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Research Paper

Drift-flux model for upward dispersed two-phase flows in a vertical rod bundle

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\textbf{ABSTRACT}

The present study investigates the drift-flux model for upward dispersed two-phase flows in a vertical rod bundle. The drift-flux model is critical to formulating the interfacial drag force in one-dimensional simulation codes. A rod bundle includes three length scales: a large outer casing width, a medium hydraulic equivalent diameter of a subchannel, and a small gap between rods. Existing research does not provide comprehensive information on an appropriate geometrical length scale for a rod bundle. The present study calculates the evolution process of the drift velocity using the two-bubble-group approach. The two-bubble-group-based formulation is simplified to calculate the drift velocity behavior at the transition between bubbly and beyond-bubbly flows. A scheme is proposed to explicitly calculate the distribution parameter and drift velocity with operating parameters. The new drift-flux correlation is validated by 317 data collected for the pressure from 0.1 to 12 MPa. The extensive evaluation identifies that an appropriate length scale characterizing a two-phase flow in a rod bundle is an outer casing width of a rod bundle. The prediction bias of the new drift-flux correlation was negligibly small, and the random error was 0.0571 in an absolute value measure and 16.1 \% in a relative value measure.

1. Introduction

The two-fluid model is used for Eulerian-Eulerian numerical simulations of the thermo-fluid behavior of the gas–liquid two-phase flow \[1\]. The two-fluid model formulates the conservation of mass, momentum, and energy for each phase. Interfacial transfer terms appear in the two-fluid model formulation to describe the transfer of mass, momentum, and energy between phases. The interfacial transfer terms allow the two-fluid model to simulate thermal and kinematic non-equilibrium. In thermo-fluid simulation codes using the one-dimensional two-fluid model, the interfacial drag force term plays a key role in the interfacial momentum transfer term \[2–4\]. The interfacial drag force term is formulated as the product of the drag coefficient and the square of the area-averaged relative velocity between gas and liquid \[5\]. In this formulation process, the concept of the drift-flux model is applied \[6\].

The drift-flux model introduces the concept of the drift velocity defined by the difference between gas velocity and mixture volumetric flux. The drift velocity characterizes kinematic non-equilibrium or relative velocity between gas and liquid phases. When the drift-flux model is applied to one-dimensional analyses, averaging the local drift-flux model over a flow channel yields a covariance term due to non-uniform distributions of void fraction, gas velocity, and liquid velocity. The covariance term is called the distribution parameter, and it is the essential parameter characterizing phase distribution in a flow channel. The drift-flux model incorporates the local information on the flow distribution into one-dimensional analyses through the distribution parameter. By accurately providing the distribution parameter and drift velocity through constitutive equations, reliable numerical simulations of the thermo-fluid behavior of the gas–liquid two-phase flow can be performed.

The drift velocity is expressed by the product of a liquid fraction and relative velocity between gas and liquid phases. In the drift velocity modeling, the relative velocity is modeled depending on the bubble-shape regime. In the case of a bubble shape in infinite media, viscous, distorted-fluid-particle, and cap bubble regimes, the drift velocity is modeled by a drag law. In the case of a bubble shape confined by a channel wall, a slug flow regime, the drift velocity is modeled by a potential flow theory. In the case of a highly deformed bubble-shape regime, a churn-turbulent-flow regime, the drift velocity is modeled by a critical Weber number. When the modeled drift velocity is applied...
to real two-phase flows, it is common to add some empirical coefficients and correction factors to compensate for the difference between the model and experimental data. Limited attempts were made to model the drift velocity at flow regime transitions. However, the drift velocity model for each bubble-shape regime can be the foundation to express the behavior of the drift velocity at the flow regime transitions.

On the other hand, the distribution parameter is determined by the distributions of void fraction and mixture volumetric flux. However, experimental databases of the void fraction and mixture volumetric flux distributions are limited. In the distribution parameter modeling, the distribution parameter is empirically determined as a slope in the drift-flux plot, mixture volumetric flux vs gas velocity.

Thermo-fluid phenomena involving gas–liquid two-phases appear in many engineering fields, including mechanical engineering, chemical engineering, nuclear engineering, petroleum engineering, and civil engineering. Circular channels, which are relatively simple flow paths, are often used in air conditioning equipment, heat pipes, oil pipelines, and plant piping systems. Kataoka and Ishii [7] showed that the drift velocity depended on the size of the flow path and they proposed a drift-flux correlation for large-diameter circular and large-rectangular channels using hydraulic equivalent diameters. The simple circular channel is geometrically characterized by a single characteristic length, such as the circular channel diameter.

Unlike circular channels, the internal structure of heat exchangers is complex, and multiple characteristic length scales exist [8]. For example, typical shell and tube type heat exchangers, steam generators, and nuclear reactor fuel assemblies are composed of a tube or rod bundle arranged in an outer casing. Length scales in a rod or tube bundle include subchannel hydraulic equivalent diameter, gap width between heat transfer tubes (or rods), and outer casing width. The characteristic length of a subchannel surrounded by multiple tubes (or rods) is expressed as a hydraulic equivalent diameter, whose magnitude is commonly categorized as a medium-size channel. In contrast, the gap width between heat transfer tubes is distinguished as a mini-channel, and the outer casing width as a large-size channel. Thus, there are three types of characteristic lengths for tube (or rod) bundle-heat exchanger systems.

