Dynamic Coding-Based Control Scheme Under Lossy Digital Network: An Optimized Time-Varying Packet Length Approach

Jiarui Li, Yugang Niu†, Member, IEEE, Daniel W. C. Ho, Fellow, IEEE

Abstract—This work proposes a design scheme of the desired controller under the lossy digital network by introducing a dynamic coding and packet-length optimization strategy. Firstly, the weighted try once-discard (WTOD) protocol is introduced to schedule the transmission of sensor nodes. The state-dependent dynamic quantizer and the encoding function with time-varying coding length are designed to improve coding accuracy significantly. Then, a feasible state-feedback controller is designed to attain that the controlled system subject to possible packet dropout is exponentially ultimately bounded in the mean square sense. Moreover, it is shown that the coding error directly affects the convergent upper bound, which is further minimized by optimizing the coding lengths. Finally, the simulation results are provided via the double-sided linear switched reluctance machine systems.

Index Terms—Lossy digital network; Dynamic encoding-decoding scheme; exponentially ultimately bounded (EUB); quantized-based control.

I. INTRODUCTION

In the past two decades, networked control systems have gained much concern since system components are often far away from each other and joined via digital channels [1]–[4]. The critical point is to convert the analog signals into a binary string before transmitting them through the digital channel [5]. One effective method is quantizing the analog signal to some constant in a finite set before mapping it into a binary string (i.e., the data packet) via the encoder. After that, the decoder converses the received string back to the analog signal. Due to the limited transmission power, some data packets may drop during transmission [6]–[8]. This process is the quantization-based encoding-decoding scheme (EDS) under the lossy network.

To date, some intriguing results have been reported in the literature [9]–[11] concerning quantization-based EDS. For example, Qiu et al. [9] investigated the uniform quantization-based EDS with a limited data rate, and Wang et al. [10] gave a concrete design procedure for the multiple-description coding scheme under the control/filtering problem to improve the coding accuracy. However, the quantizer’s parameter for the works mentioned above is static, while the dynamic one merits a higher coding precision or a more extensive coding range [12]–[14]. Generally, the quantizer’s parameter can be dynamically updated by the time-dependent [15]–[17] or state-dependent strategy [18]–[20]. The quantizer’s parameter does not need to transmit for the former since it can be set at both the encoder and decoder sides in advance. However, this strategy cannot deal well with system uncertainty or external disturbance. Specifically, the quantizer’s parameter under the time-dependent adjusting strategy as in [15], [16] may decrease to zero over time so that it would fail while a rising trend for the state signal happens. On the contrary, the state-dependent strategy can remove this obstacle. Still, both the zoomed measurement and the quantizer’s parameter had to be transmitted since the exact state signal was unknown on the decoder side. Aiming at that, Niu and Ho [18] proposed a practical adjusting rule to transmit the dynamic parameter via an integer pair, which may take values from an infinite set in theory that ignores the limitation of coding length. Recently, the above result has been extended to the case with the finite coding length in [19], and the coding accuracy is higher than the static quantization-based EDS.

Generally, the length of a packet refers to the sum of lengths from all required coding signals (e.g., the state/output signal, the quantizer’s parameter), and increasing any signal’s coding length will result in the growth of the packet length. Moreover, some communication protocols (e.g., Round-Robin protocol [21], [22], Weighted Try-Once Discard (WTOD) protocol [23], [24], and Random Access protocol [25]–[27]) may reduce packet length by transmitting less data. So far, some studies have found that channel gain [28], power level [29], and packet length [21] affect the packet loss probability. Thus, the coding length affects not only the coding error but also the packet loss probability, which will further affect the dynamic performance of the control system. Increasing the coding length simply improving the coding accuracy may deteriorate the system performance due to higher packet loss probability. However, in the existing relevant literature (e.g., [9]–[11], [19]), the coding length and the packet loss probability are usually fixed or given in advance. Up to now, few efforts have been made to investigate their relationship and consider the influence of coding error and packet loss probability on the system performance.

Based on the above discussions, selecting the proper coding lengths for the zoomed measurement and the dynamic quantiz-
er’s parameter will be an essential issue for the control design under the lossy network. Thus, in this work, we improve the coding accuracy by designing the time-varying coding lengths. The communication protocol is first introduced to reduce the packet length by selecting only one sensor node to access the network at each instant. Then, a dynamic quantization-based EDS is proposed, in which the zoomed measurement and the state-dependent quantizer parameter are encoded via different coding rules before transmission. Accordingly, the data packet consists of binary strings for the token, the quantized state signal, and the quantizer’s parameter. The relation between the packet length and the packet loss probability is detailed analyzed. The coding lengths can be optimized to minimize the adverse impact of coding errors while promising the stability of the controlled system. In particular, higher coding accuracy can be achieved under a time-varying coding length than the fixed one. The main contributions of this paper are given as follows:

1) This work proposes a dynamic quantization-based coding scheme whose coding lengths (for the zoomed measurement and the dynamic parameter) are designed comprehensively via the coding error and the packet loss on the control performance.

2) The state-dependent quantizer’s parameter takes values from a finite set via the modified adjusting rule. Meanwhile, its time-varying coding length can include more information under the same maximum coding length.

3) The relation between the maximum allowable packet loss probability and the coding error is established. Moreover, the minimum bound of the coding error in this work is much smaller than the one under the static coding scheme with the same coding range.

Notation: \( \mathbb{R}^n \) denotes the set of \( n \)-dimensional vectors. \( \Pr \{ X \} \) is the probability of the event \( X \). \( | \cdot | \) is the 1-norm, and \( \| \cdot \| \) is the Euclidean norm. \( \mathbb{E}\{\cdot\} \) is the mathematical expectation, and \( \mathbb{E}\{\cdot|\cdot\} \) is the conditional expectation. \([x]\) and \( \lfloor x \rfloor \) represent the floor function and ceil function, respectively. \( \delta(\cdot) \) is the Kronecker delta function, i.e., \( \delta(v) = 1 \) if \( v = 0 \) and \( \delta(v) = 0 \) otherwise.

II. SYSTEM DESCRIPTION AND LOSSY DIGITAL NETWORK

Consider a class of discrete-time systems as:

\[
x(k+1) = (A + \hat{A}(k))x(k) + Bu(k) + D\omega(k)
\]

(1)

where \( x(k) \in \mathbb{R}^n \) and \( u(k) \in \mathbb{R}^m \) are the state vector and the control input, respectively. \( \hat{A}(k) \) denotes the norm-bounded parameter uncertainty with \( \hat{A}(k) = EF(k)H \), where \( E \) and \( H \) are known constant matrices, and the unknown time-varying matrix \( F(k) \) satisfies \( F^T(k)F(k) \leq I \). The unknown external disturbance \( \omega(k) \) satisfies \( \| \omega(k) \| \leq \bar{\omega} \) with \( \bar{\omega} \) known constant. \( A, B, D \) are constant matrices with compatible dimensions.

In this work, we transmit the state signal through the lossy digital channel via binary phase-shift keying (BPSK). A two-value variable \( \theta(k) \in \{0,1\} \) is introduced to describe the unsuccessful/successful transmission of an \( \mathcal{X}(k) \)-bits packet. Similar to [21], [29], the packet arrival probability is

\[
\Pr\{\theta(k) = 1\} = \bar{\theta}^{\mathcal{X}(k)}, \quad \Pr\{\theta(k) = 0\} = 1 - \bar{\theta}^{\mathcal{X}(k)}
\]

(2)

with the arrival probability of a one-bit packet \( 0 < \bar{\theta} \leq 1 \).

