \) is identified using the digital twin, and a geometric solution for \( \zeta \) is developed.

Thus, physical knowledge empowers us to build robust equations with simple forms and fewer parameters (no more than two parameters to be learnt from data in each model), so that the model can be calibrated with high certainties, as demonstrated by the small 95% confidence intervals in Fig. 3 and Fig. 6(D). This approach enhances the efficiency and robustness in predicting boundary temperatures. Moreover, it provides the flexibility to make different combinations of Model T1 to T3, to adapt to various practical scenarios.

Although focusing on the soil temperature at the boundary, Models
T1-T3 are applicable to predicting soil temperature in a much broader region starting from $\zeta$ towards infinity, by specifying the independent variable $\delta$ (Model T). Besides, Model T3 provides a generic approach for evaluating the boundary of undisturbed soil, denoted by $\zeta$. This boundary is where temperature variations at any frequency vanish. Such a conceptual distance has been widely used in heat transfer modelling and other studies on soil thermodynamics. Empirical values or infinity have often been used (Abdel-Aal et al., 2021; Dürenmatt and Wanner, 2014; Figueroa et al., 2021). Our study reveals that $\zeta$ ranges from 3 m to 20 m for a normal sewer pipe, depending on pipe dimensions and burial depth (Fig. 6(A) and Table S4). Model T3 can be used to estimate $\zeta$ for any given conditions.

4.3. Applicability, limitations and future work

This study provides a generic methodology for establishing efficient boundary conditions for conductive heat transfer modelling involving the use of PDEs. These include all the heat transfer models listed in Table 1, which employ either 1D (radial) or 2D (Cartesian coordinate) PDEs (Abdel-Aal et al., 2021; Dürenmatt and Wanner, 2014; Eliason et al., 2017; Hadengue et al., 2021). Our method enables users of these models to find a minimum computational field, and the boundary temperature, which minimise the computational load while ensuring the solution accuracy. This was demonstrated in our case study (Section 3.4), where we employed the method to find the soil-layer thickness (computational field) and the temperature distribution at the soil-layer boundary for the model proposed in Dürenmatt and Wanner (2014). The efficient and accurate temperature predictions enabled by our method can support the management of heat recovery from wastewater (Saagi et al., 2021), the prediction and control of sewer corrosion (Pikaar et al., 2019), and the modelling of temperature-dependant biochemical reactions in sewers (Wanner et al., 2005). In addition to sewers, our methodology is potentially applicable to other types of buried pipes, for example, water supply pipes where the water temperature influences chlorine decay (Geng et al., 2022), and heat supply pipelines where the heat loss during heat transport should be minimised (Zheng et al., 2018).

The sewer pipe simulated here is filled either with air (gravity sewer with shallow water depth) or water (rising mains) for simplicity as demonstrated in Section 2.1, but gravity sewers typically contain both air and water with their proportions changing over time. Further research is needed to develop auxiliary models to extend this methodology to a pipe containing both water and air.

In this study, the soil thermal diffusivity is assumed constant for the ambient soil, but it changes over soil texture and moisture content in nature (Arkhangel'skaya and Lukyashchenko, 2018). The standard function for calculating thermal diffusivity (Standard) as a function of soil moisture and soil material can be integrated into the digital twin to generate datasets to train new model terms accounting for the varying soil thermal diffusivity. Also, in cases where the pipe is submerged in saturated soil (such as in wet weather) or below the groundwater table, the dominated heat transfer process may change from thermal conduction to thermal convection. In such cases, the current model is not suitable and new models are required. Further, we only considered cylindrical pipes in this study. The models should be adapted for pipes with different shapes (e.g. oval or rectangular), but the proposed methodology is applicable.

Based on Fourier analysis results, the DC and annual amplitude of source temperatures are reproducible from the past years. This feature ensures accurate inputs for Models T. However, the reproducibility may be deteriorated by irregular episodes in nature, such as persistent rainfall, snowmelt, and droughts (Hillel, 2003; Saagi et al., 2021). In such situations, our method will yield errors in predicting the boundary soil temperature. One potential approach is to regress longer-term (years) historical data of source temperatures to filter the anomalies and identify the typical DC and annual frequency features for establishing boundary conditions. In this study, we identified that the annual and daily frequencies are the most prominent low and high frequencies of the temperature profiles. This conclusion was drawn by investigating the temperatures of all capital cities in Australia, which distribute across temperate, tropical and sub-tropical zones. The dominant frequencies of the temperature in a broader range of climate conditions need to be further verified.

This work helps to reduce the computational demand in sewer heat transfer modelling by reducing the thickness of soil to be included in the model. The model our approach supports is still a mechanistic model with three independent variables, namely radius, angle, and time. The model could still be too slow for large networks or real-time applications (Cen et al., 2023; Li et al., 2022a). With the support of digital twins (Arnell et al., 2023), there could be opportunities to develop swift thermal models to be integrated with swift hydraulic models previously developed (Li et al., 2022b, 2019). This approach will potentially deliver a model that is much more efficient to solve.

5. Conclusions

In this study, a systematic methodology for establishing boundary conditions for sewer heat transfer modelling was developed and applied to model sewer wall temperature in real-life sewers. The main findings are summarised as follows:

- Fourier analysis of ground surface temperature and in-sewer fluid temperature reveals the annual and diurnal frequencies are the most prominent low and high frequencies of the temperature profiles, respectively. Semiannual variation could be significant in ground surface temperature in certain regions.
- The soil-layer thickness in sewer heat transfer modelling can be defined as the penetration depth of diurnal variation of the in-sewer fluid temperature. A simple model has now been developed to determine the soil boundary thickness.
- The soil boundary temperature can be decomposed into three components attributed to the ground surface temperature variation, in-sewer fluid temperature variation, and the difference between the average temperatures of these two heat sources. Each component can now be estimated by a physics-informed model learnt from digital twins of sewer heat transfer.
- The proposed methodology is general and deterministic, using simple and predictable inputs. It can be easily applied to scenarios with various pipe dimensions, pipe geographic locations, source temperatures, and soil thermal properties.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgements

The authors acknowledge the Reducing Sewer Corrosion through Model-supported Ventilation Control Project LP190101262 funded by the Australian Research Council (ARC), along with Melbourne Water, Water Cooperation, Urban Utilities, and DC Water, and the UQ Digital Water Initiative funded by The University of Queensland. Jiuling Li acknowledges UQ Early Career Researcher Development KnX&T Fund. Zhiguo Yuan is a Global STEM Scholar funded by the Hong Kong Special Administrative Region via the Innovation, Technology and Industry
Supplementary materials


References