The hydraulic equivalent diameter is used for friction loss and heat transfer coefficient calculations. However, an appropriate choice of characteristic length is controversial for the drift-flux correlation because the channel size relative to bubble size has a significant impact on calculating the drift velocity. When large bubbles are generated across multiple subchannels, the bubble size is larger than the subchannel hydraulic equivalent diameter and closer to the outer casing width.

Extensive efforts have been made to develop drift-flux correlations for various flow channels [9–11]. When those drift-flux correlations are applied to bundle heat exchangers, the hydraulic equivalent diameter is often used as the geometrical length scale. The choice of the characteristic length representing the thermo-fluid dynamics in the bundle significantly affects the calculated value of the drift velocity. When the mixture volumetric flux of gas–liquid two-phase flow is high, the choice of the characteristic length is not an issue. Since the mixture volume flux is much higher than the drift velocity, the effect of the drift velocity on the void fraction is negligible. In contrast, under low mixture volumetric flow conditions, the values of the mixture volumetric flux and the drift velocity are comparable, and the choice of the characteristic length is expected to significantly affect the accuracy of thermo-fluid analysis using the drift-flux correlation.

This study aims to identify an appropriate geometrical characteristic length scale for upward two-phase flow in a vertical tube (or rod) bundle geometry and validate a new drift-flux correlation with an appropriate geometrical characteristic length scale. The subchannel hydraulic equivalent diameter and outer casing width are selected as potential characteristic length scales. Five test data sets obtained in rod bundle test sections are collected from five different sources: Nuclear Power Engineering Corporation (NUPEC); Oak Ridge National Laboratory (ORNL); Japan Atomic Energy Research Institute (JAERI, currently the Japan Atomic Energy Agency, JAEA); Central Research Institute of Electric Power Industry (CRIEPI); and Purdue University. The pressure
2. Drift-flux model and key constitutive equations

2.1. One-dimensional drift-flux model

Zuber and Findlay [6] first proposed the concept of the drift velocity \( v_{\text{dir}} \) to incorporate the kinematic non-equilibrium into two-phase flow analyses. They defined the drift velocity as the difference between gas velocity \( v_g \) and mixture volumetric flux \( j \).

\[
v_{\text{dir}} = v_g - j
\]  
(1)

The alternative form of the drift velocity is the product of the liquid fraction \( f \) and the relative velocity between gas and liquid phases. For the homogeneous flow, where gas and liquid velocities are the same, the drift velocity and the relative velocity become zero.

Area-averaging Eq. (1) yields the one-dimensional drift-flux model as:

\[
\langle v_{\text{dir}} \rangle = C_0 \langle j \rangle + \langle v_g \rangle
\]  
(2)

where \( \langle \cdot \rangle \) and \( \langle | \cdot | \rangle \) indicate the area-averaged and void fraction-weighted mean quantities, respectively. The distribution parameter \( C_0 \) represents the covariance of void fraction \( \alpha \) and mixture volumetric flux \( j \) in the area-averaging form as:

\[
C_0 \equiv \langle \alpha j \rangle / \langle \alpha \rangle
\]  
(3)

The void fraction-weighted mean drift velocity is defined as:

\[
\langle v_{\text{dir}} \rangle = \langle \alpha j \rangle / \langle \alpha \rangle
\]  
(4)

Using the velocity scale \( \Delta \rho g \sigma / \rho_i^{0.25} \) non-dimensionalizes Eq. (2) as follows.

\[
\langle v_{\text{dir}} \rangle = C_0 \langle j' \rangle + \langle v_g \rangle
\]  
(5)

where

\[
\langle j' \rangle \equiv \langle j \rangle / \Delta \rho g \sigma / \rho_i^{0.25}
\]  
(6)

\[
\langle v_{\text{dir}} \rangle \equiv \langle v_g \rangle / \Delta \rho g \sigma / \rho_i^{0.25}
\]  
(7)

and

\[
\langle v_{\text{dir}} \rangle \equiv \langle v_g \rangle / \Delta \rho g \sigma / \rho_i^{0.25}
\]  
(8)

2.2. Key constitutive equations for a rod bundle

Ozaki and Hibiki [12,13] developed the drift-flux correlation for upward two-phase flows in a rod bundle. In the process of the correlation development, they did not use the conventional method, which determined the distribution parameter and drift velocity as the slope and intercept in the drift-flux plot \( \langle j \rangle \) vs \( \langle v_{\text{dir}} \rangle \). Instead, Ozaki and Hibiki first calculated the drift velocity by Hibiki and Ishii’s correlation Eq. (9) [14]. Next, they back-calculated the distribution parameters from Eq. (2) using the NUPEC data. Then, Ozaki and Hibiki proposed the distribution parameter correlation Eq. (23).