Remark 1: In [21], it was assumed that the packet length for all sensors is the same and fixed-constant, i.e., the packet length is an integer multiple of the number of selected nodes. In this work, we shall investigate how to reasonably allocate the coding length for different sensors to achieve a better control performance in Section IV-C.

Before transmitting the state signal through the lossy digital network, a crucial step is to encode the analog state to a binary string via the encoding-decoding scheme (EDS). It is clearly shown from (2) that the packet arrival probability \( \bar{\theta}^{\mathcal{X}(k)} \) shall increase with the decrease in packet length \( \mathcal{X}(k) \) (i.e., \( \bar{\theta} \geq \bar{\theta}^1 \geq \bar{\theta}^2 \geq \cdots \)). However, at the same time, it results in a bigger coding error, which shall be analyzed later in Section III-E. Apparently, either a more prominent coding error or a higher packet loss probability may harm the control performance.

Our objective is to design a dynamic coding scheme with a proper packet length, which can reduce the adverse impact of the coding error and the packet loss on the control performance.

III. WTOD PROTOCOL AND DYNAMIC QUANTIZATION-BASED CODING SCHEME

In this work, a lossy digital network is considered in Sensor-to-Controller (S/C) channel, where the more prolonged the packet, the higher the packet loss probability. Firstly, the weighted try once-discard (WTOD) protocol is introduced to reduce the packet length. Other protocols (e.g., the Round-Robin protocol and Random Access protocol) could also serve the same purpose by adopting different rules to schedule the transmission of the sensor node. Then, the selected sensor signal is encoded into the packet with \( \mathcal{X}(k) \)-bits and transmitted via the lossy network. Fig.1 shows the structure of the considered system.

For simplicity, it is assumed that the control signal \( u(k) \) and the ACK flag \( \theta(k) \) can be successfully transmitted to the actuator (or the WTOD protocol module) through the lossless network. Besides, extending the present work to the case with packet loss in both S/C and Controller-to-Actuator (C/A) channels is interesting and not difficult.
A. WTOD protocol

Under the WTOD protocol, only one sensor node can access the network at each instant. The selected sensor node is represented via the token $\varphi(k) \in \mathcal{U} \triangleq \{1, 2, \ldots, n\}$ according to the following maximum-error first policy

$$
\varphi(k) = \arg \max_{1 \leq i \leq n} (x_i(k) - \hat{x}_i(k-1)) \\
\times \Psi_i(x_i(k) - \hat{x}_i(k-1))
$$

(3)

where $\hat{x}_i(k-1)$ represents the previous signal of the $i$-th sensor node available for the controller. $\Psi_i > 0$ is the weight coefficient. Notice that this work considers three network-induced constraints: the WTOD protocol, the coding (quantization), and the packing loss. Hence, the available state signal for the controller side is given as follows:

$$
\hat{x}_i(k) = \begin{cases} \\
\bar{q}_i(x_i(k), \mu_i(k)), & \text{if } \varphi(k) = i, \theta(k) = 1, \\
\hat{x}_i(k-1), & \text{otherwise.}
\end{cases}
$$

(4)

where $\bar{q}_i(x_i(k), \mu_i(k))$ is the quantized state. That is, the available state $\hat{x}_i(k)$ only updates its value when the current WTOD protocol selects this node, and the data packet does not drop out. Otherwise, the controller will use the previously available information. Moreover, as shown in Fig. 1, both the quantized state signal $\bar{q}_i(x_i(k), \mu_i(k))$ and Acknowledgement (ACK) should be sent to the protocol module to calculate the token $\varphi(k)$ via the updated principle (3).

Furthermore, the WTOD protocol (3) may be described in the following compact form

$$
\varphi(k) = \arg \max_{1 \leq i \leq n} (x_i(k) - \hat{x}_i(k-1))^T \\
\times \bar{\Psi}_i(x_i(k) - \hat{x}_i(k-1))
$$

(5)

with $\bar{\Psi} \triangleq \text{diag}\{\Psi_1, \ldots, \Psi_n\}$ and $\bar{T}_i \triangleq \text{diag}\{\delta(i-1), \ldots, \delta(i-n)\}$. The Kronecker delta function $\delta(v) = 1$ if $v = 0$, and $\delta(v) = 0$ otherwise.

In the sequel, we shall design a dynamic quantization-based coding scheme to quantize the selected state $x_{\varphi(k)}(k)$ to some constants in the finite set and map it into a binary coding string.

B. State-dependent dynamic quantizer

Introduce the following dynamic quantizer:

$$
\bar{q}_{\varphi(k)}(x_{\varphi(k)}(k), \mu_{\varphi(k)}(k)) \triangleq q_{\varphi(k)}(x_{\varphi(k)}^*(k)) \mu_{\varphi(k)}(k)
$$

(6)

where $q_{\varphi(k)}(\cdot)$ is the uniform quantizer, $x_{\varphi(k)}(k) \triangleq x_{\varphi(k)}^*(k)/\mu_{\varphi(k)}(k)$ is the zoomed measurement, $\mu_{\varphi(k)}(k)$ is the dynamic parameter with $0 < \mu_{\varphi(k)}(k) \leq 1$.

Here, the dynamic quantizer (6) depends on the token $\varphi(k)$, which could provide more flexibility since there may be significant differences among signals from different sensors. Besides, notice that $\max_k \{\mu_{\varphi(k)}(k)\} = 1$, and then, the quantization range $M_{\varphi(k)}$ should be selected large enough (i.e., $|x_{\varphi(k)}(k)| < M_{\varphi(k)}$) so that the quantizer (6) shall not saturate. In this work, the uniform quantizer $q_{\varphi(k)}(\cdot)$ has $\alpha_{\varphi(k)}$ quantization levels, while the quantizer’s parameter $\mu_{\varphi(k)}(k)$ takes values from a finite set with $J_{\varphi(k)}$ elements (as stated later below (13)).

For the token $\varphi(k) = p$ (i.e., the $p$-th sensor $x_p(k)$ obtains the token), the interval $[-M_p, M_p]$ under the uniform quantizer $q_p(\cdot)$ is partitioned into $\alpha_p$ regions as

$$
L_{\tau_p, p} \triangleq \left[ -M_p + \frac{2M_p}{\alpha_p} (\tau_p - 1), -M_p + \frac{2M_p}{\alpha_p} \tau_p \right]
$$

(7)

with $\tau_p = 1, 2, \ldots, \alpha_p - 1$ and

$$
L_{\alpha_p, p} \triangleq \left[ M_p - \frac{2M_p}{\alpha_p}, M_p \right].
$$

(8)

If the zoomed measurement $x^*_p(k) \in L_{\tau_p, p}$, the middle of the interval $L_{\tau_p, p}$ is taken as the quantization level $q_p(x^*_p(k))$, i.e.,

$$
q_p(x^*_p(k)) = -M_p + (2\tau_p - 1)M_p/\alpha_p.
$$

(9)

Thus, if the zoomed measurement $x^*_p(k)$ is located within the interval $[-M_p, M_p]$, its quantization error satisfies

$$
|q_p(x^*_p(k)) - x^*_p(k)| \leq \Delta_p,
$$

(10)

with the bound of quantization error as

$$
\Delta_p = M_p/\alpha_p.
$$

(11)

Compared with the conventional static quantizer’s parameter ($\mu_p(k) = 1$), the dynamic one may decrease the quantization error under the same quantization range $M_p$, i.e., reducing from $\Delta_p$ to $\Delta_p\mu_p(k)$ with $0 < \mu_p(k) \leq 1$ (seen from (6) and (11)). In this work, we shall design the quantizer’s parameter $\mu_p(k)$ dependent on the state $x_p(k)$. Different from the time-dependent parameter as in [13], [15], the dynamic quantizer (6) shall not saturate when the state $x_p(k)$ suddenly increases due to the parameter uncertainty $\bar{A}(k)$ or disturbance $\omega(k)$ in the system model (1). Since the original state signal $x_p(k)$ is unavailable for the decoder, we need to transmit the state-dependent quantizer’s parameter $\mu_p(k)$ for decoding.