Hibiki and Ishii’s correlation for drift velocity [12,13]:

\[
\langle v_{\text{dir}} \rangle = \left( \langle v_{\text{dir}} \rangle \rangle e^{-1.39j} + \langle v_g \rangle \right) \left( 1 - e^{-1.39j} \right)
\]  
(9)

where the subscripts B and Kl,w indicate the drift velocity correlation for bubbly flow Eq. (11) [9] and Kataoka and Ishii’s correlation for drift velocity Eqs. (19)-(22) [7], respectively. The subscript \( w \) is used to emphasize that the outer casing width of a rod bundle \( w \) is utilized as the characteristic length scale. It should be noted here that the square outer casing width of a bundle is identical to the hydraulic equivalent diameter for the outer casing only. The non-dimensionalized superficial gas velocity is defined by:

\[
\langle j' \rangle \equiv \langle j \rangle / \Delta \rho g \sigma / \rho_i^{0.25}
\]  
(10)

Drift velocity correlation for bubbly flow [9]:

\[
\langle v_{\text{dir}} \rangle = \sqrt{2} (1 - \langle j \rangle)^{0.75}
\]  
(11)

Original Kataoka and Ishii’s correlation for drift velocity using \( D_h \) as characteristic length [7]:

Low viscosity systems (\( N_{fl} \leq 2 \times 10^{-3} \)):

\[
\langle v_{\text{dir}} \rangle = 0.0019D_h^{0.809}N_f^{0.157}N_{fl}^{0.562} \text{ for } D_h \leq 30, \tag{12}\n\]

\[
\langle v_{\text{dir}} \rangle = 0.030N_f^{0.157}N_{fl}^{0.562} \text{ for } D_h \geq 30. \tag{13}\n\]

The non-dimensional flow channel size is defined by:

\[
D_h = D_h / La
\]  
(14)

where Laplace length scale \( La \) is defined by:

\[
La \equiv \sigma / \Delta \rho g \sigma / \rho_i^{0.5}
\]  
(15)

The density ratio and viscosity number are defined by Eqs. (16) and (17), respectively.

\[
N_f \equiv \rho_2 / \rho_1
\]  
(16)

\[
N_{fl} \equiv \mu_1 / (\rho_1 \sigma La)^{0.5}
\]  
(17)

High viscosity systems (\( N_{fl} \geq 2 \times 10^{-3} \)):

\[
\langle v_{\text{dir}} \rangle = 0.92N_f^{0.157} \text{ for } D_h \geq 30. \tag{18}\n\]

Modified Kataoka and Ishii’s correlation for drift velocity using \( w \) as characteristic length [12,13]:

Ozaki and Hibiki [12,13] recommended the outer casing width of a rod bundle \( w \) for the characteristic length when Kataoka and Ishii’s correlation is applied to a rod bundle. Therefore, the following are obtained.

Low viscosity systems (\( N_{fl} \leq 2 \times 10^{-3} \)):

\[
\langle v_{\text{dir}} \rangle = 0.0019D_h^{0.809}N_f^{0.157}N_{fl}^{0.562} \text{ for } D_h \leq 30, \tag{19}\n\]

\[
\langle v_{\text{dir}} \rangle = 0.030N_f^{0.157}N_{fl}^{0.562} \text{ for } D_h \geq 30. \tag{20}\n\]

The non-dimensional bundle casing size is defined by:

\[
D_w = w / La
\]  
(21)

High viscosity systems (\( N_{fl} \geq 2 \times 10^{-3} \)):

\[
\langle v_{\text{dir}} \rangle = 0.92N_f^{0.157} \text{ for } D_w \geq 30. \tag{22}\n\]

Ozaki and Hibiki’s correlation for distribution parameter in rod bundles [12,13]:

\[
C_0 = 1.1 - 0.1w^{0.5}
\]  
(23)

Equation (23) is applicable to bulk boiling two-phase flow in a rod bundle. Hibiki et al. [15] analytically demonstrated that the distribution parameter was reduced to zero as the void fraction approached zero.
Ishii [9] proposed the following factor for subcooled boiling flows in circular and rectangular channels $\xi$.

$$\xi = 1 - \exp(-18\langle\alpha\rangle)$$

The values of the factor given by Eq. (24) at $\langle\alpha\rangle = 0$ and 0.15 are 0 and 0.93. Eq. (24) gives reasonable values at the onset of subcooled boiling and at the transition between subcooled and bulk boiling flows. Ozaki and Hibiki applied the bubble-layer thickness model to analytically derive the distribution parameter for subcooled boiling flows in a rod bundle. Thus, the distribution parameter for a rod bundle for subcooled boiling flow is given by:

$$C_0 = (1.1 - 0.1N_{\infty}^3)(1 - \exp(-12.1\langle\alpha\rangle^{0.7}))$$

for $D_0/P_0 = 0.7 - 0.9$.

The void fraction in Eq. (25) can be calculated by Eqs. (23) and (26) to avoid an iteration process.

Drift velocity correlation for churn flow [9]:

$$\langle\langle v_g + gj, CT \rangle\rangle = \bar{\bar{v}}^2$$

3. Advanced drift-flux correlation for a rod bundle

3.1. Current issues in Ozaki and Hibiki’s drift-flux correlation for a rod bundle

Ozaki and Hibiki’s drift-flux correlation, the most advanced correlation for a rod bundle, was validated using steam-water two-phase flow data collected in a prototypic $8 \times 8$ rod bundle for high-pressure conditions up to 8.6 MPa (i.e., NUPEC data) [16–18]. Since most of the NUPEC data were collected for high superficial gas velocity conditions, the drift velocity was calculated for the conditions where $C_0\langle f^+\rangle \gg \langle\langle v_g^+\rangle\rangle$ and $\langle\langle v_g^+\rangle\rangle \cong \langle\langle v_g^+; KL, w\rangle\rangle$ in the process of correlation development. The NUPEC data were suitable for evaluating the overall performance of Ozaki and Hibiki’s drift-flux correlation. However, the NUPEC data were not fully appropriate for validating the assumed characteristic length scale for a rod bundle. They were also not adequate for validating the assumed transition behavior of the drift velocity between bubbly and beyond-bubbly flows, Eq. (9). The fundamental reasoning used to select the outer casing width of a rod bundle as the characteristic length scale and the modeling philosophy for the transition behavior of the drift velocity between bubbly and beyond-bubbly flows are discussed in the present section.