Accordingly, we design the adjusting rule as

$$
\mu_p(k) = \begin{cases} \\
\beta_p^\text{up}, & \text{if } |x_p(k)| \geq M_p\mu_p^\text{up}, \\
\beta_p^\text{down}, & \text{if } |x_p(k)| < M_p\mu_p^\text{down},
\end{cases}
$$

(12)

where $\beta_p$ is a given constant with $0 < \beta_p < 1$, $\mu_p^\text{down} \triangleq \min_k \{\mu_p(k)\} = \beta_p^\text{up}$ is defined as the minimum dynamic parameter, the superscript $\text{up}(k)$ is given as

$$
\text{up}(k) \triangleq \max_{v \in \mathcal{V}_p} \{v | |x_p(k)| \leq M_p\beta_p^\text{up}\},
$$

(13)

and $\mathcal{V}_p \triangleq \{0, 1, \ldots, \text{up}(k)\}$. The specific value of $\text{up}(k)$ shall be given via the maximum coding length of the quantizer’s parameter later in (23). Then, it is known from the set $\mathcal{V}_p$ that the dynamic parameter $\mu_p(k)$ could only take finite values (i.e., $\mu_p(k) \equiv \text{up}(k)$) under the proposed adjusting rule (12). Moreover, when $|x_p(k)| \geq M_p\mu_p^\text{up}$, it can be calculated from (12) and (13) that

$$
M_p\mu_p^\text{up} \beta_p < |x_p(k)| \leq M_p\mu_p^\text{up},
$$

(14)

which implies that the quantizer (6) shall not saturate under the designed adjusting rule (12), i.e.,

$$
|\bar{q}_p(x_p(k), \mu_p(k)) - x_p(k)| \leq \Delta_p\mu_p(k),
$$

(15)
for \( x_p(k) \in \left[ -M_p \mu_p(k), M_p \mu_p(k) \right] \).

Remark 2: It is seen from (12) and (13) that the state-dependent dynamic parameter \( \mu_p(k) \) shall become smaller when the state \( x_p(k) \) approaches the origin. To be specific, we have \( \mu_p(k) = \beta_p^h \) for \( x_p(k) \in [ -M_p \beta_p^h, -M_p \beta_p^{h+1} ) \cup ( M_p \beta_p^{h+1}, M_p \beta_p^h ] \) \((h = 0, 1, \ldots, \bar{\tau}_p - 1)\), and \( \mu_p(k) = \beta_1^p \) for \( x_p(k) \in [ -M_p \beta_p^p, M_p \beta_p^p ] \).

Remark 3: In [19], the adjusting rule (28) can be regarded as a special case of \( \beta_p = 0.5 \). Obviously, it provides more flexibility to the quantizer (6) by selecting different \( \beta_p \), and how the value of \( \beta_p \) affects the coding error will be discussed in Section III-E. Moreover, the expression (13) shows a more direct and convenient method to determine the value of \( \bar{\tau}_p \) than the expressions (29)-(31) in [16]. This work simplifies the calculation procedure, and the same effect can be achieved.

In the following, the partition number \( \alpha_p \), and the number \( \beta_p \) of elements in the set \( \mathcal{V}_p \) shall be decided by the binary coding lengths later.

C. Encoding scheme with time-varying coding length

In this subsection, we shall first give the encoding scheme for the quantized zoomed measurement \( x_p^*(k) \) in (9) and the quantizer’s parameter \( \mu_p(k) \) in (12). The encoding functions for \( x_p^*(k) \) and \( \mu_p(k) \) are characterized as, respectively:

\[
\begin{align*}
\tilde{x}_p(k) &= f_p(x_p^*(k)), \\
\tilde{\mu}_p(k) &= h_p(\mu_p(k)),
\end{align*}
\]

(16)

with

\[
f_p(x_p^*(k)) = \tau_p,
\]

if \( x_p^*(k) \) lies in the \( \tau_p \)-th interval i.e., \( L_{\tau_p} \), and

\[
h_p(\mu_p(k)) = \begin{cases} 
\varnothing, & \text{if } M_p \beta_p < |x_p(k)| \leq M_p, \\
h_1, & \text{if } M_p \beta_p^{h+1} < |x_p(k)| \leq M_p \beta_p^h \quad (h = 1, 2, \ldots, \bar{\tau}_p - 1), \\
\bar{\tau}_p, & \text{if } |x_p(k)| < M_p \beta_p^p.
\end{cases}
\]

(18)

Here, the encoding function \( h_p(\mu_p(k)) = \varnothing \) means that we do not need to code in this case.

Then, the length of the transmitted data packet includes three coding lengths, i.e., the token \( \varphi(k) = p \), the zoomed measurement \( \tilde{x}_p(k) \), and the dynamic parameter \( \tilde{\mu}_p(k) \) (as shown in Fig. 2). The corresponding coding lengths are denoted as \( X \)-bits, \( Y_p \)-bits and \( Z_p \)-bits. The whole packet length \( \lambda_p(k) \)-bits is given as:

\[
\lambda_p(k) = X + Y_p + Z_p(k).
\]

(19)

Table I. Number of elements under different coding lengths

<table>
<thead>
<tr>
<th>Coding length ( Z_p(k) )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>\ldots</th>
<th>\text{( Z_p )}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of values</td>
<td>1</td>
<td>( 2^1 )</td>
<td>( 2^2 )</td>
<td>\ldots</td>
<td>( 2^{\text{( Z_p )}} )</td>
</tr>
</tbody>
</table>

(22)

Therefore, the time-varying coding length for \( \tilde{\mu}_p(k) \) can be calculated from (18) and Table I that

\[
Z_p(k) = \begin{cases} 
0, & \text{if } \tilde{\mu}_p(k) \in \varnothing, \\
\ell, & \text{if } \tilde{\mu}_p(k) \in \mathcal{V}_\ell,
\end{cases}
\]

with \( \mathcal{V}_\ell \triangleq \{ 2^\ell - 1, 2^\ell, \ldots, 2^{\ell+1} - 2 \} \) \((\ell = 1, 2, \ldots, \text{\( Z_p \)})

Remark 4: Compared with the fixed-length dynamic coding scheme, the time-varying coding length \( \lambda_p(k) \) could take more values under the same maximum coding length \( \text{\( Z_p \)} \), i.e., \( 1 + 2^1 + \ldots + 2^{\text{\( Z_p \)}} = 2^{Z_p+1} - 1 \geq 2^{\text{\( Z_p \)}}. \) On the other hand, the packet length \( \lambda_p(k) \) gradually increases, not always with the maximum packet length through the whole transmission, when the quantizer’s parameter \( \mu_p(k) \) decreases (i.e., the state \( x_p(k) \) approaches the origin).