3.2. Characteristic length scale for a rod bundle

The characteristic length in Kataoka and Ishii’s correlation [7] was hydraulic equivalent diameter because the correlation was originally developed for simple geometrical channels, such as circular and rectangular channels. In these channel geometries, the characteristic length scale was uniquely determined. The selection of the length scale characterizing two-phase flow behavior in a rod bundle is not straightforward.

Fig. 1 shows the typical two-phase flow behavior in an $8 \times 8$ rod bundle visualized for the pressure of 0.1 MPa at Purdue University [19]. Fig. 2 shows the top cross-sectional view of the $8 \times 8$ rod bundle test section [19]. Figs. 3 and 4 show the distributions of void fraction and gas velocity observed in bubbly and beyond-bubbly (or cap turbulent) flows, respectively, measured by a four-sensor conductivity probe [19]. The distributions are displayed along the x-axis at $y = 0$ in Fig. 2. The $x$ is the distance measured from the bundle center along the direction parallel to the outer casing of the rod bundle, and $w$ is the width of the outer casing of the rod bundle. The measurement was performed at the subchannel center and the minimum gap between two rods. The measurement locations are depicted by red dots in Fig. 2. The measurement error of the...
A four-sensor conductivity probe is reported to be ±10%. Group-1 and group-2, designated in Figs. 3 and 4, originate from a two-group approach to treating bubbles in two groups; such an approach is introduced in developing a two-group interfacial area transport equation [1]. The bubbles are grouped based on their transport characteristics, i.e., drag coefficient. Bubbles in spherical and distorted particle regimes are referred to as group-1 bubbles. Bubbles in cap and slug bubble regimes are referred to as group-2 bubbles. Group-1 bubbles are dominant in bubbly flow, whereas group-2 bubbles are formed in beyond-bubbly flow.

Fig. 3 shows the distributions of void fraction and gas velocity for bubbly flow. A pattern, such as a high void fraction at the gap and a low void fraction at the subchannel center, is repeated along the x-axis at y = 0. Another pattern, such as a high gas velocity at the gap and a low gas velocity at the subchannel center, is also repeated along the x-axis at y = 0. Fig. 3 shows that the characteristic length scale for bubbly flow may be the subchannel length scale, such as the subchannel hydraulic equivalent diameter. Fig. 4 shows the distributions of void fraction and gas velocity for beyond-bubbly flow. A global pattern is observed, such as power-law distributions for group-2 void fraction and gas velocity. The differences in void fraction and gas velocity between the gap and subchannel center are relatively small, which indicates that bubbles span over several subchannels in the rod bundle. The characteristic length scale for beyond-bubbly flow may be approximated by the outer casing width of the rod bundle.

Yang et al. [20] measured area-averaged void fractions for air–water two-phase flows in an 8 × 8 rod bundle for pressures of 0.1 and 0.3 MPa. The measurement was performed at four axial elevations. Their discussion on an appropriate characteristic length scale in a rod bundle is summarized as follows.

- The hydraulic equivalent diameter of a subchannel surrounded by rods was an appropriate length scale for bubbly flow because the motion of small bubbles composed of bubbly flow would be restricted within a subchannel.
- As the void fraction increased, the coalescence of small bubbles produced large bubbles spanning over several subchannels. For beyond-bubbly flow, the subchannel hydraulic equivalent diameter would no longer be valid to characterize the flow behavior. The outer casing width of a rod bundle was an appropriate length scale for beyond-bubbly flow.

Yang et al. calculated gas velocity \( \langle \langle v_G \rangle \rangle \) using Kataoka and Ishii’s correlation with two different length scales, such as subchannel hydraulic equivalent diameter and outer casing width of a rod bundle. Fig. 5(a) and 5(b) show the comparison results for pressures of 0.1 and 0.3 MPa, respectively [20]. Black solid and red dashed lines indicate the calculation results by Kataoka and Ishii’s correlation with the outer casing width and subchannel hydraulic equivalent diameter, respectively. They utilized \( C_0 = 1.2 \) in the calculation. The green dotted line indicates the calculated result by Ishii’s churn flow correlation of the drift velocity Eq. (26) [9] and \( C_0 = 1.2 \). Blue chain and orange double chain lines indicate the calculated results by Bestion’s correlation [21], Eq. (27) with two different distribution parameters \( C_0 = 1.2 \) and 1.0.

Bestion’s correlation for drift velocity [21]:

---

**Fig. 3.** Distributions of (a) void fraction and (b) gas velocity of bubbly flow along bundle center line parallel to the outer casing of rod bundle at \( z/D_H = 200 \). Flow conditions: \( \langle j_g \rangle = 0.02 \text{ m/s} \) and \( \langle j_f \rangle = 1.0 \text{ m/s} \) [19].