D. Decoding scheme

After transmitting the data packet through the lossy digital network, the corresponding decoding function at the controller side is constructed as:

\[
\tilde{\mu}_p(k) = \begin{cases} 
0, & \text{if } \tilde{\mu}_p(k) \in \varnothing, \\
\mu_p(k), & \text{if } \tilde{\mu}_p(k) \in \{ 1, 2, \ldots, \bar{\tau}_p \}.
\end{cases}
\]

(23)

\[
\tilde{\mu}_p(k) = \theta(k)\left( -M_p + \frac{(2\tilde{x}_p(k) - 1)M_p}{\alpha_p} \right) \tilde{\mu}_p(k)
\]

(25)

with

Fig. 2. An example of the transmitted binary data packet.
Utilizing the expressions (9) and (12), the decoder (25) can be rewritten as
\[ \hat{x}_p(k) = \theta(k)\tilde{q}_p(x_p(k), \mu_p(k)). \] (26)

Then, the coding error can be denoted as
\[ e_p(k) = \tilde{q}_p(x_p(k), \mu_p(k)) - x_p(k). \] (27)

E. Relation between the minimum allowable packet arrival probability \( \tilde{\theta}_0 \) and the coding error \( e_p(k) \)

Suppose the minimum allowable packet arrival probability is \( \tilde{\theta}_0 \). According to the expression (19) and the upper bound \( \tilde{Z}_p \) of coding length for \( \tilde{\mu}_p(k) \), the maximum packet length is given as
\[ \tilde{X}_p = X + Y_p + \tilde{Z}_p. \] (28)

To meet the requirement \( \tilde{\beta}^\text{Wv}(k) \geq \tilde{\theta}_0 \), we should set
\[ \tilde{X}_p = \lfloor \log_2 \tilde{\theta}_0 \rfloor. \] (29)

Denoting \( \tilde{\lambda}_p \triangleq \tilde{X}_p - X \), we have from (28)
\[ Y_p + \tilde{Z}_p = \tilde{\lambda}_p = \lfloor \log_2 \tilde{\theta}_0 \rfloor - \lfloor \log_2 n \rfloor. \] (30)

In the sequel, we shall analyze the coding error \( e_p(k) \) under the given minimum allowable packet arrival probability \( \tilde{\theta}_0 \) via three different selection schemes for coding length.

Case I: The uniform quantization-based coding scheme is with \( \tilde{Z}_p = 0 \), and then, we have \( Y_p = \tilde{\lambda}_p \) from (30). In this case, \( \tilde{\mu}_p(k) = 1 \) and \( \tilde{q}_p(x_p(k), \mu_p(k)) = q_p(x_p(k)) \).

The coding error \( e_p(k) \) (the same as its minimum bound) satisfies
\[ |e_p(k)| \leq \Delta_p = 2^{-\tilde{\lambda}_p}M_p. \] (31)

where the coding length \( \tilde{\lambda}_p \) is decided by the minimum allowable packet arrival probability \( \tilde{\theta}_0 \) in (30).

Case II: The logarithmic quantization-based coding scheme is with \( Y_p = 1 \), and then, we have \( \tilde{Z}_p = \tilde{\lambda}_p - 1 \) \( (\tilde{Z}_p \geq 1) \) from (30). In this case, we have \( \Delta_p = \frac{1}{2}M_p \), and the corresponding quantizer (6) can be written as:
\[
\tilde{q}_p(x_p(k), \mu_p(k)) = \begin{cases} \frac{1}{2}M_p\beta_p^r, & \text{if } x_p(k) \in (M_p\beta_p^{r+1}, M_p\beta_p^r], \\ \frac{1}{2}M_p\beta_p^{r-1}, & \text{if } x_p(k) \in [0, M_p\beta_p^r], \\ -\frac{1}{2}M_p\beta_p^{r+1}, & \text{if } x_p(k) \in [-M_p\beta_p^{r+1}, 0), \\ -\frac{1}{2}M_p\beta_p^{r}, & \text{if } x_p(k) \in (-M_p\beta_p^{r}, -M_p\beta_p^{r-1}). \end{cases}
\] (32)

with \( r = 0, 1, \ldots, \tilde{r}_p \) and \( \tilde{r}_p = 2^{\tilde{\lambda}_p} - 2 \). Here, the parameter \( \beta_p \) with \( 0 < \beta_p < 1 \) can be regarded as the quantization density in the traditional logarithmic quantizer.

For \( M_p\beta_p^r \leq |x_p(k)| \leq M_p \), the coding error can be obtained from (14) that
\[ |e_p(k)| \leq \Delta_p\mu_p(k) < \frac{1}{2\beta_p}|x_p(k)|. \] (33)

For \( |x_p(k)| < M_p\beta_p^r \), one has the minimum bound of the coding error from (12) and (23) that
\[ |e_p(k)| \leq \Delta_p\beta_p^r = \frac{1}{2}M_p\beta_p^{2\tilde{\lambda}_p - 2}. \] (34)

Remark 5: When the encoded signal \( x_p(k) \) is far from the origin, the coding error \( e_p(k) \) under Case II may be larger than the one under Case I. On the other hand, if the quantization density \( \beta_p \) is set as
\[ 0 < \beta_p < 2^{(1-\tilde{k}_p)/(2^{\tilde{\lambda}_p} - 2)}, \] (35)

the minimum bound of the coding error, \( \Delta_p\beta_p^r \) in (34), could be smaller than the one in (31), i.e.,
\[ \frac{1}{2}M_p\beta_p^{2\tilde{\lambda}_p - 2} < 2^{-\tilde{k}_p}M_p. \]

Case III: The proposed dynamic coding scheme is with \( Y_p + \tilde{Z}_p = \lfloor \log_2 \tilde{\theta}_0 \rfloor - \lfloor \log_2 n \rfloor \), the relation (14) holds, and then, one obtains from (14) and (21) that
\[ |e_p(k)| \leq \Delta_p\mu_p(k) = \frac{\Delta_p}{M_p\beta_p}|x_p(k)| = 2^{-\tilde{\lambda}_p}\beta_p|x_p(k)|. \] (36)

with \( \beta_p = 1/\sqrt{\beta_p} \).

For \( |x_p(k)| < M_p\mu_p \), the relation (14) does not hold. It can be calculated from (21) and (23) that the minimum bound of the coding error is
\[ |e_p(k)| \leq \Delta_p\mu_p = 2^{-\tilde{\lambda}_p}\beta_p^{2\tilde{\lambda}_p - 1}M_p. \] (37)

Remark 6: All of the coding lengths in the above cases are under the relation (30), and Case I is with \( \tilde{Z}_p = 0 \) while Case II is with \( Y_p = 1 \). Hence, the proposed dynamic coding scheme in Case III has a more general form, which could cover both the uniform and logarithmic quantization-based coding schemes. Moreover, compared with the expressions (31), (33)-(34) and (36)-(37), the proposed coding scheme in Case III enables to balance of the coding error \( e_p(k) \) far from the origin and the one near the origin by designing the coding lengths \( Y_p \) and \( \tilde{Z}_p \).

IV. CONTROLLER DESIGN UNDER DYNAMIC CODING

As discussed in Section III, we first utilize the WTOD protocol to decide which sensor node can access to transmit. Then, the selected signal \( x_p(k) \) is encoded into a binary string with \( \tilde{X}_p(k) \)-bits and transmitted through the lossy network. Hence, only the decoded signal \( \hat{x}_p(k) \) is available at the controller side. Now, our objective is to design the feasible state-feedback controller under the proposed dynamic coding scheme with time-varying packet length so that the controlled system, subject to possible packet dropout, is exponentially ultimately bounded in the mean square sense.

A. Controller design

The controller may utilize its previous value if the sensor node is not chosen or the data packet is lost during transmission. Utilizing (26) and (27), a compact form for the available signal \( \hat{x}(k) \) at the controller side is
\[ \hat{x}(k) = \theta(k)(Y_p\hat{x}(k) + \tilde{e}_p(k)) + (I - \theta(k)\mathbf{Y}_p)\hat{x}(k - 1) \] (38)
with \( \tilde{e}_p(k) \equiv \mathbf{Y}_p e(k) \) and \( e(k) \equiv [e_1(k), \ldots, e_n(k)]^T \).
Then, we design the token-dependent control law as:

$$u(k) = K_{\varphi}(k)\hat{x}(k)$$  \hspace{1cm} (39)

with \(\varphi(k) \triangleq \theta(k)\varphi(k)\).

Substituting (38) and (39) into the system (1), we have the closed-loop system for \(\varphi(k) = p\):

$$x(k + 1) = [A + \bar{A}(k) + \theta(k)BK_p\Upsilon_p]x(k) + [\bar{\theta}(k)BK_p(I - \Upsilon_p) + (1 - \theta(k))BK_0]x(k - 1) + \theta(k)BK_p\hat{e}_p(k) + D\omega(k).$$  \hspace{1cm} (40)

Notice that the token \(\varphi(k)\) is unavailable for the controller when the packet drops. Here, we adopt the control gain \(K_0\), not the token-dependent one \(K_{\varphi}(k)\), once the packet is not received.

B. Stability analysis

The definition of exponentially ultimately bounded (EUB) and convergent upper bound (CUB) are first given below.

**Definition 1:** The closed-loop system (40) is said to be EUB in the mean square sense, if there exist constants \(0 \leq \gamma < 1, c_1 > 0, c_2 \geq 0\) satisfying

$$\mathcal{E}\{||\bar{x}(k)||^2\} \leq c_1\gamma^k||\bar{x}(0)||^2 + c_2, \ \forall k > 0$$  \hspace{1cm} (41)

where \(\bar{x}(0)\) is the initial condition, and \(c_2\) is the CUB of \(\mathcal{E}\{||\bar{x}(k)||^2\}\).

Notice that the closed-loop system (40) depends on the stochastic variable \(\theta(k)\). Then, we shall analyze the stochastic stability of the closed-loop system under the lossy network.

**Theorem 1:** Consider the closed-loop system (40) under the dynamic coding scheme with the given coding lengths \(\Upsilon_p(k)\) and \(\bar{Z}_p(k)\). For scalars \(0 < \gamma < 1, \rho > 0, \eta > 0\), if there exist matrices \(P > 0, Q > 0, K_p, K_0\) and scalars \(\kappa_{p,s} > 0\) (s = 1, \ldots, n) satisfying the following matrix inequalities for any \(p \in \mathcal{U}\) and \(\lambda_p(k) \in \{X + Y_p, X + Y_p + 1, \ldots, X + Y_p + Z_p\}:

$$\Xi_p,\lambda_p(k) \triangleq \Lambda_p + \Pi_p,\lambda_p(k) < 0$$  \hspace{1cm} (42)

with

\[
\begin{align*}
\Lambda_p &\triangleq \text{diag}(\mathcal{M}_p + N_p, -\rho I, -\eta I), \\
\Pi_p,\lambda_p(k) &\triangleq \Lambda^T_p,\lambda_p(k)P\Lambda_p,\lambda_p(k) + C^T_p,\lambda_p(k)QC_p,\lambda_p(k) \\
&\quad + \hat{\theta}(\lambda_p(k))(B^T_pP\bar{P}_p + D^T_pQ\bar{Q}_p), \\
\mathcal{M}_p &\triangleq \text{diag}\{-\gamma P, -\gamma Q\} + \mathcal{I}^T\bar{P}\sum_{s=1}^n\kappa_{p,s}(\Upsilon_p - \Upsilon_s)\mathcal{I},
\end{align*}
\]

\[
\begin{align*}
N_p &\triangleq \text{diag}(2^{-2Y_p}\bar{\beta}^2_p\rho\Upsilon_p, 0), \\
A_{p,\lambda_p(k)} &\triangleq \left[ [A_{p,\lambda_p(k)} F_{p,\lambda_p(k) D} \right], \\
B_p &\triangleq \left[ B_p BK_p 0 \right], \\
C_{p,\lambda_p(k)} &\triangleq \left[ C_{p,\lambda_p(k)} \hat{\theta}\lambda_p(k)I 0 \right], \\
D_p &\triangleq \left[ D_p I 0 \right],
\end{align*}
\]

$$\mathcal{C}_{p,\lambda_p(k)} \triangleq \left[ A + \bar{A}(k) + \hat{\theta}\lambda_p(k)BK_p\Upsilon_p + \hat{\theta}\lambda_p(k)BK_p(I - \Upsilon_p) + (1 - \hat{\theta}\lambda_p(k))BK_0 \right],$$

$$\mathcal{D}_p \triangleq \left[ BK_p(I - \Upsilon_p) - BK_0 \right],$$

then the closed-loop system (40) is EUB in the mean square sense, and the CUB of \(\mathcal{E}\{||\bar{x}(k)||^2\}\) is

$$c_2 = \max_{p \in \mathcal{U}} \left\{ \frac{\rho (2 - Y_p)\beta^2_p (2^{2p-1}) M_p^2 + \eta \bar{\omega}^2}{(1 - \gamma)P_{\min}} \right\}$$  \hspace{1cm} (43)

with \(P_{\min} \triangleq \lambda_{\min}\{\text{diag}(P, Q)\}, P_{\max} \triangleq \lambda_{\max}\{\text{diag}(P, Q)\}\).

**Proof:** Choose the Lyapunov functional candidate as

$$V(k) \triangleq x^T(k)(P_x(k) + \bar{x}^T(k - 1)Q\bar{x}(k - 1)).$$  \hspace{1cm} (44)

Denote \(\tilde{x}(k) \triangleq [x^T(k), \bar{x}^T(k - 1)]^T\). Then, we have from (38) and (40) that

$$\mathcal{E}\{V(k + 1)||\bar{x}(k)||\} - \gamma V(k)$$

$$= \left[ A_{p,\lambda_p(k)} \tilde{x}(k) + F_{p,\lambda_p(k)}e_p(k) + D\omega(k) \right]^TP + \mathcal{E}\{\left(\theta(k) - \hat{\theta}\lambda_p(k)^2\right)^2[B_p\tilde{e}_p(k) + BK_p\hat{e}_p(k) + D\omega(k)] \}$$

$$+ \left[C_{p,\lambda_p(k)} \tilde{x}(k) + \hat{\theta}\lambda_p(k)^2\epsilon_p(k) \right]^TQ \left[ C_{p,\lambda_p(k)} \tilde{x}(k) + \hat{\theta}\lambda_p(k)^2\epsilon_p(k) \right]$$

$$\leq \mathcal{L}_p,\lambda_p(k)^T\bar{P}\sum_{s=1}^n\kappa_{p,s}(\Upsilon_p - \Upsilon_s)\mathcal{I} \bar{x}(k)$$  \hspace{1cm} (45)