**Fig. 4.** Distributions of (a) void fraction and (b) gas velocity of cap turbulent flow along bundle center line parallel to the outer casing of rod bundle at \( z/D_H = 200 \). Flow conditions: \( \langle j_g \rangle = 0.5 \text{ m/s} \) and \( \langle j_f \rangle = 0.2 \text{ m/s} \) [19].
\[ (v_{gi,b}) = 0.188N_{\rho}^{-0.3}D_H^{0.5} \] (27)

As shown in Fig. 5, the data collected at \( z/D_h = 89 \), where \( z \) is the axial distance from the test section inlet, are significantly different from the data collected at other measurement elevations. A spacer grid located near \( z/D_h = 89 \) disrupted the two-phase flows resulting in bubble redistribution. Due to the highly scattered data, Yang et al. could not demonstrate the validity of the above speculation on the appropriate length scale with sufficient confidence. Bestion’s correlation significantly overestimated the drift velocity and failed to predict the data.

3.3. Transition behavior of the drift velocity between bubbly and beyond-bubbly flow

The drift velocity correlation Eq. (9) was developed based on the following observations by Kataoka and Ishii [7].

- The drift-flux correlation with the drift velocity calculated by the bubbly or churn flow correlation agreed with data in the range of \( \alpha_g \leq 0.5 \).
- For higher \( \alpha_g \), the drift velocity approached the value for beyond-bubbly flow.

The weighting factor \( e^{-1.39\alpha_g} \) in Eq. (9) was determined by assuming the weighting factor of 0.5 at \( \alpha_g = 0.5 \) [14]. The assumed transition characteristics of the drift velocity between bubbly and beyond-bubbly flows were not validated experimentally.

Recently, Hibiki and Tsukamoto [22] revisited the transition behavior modeling of the drift velocity between bubbly and beyond-bubbly flows in medium-to-large circular channels. They applied a rigorous two-group drift-flux model to explain the dynamic behavior of the drift velocity at the transition between bubbly and beyond-bubbly flows. Their analytical derivation demonstrated that the sum of the group-1 and group-2 drift velocities with a weighting function could express the dynamic behavior of the drift velocity at the transition. The weighting function was a function of the ratio of group-1 void fraction and total void fraction. Hibiki and Tsukamoto approximated the weighting function by a function of total void fraction. They suggested that the void fraction was a more suitable parameter to describe the drift velocity behavior than the non-dimensional superficial gas velocity was.

The experimentally validated drift velocity correlation is expressed by [22]:

\[ (v_{gi,b}) = (v_{gi,b})e^{-60.63(\alpha_g)^{0.87}} + (v_{gi,KI})\left(1 - e^{-60.63(\alpha_g)^{0.87}}\right) \] (28)

Since Eq. (28) was given as a function of the void fraction, the drift velocity could not be calculated solely by operating parameters, such as \( \langle j_g \rangle \) and \( \langle j_f \rangle \). The following approximation was introduced to eliminate the dependence of Eq. (28) on the void fraction as [22]:

\[ (v_{gi}) = \sqrt{2}(1 - (\alpha_{KI}))^{1.75}e^{-60.63(\alpha_{KI})^{2.87}} + (v_{gi,KI})\left(1 - e^{-60.63(\alpha_{KI})^{2.87}}\right) \] (29)

where \( (\alpha_{KI}) \) is the void fraction calculated by Kataoka and Ishii’s correlation for circular channels. The distribution parameter for circular channels is given by [22]:

\[ C_0 = 1.2 - 0.2N_{\rho}^{0.5} \] (30)

Equation (29) avoids an iteration calculation to compute the void fraction using the drift-flux correlation and allows for calculating the drift velocity using operating conditions \( \langle j_g \rangle \) and \( \langle j_f \rangle \) explicitly. Equation (29) was validated by the data collected in medium-to-large circular channels.

The above model to describe the transition behavior of the drift velocity between bubbly and beyond-bubbly flows is tested for rod bundle geometries in section 4. The explicit form of the drift velocity correlation for a rod bundle is:

\[ (v_{gi}) = \sqrt{2}(1 - (\alpha_{KI,RR}))^{1.75}e^{-60.63(\alpha_{KI,RR})^{2.87}} + (v_{gi,KI})\left(1 - e^{-60.63(\alpha_{KI,RR})^{2.87}}\right) \] (31)

where \( (\alpha_{KI,RR}) \) is the void fraction calculated by Eq. (23) for the distribution parameter as:

\[ (\alpha_{KI,RR}) = \frac{\langle j_g \rangle}{(1.1 - 0.1N_{\rho}^{0.5})(\langle j_f \rangle) + (\langle j_{KI,RR} \rangle)} \] (32)

Fig. 6(a) depicts the calculation scheme of Ozaki and Hibiki’s correlation [12,13]. Fig. 6(b) depicts the calculation scheme of the new drift-flux correlation. The underlined parts highlighted in red are the original contributions proposed by the present study. The equation set of the new drift-flux correlation is summarized in Table 1. In Fig. 6(b) and Table 1, the outer casing width for a rod bundle is used as the length scale recommended by the present study. Thus, \( (\alpha_{KI,RR}) = (\alpha_{KI,RR}) \) in Eq. (31) and (32). In section 4, two characteristic length scales, sub-channel hydraulic equivalent diameter \( D_h \) and outer casing width \( w \), are used in calculating \( (\alpha_{KI,RR}) \) to discuss the effect of the length scale selection on drift-flux predictions.
Table 1  
Equation set of the new drift-flux correlation applicable to dispersed two-phase flows in a rod bundle.

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<td>Low viscosity systems (( N_{s} &lt; 2 \times 10^{-3} )):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \langle \psi \rangle_{\text{cal}} = 0.0014N_{s}^{0.57} \psi_{\text{cal}}^{0.57}N_{s}^{0.56} ]</td>
<td>Eq. (19) [7,12]</td>
<td>determined by the outer casing width of the rod bundle was identified as an appropriate length scale for (a) prediction.</td>
</tr>
<tr>
<td>for ( D_{s}^{\text{c}} \leq 30 )</td>
<td>Eq. (20) [7,12]</td>
<td></td>
</tr>
<tr>
<td>[ \langle \psi \rangle_{\text{exp}} = 0.030N_{s}^{0.57} \psi_{\text{cal}}^{0.57}N_{s}^{0.56} ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>for ( D_{s}^{\text{c}} \geq 30 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High viscosity systems (( N_{s} &gt; 2 \times 10^{-3} )):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \langle \psi \rangle_{\text{cal}} = 0.92N_{s}^{0.57} ]</td>
<td>Eq. (22) [7,12]</td>
<td></td>
</tr>
<tr>
<td>for ( D_{s}^{\text{c}} \geq 30 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.4. Statistical parameters to examine the prediction accuracy of new drift-flux correlation

Section 4 examines the validation of the new drift-flux correlation for a rod bundle, Eqs. (23) and (31). Four statistical parameters are introduced here to examine the prediction accuracy.

Mean error \( m_{\text{rel}} \)

\[ m_{\text{rel}} = \frac{1}{N} \sum_{i=1}^{N} (\psi_{\text{cal}} - \psi_{\text{exp}}) \]  

(33)

Standard deviation \( s_{\text{rel}} \)

\[ s_{\text{rel}} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (\psi_{\text{cal}} - \psi_{\text{exp}} - m_{\text{rel}})^2} \]  

(34)

Mean relative error \( m_{\text{rel}} \)

\[ m_{\text{rel}} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\psi_{\text{cal}} - \psi_{\text{exp}}}{\psi_{\text{exp}}} \right) 	imes 100 \]  

(35)

Standard relative deviation \( s_{\text{rel}} \)

\[ s_{\text{rel}} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left( \frac{\psi_{\text{cal}} - \psi_{\text{exp}} - m_{\text{rel}}}{\psi_{\text{exp}}} \right)^2} \times 100 \]  

(36)

The mean error \( m_{\text{rel}} \) and mean relative error \( m_{\text{rel}} \), are the indices to describe the bias of an equation. Positive and negative values indicate the tendency of overestimation and underestimation, respectively. The standard deviation \( s_{\text{rel}} \) and standard relative deviation \( s_{\text{rel}} \) are the indices to describe the random error of an equation. Large values indicate the tendency to scatter around the predicted value.

4. Validation of advanced drift-flux correlation for a rod bundle

This section examines the effect of the length scale calculation on drift-flux predictions and the validation of the new drift-flux correlation for a rod bundle, Eqs. (23) and (31). For this purpose, five databases are used: diabatic NUPEC database [16–18], diabatic ORNL database [23], diabatic JAERI (currently Japan Atomic Energy Agency) database [24], adiabatic CRIEPI database [25,26], and adiabatic Purdue University database [20]. The databases provide the gas velocity and mixture volumetric flux data. Figs. 2 and 7 show the geometries of the test bundles with the outer casing width used in the validation process. It should be noted here that the outer casing width for the JAERI test bundle is the inner diameter of the outer circular casing. The JAERI test used a partial rod bundle of simulated 17 × 17 rod bundle in PWR. Tables 2 and 3 summarize the bundle geometries and test conditions, respectively. The collected databases cover a wide range of geometrical and test conditions as follows: rod diameter-to-pitch ratio from 0.644 to 0.769; subchannel hydraulic equivalent diameter from 11.5 to 21.3 mm; outer casing width from 68.0 to 140 mm, pressure from 0.1 to 12 MPa; and mass flux from 0 to 2000 kg/m²s. The above five databases include 317 data used for the model validation.

Validation using the NUPEC database.

Fig. 8 compares the new drift-flux correlation of Eqs. (23) and (31) with the NUPEC data. The abscissa and ordinate axes indicate the non-dimensionalized mixture volumetric flux Eq. (7) and gas velocity Eq. (6), respectively. Fig. 8(a), 8(b), 8(c), and 8(d) are the drift-flux plots for pressure of 1.0, 4.0, 7.2, and 8.6 MPa, respectively. Fig. 9(a), 9(b), 9(c), and 9(d) are, respectively, expanded scale plots of Fig. 8(a), 8(b), 8(c), and 8(d). The expanded scale plots focus on a low mixture volumetric flux.
Excluding the two lines with the legends of length scale $D_H$ and churn flow, the other three lines (black solid, red dashed and blue dotted lines) indicate the values calculated by Eqs. (23) and (31) using the outer casing width of the rod bundle $w$ as the length scale in Eq. (31). The green chain line marked as length scale $D_H$ shows the values calculated by Eqs. (23) and (12) using the subchannel hydraulic equivalent diameter as the length scale $D_H$. The green chain line approximately indicates the values calculated by Eqs. (23) and (31) using the hydraulic equivalent diameter $D_H$ as the length scale in Eq. (31) because $\langle \langle v^s_{ji} \rangle \rangle \approx \langle \langle v^c_{ji} \rangle \rangle$. The cyan double chain line marked as churn flow shows the values calculated by Eqs. (23) and (26).