According to the token selection principle (5), the following inequality holds for any \(s \in \mathcal{U} = \{1, 2, \ldots, n\}\)

$$\bar{x}^T(k)\mathcal{I}^T\bar{P}(\Upsilon_p - \Upsilon_s)\mathcal{I} \bar{x}(k) \geq 0.$$  \hspace{1cm} (46)

Besides, due to \(\mathcal{E}\{\left(\theta(k) - \hat{\theta}(\lambda_p(k))^2\right)^2\} = \hat{\theta}(\lambda_p(k) - \hat{\theta}(\lambda_p(k)^2)\epsilon_p(k)\epsilon_p(k))\) and \(||(\omega(k))|| \leq \bar{\omega},\) it follows from (45) and (46) that

$$\mathcal{E}\{V(k + 1)||\bar{x}(k)||\} - \gamma V(k)$$

$$\leq \bar{x}^T(k)\Pi_p,\lambda_p(k)\tilde{x}(k) - \gamma \bar{x}^T(k)\Pi_p,\lambda_p(k)\tilde{x}(k) \leq 0.$$  \hspace{1cm} (47)

with \(\tilde{x}(k) \triangleq [x^T(k), \bar{x}^T(k), \omega^T(k), x^T(k)]^T\).

Next, we shall analyze the stability of the closed-loop system (40) under different packet lengths \(\lambda_p(k) \in \{X + Y_p, X + Y_p + 1, \ldots, X + Y_p + Z_p\}\).

Case (i): For the packet length \(\lambda_p(k) \in \{X + Y_p, X + Y_p + 1, \ldots, X + Y_p + Z_p\},\) (i.e., the coding lengths \(Z_p(k) \in \{0, 1, \ldots, Z_p - 1\}\)), we have \(x_p(k) \in [ - M_p, -M_p\beta^{2p-2}_p \cup \{M_p\beta^{2p-2}_p, M_p\}].\) From the expression (36), we have

$$||e_p(k)||^2 = ||e_p(k)||^2 \leq 2^{-2Y_p}\bar{\beta}^2_{p} \rho M_p x(k).$$  \hspace{1cm} (48)

By means of (48), one yields from the expression (47)

$$\mathcal{E}\{V(k + 1)||\bar{x}(k)||\} - \gamma V(k)$$
\[ \leq \tilde{x}^T(k)\Pi_{p,X_p(k)}\tilde{x}(k) + \tilde{x}^T(k)I^T\Psi \sum_{s=1}^n \kappa_{p,s}(\Upsilon_p - \Upsilon_s) \times \mathcal{L}\tilde{x}(k) = \gamma \tilde{x}^T(k) \mathbf{1} + \eta \omega^2 = \tilde{x}^T(k)\Xi_{p,x_p(k)}\tilde{x}(k) + \eta \omega^2. \]  

(49)

If the condition (42) holds for \( X_p(k) \in \{ X + Y_p, X + Y_p + 1, \ldots, X + Y_p + \mathbb{Z}_p \} \), we can obtain the following inequality by taking mathematical expectation for both sides of (49) as

\[ \mathcal{E}\{V(k+1)\} - \mathcal{E}\{V(k)\} \leq (\gamma - 1)\mathcal{E}\{V(k)\} + \eta \omega^2. \]  

(50)

Denote \( \tilde{\gamma} \triangleq 1 / (\gamma - 1) > 1 \), and it can be verified that

\[ \tilde{\gamma}^k \mathcal{E}\{V(k+1)\} - \tilde{\gamma}^k \mathcal{E}\{V(k)\} = \tilde{\gamma}^k \mathcal{E}\{V(k+1)\} - \tilde{\gamma}^{k+1} \mathcal{E}\{V(k+1)\} + \tilde{\gamma}^k (1 - \gamma) \mathcal{E}\{V(k)\} \leq \tilde{\gamma}^k \omega^2 + \tilde{\gamma}^{k+1} \eta \omega^2 = \tilde{\gamma}^{k+1} \eta \omega^2. \]  

(51)

Then, summing up both sides of (51) with respect to \( k \) from 0 to \( k_0 - 1 \), we have for \( k_0 \geq 1 \)

\[ \mathcal{E}\{V(k_0)\} \leq \gamma^{k_0} \left( \mathcal{E}\{V(0)\} - \frac{\eta \omega^2}{1 - \gamma} \right) + \frac{\eta \omega^2}{1 - \gamma}. \]  

(52)

from which we can further obtain

\[ \mathcal{E}\{\|\tilde{x}(k)\|^2\} \leq \frac{\mathcal{P}_{\text{max}}}{\mathcal{P}_{\text{min}}} \gamma^k \mathcal{E}\{\|\tilde{x}(0)\|^2\} + \frac{\eta \omega^2}{(1 - \gamma) \mathcal{P}_{\text{min}}}. \]  

(53)

Case (ii): For \( X_p(k) = X + Y_p + \mathbb{Z}_p \) (i.e., \( Z_p(k) = \mathbb{Z}_p \)), we shall discuss under two independent intervals.

Firstly, for \( x_p(k) \in [-M_p \beta_p^2 \mathbb{Z}_p, \ldots, -M_p \beta_p^2 \mathbb{Z}_p + M_p \beta_p^2 \mathbb{Z}_p\] \( p \), the relation (14) still holds. The proof procedure and condition (42) (for \( X_p(k) = X + Y_p + \mathbb{Z}_p \)) are the same in Case (i).

Secondly, for \( x_p(k) \in [-M_p \beta_p^2 \mathbb{Z}_p, \ldots, -M_p \beta_p^2 \mathbb{Z}_p] \), the relation (14) does not hold, and then, we have from (37)

\[ \|\tilde{e}(k)\|^2 = \left. \mathcal{E}\{\|\tilde{x}(k)\|^2\} \right|_{\|\tilde{x}(k)\|^2 \leq (2 - Y_p \beta_p^2 (2 \mathcal{Z}_p - 1) M_p)^2}. \]  

(54)

Utilizing the expressions (47) and (54), one yields

\[ \mathcal{E}\{V(k+1)\} - \gamma V(k) \]

\[ \leq \tilde{x}^T(k)\Pi_{p,X_p(k)}\tilde{x}(k) + \tilde{x}^T(k)I^T\Psi \sum_{s=1}^n \kappa_{p,s}(\Upsilon_p - \Upsilon_s) \times \mathcal{L}\tilde{x}(k) = \gamma \tilde{x}^T(k) \mathbf{1} + \eta \omega^2 = \tilde{x}^T(k)\Xi_{p,x_p(k)}\tilde{x}(k) + \eta \omega^2 + \rho (2 - Y_p \beta_p^2 (2 \mathcal{Z}_p - 1) M_p)^2 + \eta \omega^2 \]  

with \( \Xi_{p,x_p(k)} \triangleq \Lambda_p + \Pi_{p,x_p(k)} \) and \( \Lambda_p \triangleq \delta (M_p, -\rho I, -\eta I) \).

Then, it is easily shown that \( \Xi_{p,x_p(k)} < 0 \) can be promised by \( \Xi_{p,x_p(k)} < 0 \).

Along the similar derivation as (50)-(53), we have

\[ \mathcal{E}\{\|\tilde{x}(k)\|^2\} \leq \frac{\mathcal{P}_{\text{max}}}{\mathcal{P}_{\text{min}}} \gamma^k \mathcal{E}\{\|\tilde{x}(0)\|^2\} + \frac{\eta \omega^2}{(1 - \gamma) \mathcal{P}_{\text{min}}}. \]

(55)

Combining the results in (i) and (ii), together, if the inequalities (42) hold for \( X_p(k) \in \{ X + Y_p, X + Y_p + 1, \ldots, X + Y_p + \mathbb{Z}_p \} \), the closed-loop system (40) is EUB in the mean square sense according to Definition 1.

Now, we shall give a feasible solving algorithm based on the linear matrix inequality (LMI) method to guarantee stability conditions (42) in Theorem 1.

**Theorem 2:** For given coding lengths \( Y_p \) and \( \mathbb{Z}_p \), scalars \( 0 < \gamma < 1 \), \( \rho > 0 \) and \( \eta > 0 \), if there exist matrices \( P > 0 \), \( Q > 0 \), \( K_p \), \( K_0 \) and scalars \( \varepsilon > 0 \), \( \kappa_{p,s} > 0 \) \((s = 1, \ldots, n)\) satisfying the following LMIs for any \( p \in \mathcal{U} \) and \( X_p(k) \in \{ X + Y_p, X + Y_p + 1, \ldots, X + Y_p + \mathbb{Z}_p \} \)

\[ \begin{bmatrix} \Omega_p^{(1,1)} & \ast & \ast \\ \Omega_p^{(2,1)} & \Omega_p^{(1,2)} & \ast \\ 0 & \Omega_p^{(2,2)} & -\varepsilon \mathcal{I} \end{bmatrix} < 0 \]  

(57)

with

\[ \Omega_p^{(1,1)} \triangleq \delta (\Theta^{(1,1)}, -\rho I, -\eta I), \]  

\[ \Omega_p^{(2,1)} \triangleq \delta (\Theta^{(2,1)}, \Theta^{(2,2)}), \]  

\[ \Omega_p^{(1,2)} \triangleq \delta (P - 2T \bar{\gamma}^{-1} X_p(k) (P - 2I), -Q, -\bar{T} \bar{\gamma}^{-1} X_p(k) Q), \]  

\[ \Omega_p^{(2,2)} \triangleq \left[ \begin{array}{c} E^T \\ 0 \\ 0 \\ \Phi_p^{(1,1)} \\ \Phi_p^{(2,1)} \end{array} \right], \]  

\[ \Phi_p^{(1,1)} \triangleq \left[ \begin{array}{c} \Phi_p^{(5,1)} \\ \Phi_p^{(5,2)} \\ \Phi_p^{(6,1)} \\ \Phi_p^{(6,2)} \end{array} \right], \]  

\[ \Phi_p^{(2,1)} \triangleq \left[ \begin{array}{c} \Phi_p^{(7,1)} \\ \Phi_p^{(7,2)} \end{array} \right], \]  

\[ \Phi_p^{(4,1)} \triangleq \left[ \begin{array}{c} \bar{\gamma} X_p(k) \bar{B} K_p \\ \bar{D} \end{array} \right], \]  

\[ \Phi_p^{(4,2)} \triangleq \left[ \begin{array}{c} \bar{T} X_p(k) \bar{B} K_p \\ \bar{T} X_p(k) Q \end{array} \right], \]  

\[ \Phi_p^{(1,1)} \triangleq -\gamma P + \bar{\Psi} \sum_{s=1}^n \kappa_{p,s}(\Upsilon_p - \Upsilon_s) + 2 - 2Y_p \beta_p^2 (2 \mathcal{Z}_p - 1) M_p + \eta \omega^2. \]  

(56)
Proof: Notice that the inequality
\[(I - P)P^{-1}(I - P)^T = P + P^{-1} - 2I \geq 0,\]
holds for any \(P \geq 0\), and then, we have \(-P^{-1} \leq P - 2I\). Hence, according to Schur’s complement, it is easily verified that the inequalities (42) can be ensured by the LMIs (57). ■

C. Implementation method

In the previous subsection, the coding lengths \(Y_p\) and \(Z_p\) are pre-given in Theorem 2. However, as we pointed out in section III, different selections for the coding lengths \(Y_p\) and \(Z_p\) may significantly affect the control performance. Moreover, as seen from the LMIs (57), either too small a coding length \(Y_p\) or too low a packet arrival probability \(\tilde{e}^{X_p(k)}\) may make these LMIs being infeasible.

This subsection provides an implementation method to determine the maximum packet length \(\overline{X}_p\) and find the optimal combination of the coding lengths \(Y_p\) and \(Z_p\) by minimizing the coding error \(\Delta_p\mu_p\) in (37). After that, the gains \(K_p\) \((p = 1, 2, \ldots, n)\), \(K_0\) for the controller (39) can be achieved by solving the LMIs (57). The specific steps are given as follows:

Step 1: Set \(\overline{Z}_p = 0\). We have the packet length \(\overline{X}_p = X + Y_p\). Solve the LMIs (57) for all \(p \in U\), starting with \(Y_p = 1\)-bit, and adding 1-bit to \(Y_p\) in turn, we can then achieve the feasible area of the LMIs (57). Thus, all the feasible packet lengths \(\overline{X}_p\) can be found. This is also the case for the uniform coding scheme (Case I) with the minimum bound of the coding error \(|e_p(k)| \leq 2(X - \overline{X}_p)M_p\).

Step 2: Take the feasible packet lengths \(\overline{X}_p\) in Step 1. Solve the LMIs (57) again, starting with \(\overline{Z}_p = 0\) and adding 1-bit to \(\overline{Z}_p\) in turn until \(\overline{Z}_p = \overline{X}_p - X - 1\). If these LMIs are feasible under the combinations of coding lengths \(Y_p\) and \(Z_p\), we shall calculate the following value:

\[
\min_{Y_p + Z_p} \max_{p \in U} \left\{ 2^{-P} \beta_p^2 (2^{P-1}) \right\}.
\]

The optimal combination of \(Y_p\) and \(Z_p\) is to minimize the index (59) such that the minimum bound of the coding error \(\Delta_p\mu_p\) in (37) can be further reduced.

V. ILLUSTRATIVE EXAMPLE

Consider the double-sided linear switched reluctance machine systems [30], whose parameters are obtained by the online least-squares identification method with a sampling time \(T = 0.001\:

\[
A = \begin{bmatrix} 1.0004 & 0.0909 \\ 0.0885 & 0.8247 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0047 \\ 0.0909 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},
\]

\[
E = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, \quad H = \begin{bmatrix} -0.05 \\ 0.1 \end{bmatrix}, \quad F(k) = \cos(k).
\]

Assume that the arrival probability of a one-bit packet is \(\theta = 0.9\). The weight coefficients of WTDOL protocol are set as \(\Psi_1 = \Psi_2 = 1\), and the constants \(\beta_p\) \((p = 1, 2)\) for the dynamic parameter \(\mu_p(k)\) are set as \(\beta_1 = \beta_2 = 0.2\). Besides, we select the constants \(\gamma = 0.98\), \(\rho = 1\) and \(\eta = 1\).