A superficial liquid velocity condition for each line is given in Figs. 8.

---

**Table 2**

Geometrical information of test sections in databases used for model validation.

<table>
<thead>
<tr>
<th>Database Source</th>
<th>Fluid System</th>
<th>Bundle Size</th>
<th>$D_0$ [mm]</th>
<th>$P_0$ [mm]</th>
<th>$D_0/P_0$ [-]</th>
<th>$D_H$ [mm]</th>
<th>$w$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUPEC Steam-Water</td>
<td>$8 \times 8$</td>
<td>12.3</td>
<td>16.2</td>
<td>0.759</td>
<td>14.9</td>
<td>132.5</td>
<td></td>
</tr>
<tr>
<td>ORNL Steam-Water</td>
<td>$8 \times 8$</td>
<td>9.5</td>
<td>12.7</td>
<td>0.748</td>
<td>12.1</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>JAERI Steam-Water</td>
<td>32</td>
<td>9.5</td>
<td>12.6</td>
<td>0.754</td>
<td>11.8</td>
<td>75.6</td>
<td></td>
</tr>
<tr>
<td>CRIEPI Steam-Water</td>
<td>$5 \times 5$</td>
<td>10.0</td>
<td>13.0</td>
<td>0.769</td>
<td>11.5</td>
<td>68.0</td>
<td></td>
</tr>
<tr>
<td>Purdue University Air-Water</td>
<td>$8 \times 8$</td>
<td>10.3</td>
<td>16.0</td>
<td>0.644</td>
<td>21.3</td>
<td>140</td>
<td></td>
</tr>
</tbody>
</table>

---

**Table 3**

Test conditions of databases used for model validation.

<table>
<thead>
<tr>
<th>Database Source</th>
<th>Fluid System</th>
<th>$P$ [MPa]</th>
<th>$G$ [kg/m²s]</th>
<th>$\langle j^L_x \rangle$ [m/s]</th>
<th>$\langle j^L_y \rangle$ [m/s]</th>
<th>$\langle \alpha \rangle$ [m/s]</th>
<th>Average measurement (α) error [%]</th>
<th>Maximum measurement (α) error [%]</th>
<th>No. of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUPEC Steam-Water</td>
<td>1.0-8.6</td>
<td>280-2000</td>
<td>0.295-31.1</td>
<td>0.275-2.55</td>
<td>0.280-0.871</td>
<td>2</td>
<td>N.A.</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>ORNL Steam-Water</td>
<td>3.9-8.1</td>
<td>3.1-29.3</td>
<td>0.03-3.56</td>
<td>0.00-0.11</td>
<td>0.07-0.82</td>
<td>6.63</td>
<td>42.9</td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>JAERI Steam-Water</td>
<td>3-12</td>
<td>5-100</td>
<td>0.07-4.99</td>
<td>0.000-0.126</td>
<td>0.04-0.92</td>
<td>7.16</td>
<td>33.3</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>CRIEPI Steam-Water</td>
<td>0.25-7.5</td>
<td>0-500</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
<td>5.85</td>
<td>24.7</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>Purdue University Air-Water</td>
<td>0.1-0.3</td>
<td>87.8-1359</td>
<td>0.065-8.33</td>
<td>0.088-1.4</td>
<td>0.25-0.88</td>
<td>10</td>
<td>N.A.</td>
<td>57</td>
<td></td>
</tr>
</tbody>
</table>
and 9. For example, the non-dimensionalized superficial liquid velocity for the solid black line in Fig. 8(a) is $\langle j \rangle = 2.04$. The superficial liquid velocity condition for each data symbol corresponds to the condition for the line of the same color and the same order starting from the top. For example, the non-dimensionalized superficial liquid velocity for the open circle symbol in Fig. 8(a) is $\langle j \rangle = 2.04$. The range of void fraction for each superficial liquid velocity condition is similarly given in the figure. For example, the void fraction $\langle \alpha \rangle$ ranges from 0.69 to 0.84 in the test condition indicated by the open circle symbol in Fig. 8(a).

The drift-flux plots given in Fig. 8(a), 8(b), 8(c), and 8(d) and their magnified plots, Fig. 9(a), 9(b), 9(c), and 9(d), demonstrate that the new drift-flux correlation using the outer casing width of a rod bundle agrees with the data in a wide range of mixture volumetric fluxes. The other two correlations, denoted by length scale $D_H$ and churn flow, tend to calculate a lower gas velocity than that of the new drift-flux correlation because the values of the drift velocity calculated by the other two correlations are lower than the value of the drift velocity calculated by the new drift-flux correlation. However, the differences in the calculated gas velocity among the three drift-flux correlations for the condition of $C_0 \langle j \rangle \gg \langle \langle v_g \rangle \rangle$ are small because the effect of the drift velocity on the gas velocity is insignificant for the condition. The slopes of the drift-flux plots for the condition of $C_0 \langle j \rangle \gg \langle \langle v_g \rangle \rangle$ correspond to the distribution parameter values. The drift-flux plots given in Fig. 8(a), 8(b), 8(c), and 8(d) demonstrate the validity of Eq. (23) because the slopes agree with Eq. (23).