Fig. 3 shows how the maximum packet lengths \(\overline{X}_p\) affect the feasible area of the LMIs (57) with \(\overline{Z}_p = 0\) (i.e., \(Y_p = \overline{X}_p - X\)) (Step 1). Then, we shall search for the optimal combination of coding lengths \(Y_p\) and \(Z_p\) in this feasible area to minimize the index (59). Solving the LMIs (57) again through the search algorithm for the maximum feasible coding length \(\overline{X}_p\) (all the yellow blocks in Fig. 3) (Step 2), we can find the optimal combination of coding lengths as \(Y_1 = 8\)-bits, \(Y_2 = 5\)-bits and \(Z_1 = Z_2 = 2\)-bits under the maximum packet lengths \(\overline{X}_1 = 11\)-bits and \(\overline{X}_2 = 8\)-bits (the red box in Fig. 4). To be specific, Fig. 4 shows how the different combinations of coding lengths under one maximum feasible coding length (the yellow block \(\overline{X}_1 = 11\)-bits and \(\overline{X}_2 = 8\)-bits in Fig. 3) affect the value of the index (59) (the blue numbers in Fig. 4). It is seen that the number in the red box is minimum, i.e., the optimal combination of coding lengths \(Y_p\) and \(Z_p\). The control gain matrices are obtained as \(K_0 = \begin{bmatrix} 0.3806 & 0.0518 \end{bmatrix}, K_1 = \begin{bmatrix} -4.1451 & -0.2585 \end{bmatrix}, K_2 = \begin{bmatrix} -2.4507 & -3.6383 \end{bmatrix}\).

Under the initial state \(x(0) = [1, -1]^T\), the quantization ranges are selected as \(M_1 = M_2 = 2\). Hence, the minimum bounds of the coding error for the dynamic coding scheme can be calculated from (37) that

\[
\Delta_{\bar{X}_1}\mu_1 = 2^{-Y_1} \beta_{\bar{X}_1}(2^{P-1}) M_1 = 0.5 \times 10^{-6},
\]
\[
\Delta_2 \mu_2 = 2^{-\frac{\beta_2}{2}} (2^{\frac{\beta_2}{2}} - 1) M_2 = 0.4 \times 10^{-5}.
\]

Firstly, we do 30 individual experiments with \(\omega(k) = 0.1 \sin(k)\) as shown in Fig. 5. It can be seen that the selected quantization range \(\bar{M}_p\) is large enough to promise \(|x_p(k)| < M_p\). Meanwhile, the proposed control law can guarantee the stability of the controlled system.

Then, we set the external disturbance \(\omega(k) = 0\) to observe the influence of the coding error on the system performance. Figs. 6(a)-6(b) show that the closed-loop system states are ultimately bounded due to the unavoidable coding error, especially the bound not being very large under our proposed dynamic coding scheme. Fig. 6(c) illustrates that the dynamic parameter \(\mu_p(k)\) adjusts depending on the selected state \(x_p(k)\), and the smaller \(\mu_p(k)\) is chosen when \(x_p(k)\) trends to zero. Besides, \(\mu_p(k)\) could take at most 7 values since \(\bar{M}_p = 2^{2+1} - 1 = 7\). Fig. 6(d) shows that the packet length \(X_p(k)\) is time-varying with the token \(p = 1\) or \(p = 2\) and the value of the dynamic parameter \(\mu_p(k)\).

Next, we give the simulation results under the external disturbance \(\omega(k) = 0.1 \sin(k)\). As a result, the packet length \(X_p(k)\) in Fig. 7(d) is generally smaller than the one in Fig. 6(d). Thus, fewer packets drop when the system undergoes more significant disturbance under the same network condition.

Furthermore, exploring how the dynamic parameter improves system performance would be interesting. For the static coding scheme with the same maximum packet length \(\bar{M}_p\), the bounds of coding error can be calculated from (31) that
\[
\Delta_1 \mu_1 = 2^{-\frac{\beta_1}{2}} M_1 \approx 0.195 \times 10^{-2},
\]
\[
\Delta_2 \mu_2 = 2^{-\frac{\beta_2}{2}} M_2 \approx 0.156 \times 10^{-1}.
\]

It is clearly shown from expressions (60)-(61) and (62)-(63) that the minimum bound of the coding error \(\Delta_p \mu_p\) under the dynamic coding scheme is much smaller than the one under the static coding scheme. Intuitively, the simulation results under the static coding scheme (i.e., \(\mu_p(k) = 1\)) with \(\bar{M}_1 = 11\) bits and \(\bar{M}_2 = 8\) bits are given in Fig. 8. Compared with Figs. 6(a) & 8(a), the ultimate bound of the state \(x(k)\) under the dynamic coding scheme is much smaller than the one under the static scheme. Hence, the dynamic parameter \(\mu_p(k)\) could effectively reduce the coding error near the origin.
VI. CONCLUSION

The packet loss probability shall substantially increase with the growth of packet length under the fixed transmission power level, while the coding error will gradually decrease simultaneously. This work has comprehensively considered the adverse impacts of the coding error and the packet loss on the control performance. Firstly, a dynamic quantization-based coding scheme has been proposed under the lossy network, which has a higher coding accuracy than the static one. The state-dependent quantizer’s parameter is encoded and transmitted together with the state signal to avoid the dynamic coding scheme saturating caused by the system uncertainty and the external disturbance. Meanwhile, the time-varying coding length helps enlarge the number of elements when the encoded quantizer’s parameter could take under the same maximum length. Then, the state-feedback control problem was investigated under the proposed coding scheme. Moreover, the two-step implementation method for choosing the optimal coding lengths has been given to minimize the coding error and ensure the controller is feasible simultaneously.

REFERENCES


Yugang Niu received the M.Sc. and Ph.D. degrees in control science and engineering from Nanjing University of Science and Technology, Nanjing, China, in 1992 and 2001, respectively.

In 2003, he joined the School of Information Science and Engineering, East China University of Science and Technology, and is currently a Professor. His research interests include stochastic systems, sliding mode control, Markovian jump systems, networked control systems, wireless sensor networks, and smart grids.

Dr. Niu is currently an Associate Editor for several international journals, including Information Sciences, Neurocomputing, IET Control Theory & Applications, Journal of The Franklin Institute, and International Journal of System Sciences. He is also a member of Conference Editorial Board of IEEE Control Systems Society.

Daniel W. C. Ho (Fellow, IEEE) received the B.S., M.S., and Ph.D. degrees in mathematics from the University of Salford, Salford, U.K., in 1980, 1982, and 1986, respectively.

From 1985 to 1988, he was a Research Fellow with Industrial Control Unit, University of Strathclyde, Glasgow, U.K. In 1989, he joined the City University of Hong Kong, Hong Kong, where he is currently a Chair Professor of applied mathematics, and an Associate Dean with the College of Science. He has authored or coauthored more than 260 publications in scientific journals. His current research interests include control and estimation theory, complex dynamical distributed networks, multiagent systems, and stochastic systems.

Prof. Ho was the recipient of the Chang Jiang Chair Professor Awarded by the Ministry of Education, China, in 2012, and ISI Highly Cited Researchers Award in Engineering by Clarivate Analytics from 2014 to 2022. He has been on the Editorial Board of a number of journals, including the IEEE Transactions on Neural Networks and Learning Systems, IET Control Theory & Applications, Journal of the Franklin Institute, and Asian Journal of Control.