In thermal–hydraulic analyses, a correlation developed for a circular channel is often applied to other geometries adopting the concept of hydraulic equivalent diameter. If the distribution parameter correlation for a circular channel given by Eq. (30) is applied to a rod bundle, the drift-flux correlation overestimates the gas velocity by 10% and underestimates the void fraction by 10%.

Since the NUPEC test primarily focused on the void fraction measurement in the beyond-bubbly flow region, only limited data in the bubbly flow region are available. The NUPEC test data are not suitable for examining the validity of Eq. (31), which models the transition behavior of the drift velocity between bubbly and beyond-bubbly flows. However, the data collected in the low mixture volumetric flux region that are shown in Fig. 9(a), 9(b), 9(c), and 9(d) are adequate for examining the suitable length scale to be selected in the drift velocity correlation.

The drift-flux correlation for churn flow provides the baseline for the assessment. For nearly zero void fraction, the values of the drift velocity for all three cases (Eq. (31) with $w$, Eq. (12) with $D_H$ (or Eq. (31) with $D_H$), and Eq. (26)) are the same as the value for churn flow calculated by Eq. (26). The lines obtained using the drift velocity calculated by Eq. (31) with $w$ show that the apparent slope at the transition between bubbly and beyond-bubbly flows gradually increases and decreases to reach the constant value Eq. (23) as the mixture volumetric flux increases. The observed behavior in the drift-flux plot corresponds to the transition behavior of the drift velocity between bubbly and beyond-bubbly flows. Since the cap bubble drift velocity is much higher than the churn flow drift velocity, the transition behavior is magnified in the drift-flux plot. The value of the drift velocity calculated by Eq. (12) with $D_H$ is much lower than that by Eq. (20) with $w$. The value of the drift velocity calculated by Eq. (12) with $D_H$ (or Eq. (31) with $D_H$) is similar to the value of the drift velocity for churn flow. The two drift-flux correlations significantly underestimate the gas velocity due to the underestimated drift velocity. Fig. 9(a), 9(b), 9(c), and 9(d) suggest that the outer casing width of a rod bundle is an adequate length scale for drift velocity. This conclusion is consistent with the observed local
distributions of two-phase flow parameters shown in Figs. 3 and 4.

The values of the statistical indices for void fraction and gas velocity predicted by the new drift-flux correlation and the drift-flux correlation with the drift velocity correlation for churn flow are summarized in Tables 4 and 5. The new drift-flux correlation (Eqs. (23) and (31) with \( \omega \)) reproduces the data with a negligible bias and reasonable random uncertainty. However, the drift-flux correlation based on hydraulic equivalent diameter (Eqs. (23) and (31) with \( D_{HE} \)) overestimates the void fraction due to the underestimated drift velocity. The comparison between the new drift-flux correlation and the NUPEC data leads to the following essential conclusions.

- The distribution parameter for a rod bundle is represented by Eq. (23).
- The appropriate length scale for a rod bundle is the outer casing width of a rod bundle.
- Validation using the ORNL database

Fig. 10 compares the new drift-flux correlation of Eqs. (23) and (31) with the NUPEC data as the drift-flux plots for pressures of 4.0 and 8.0 MPa, respectively. It should be noted that the void fraction was measured by a differential pressure cell, assuming negligible frictional pressure loss. The assumption may cause overestimation of the measured void fraction or underestimation of the gas velocity for high velocity conditions.

Fig. 10 shows excellent agreement between the new drift-flux correlation with \( w \) and the data for \( 0 \leq \langle \dot{j}^+ \rangle \leq 1.5 \), where the flow regime transitions from bubbly to beyond-bubbly flows. The assumption of

<table>
<thead>
<tr>
<th>Database Source</th>
<th>Void Fraction</th>
<th>Gas Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( m_1 [-] )</td>
<td>( m_{a1} [%] )</td>
</tr>
<tr>
<td>NUPEC</td>
<td>0.00175</td>
<td>0.0212</td>
</tr>
<tr>
<td>ORNL</td>
<td>-0.0505</td>
<td>0.0196</td>
</tr>
<tr>
<td>JAERI</td>
<td>-0.0145</td>
<td>0.0550</td>
</tr>
<tr>
<td>CRIEPI</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Purdue University</td>
<td>0.0292</td>
<td>0.0790</td>
</tr>
<tr>
<td>Average of All Data</td>
<td>-0.00811</td>
<td>0.0571</td>
</tr>
</tbody>
</table>

Fig. 9. Drift-flux plots for NUPEC data at low mixture volumetric flux conditions (a) 1.0 MPa; (b) 4.0 MPa; (c) 7.2 MPa; (d) 8.6 MPa.
negligible frictional pressure loss is valid for the condition. The data for the void fraction higher than 0.7 are lower than the new drift-flux correlation. The plausible reasons are: (1) a deteriorated measurement accuracy due to the assumption of negligible frictional pressure loss; (2) a decrease in the distribution parameter for the high void fraction condition ($\alpha_t \approx 1$ as $\langle \alpha \rangle \rightarrow 1$); and (3) a decrease in the drift velocity due to the flow regime transition to separated flow ($\langle \alpha \rangle \rightarrow 1$).

The distribution parameter approaches unity as the void fraction approaches unity. The distribution parameter for dispersed two-phase flow should take a value of about 1.0, calculated for the pressure of 4.0 and 8.0 MPa and approximately 1 for annular flow. The decreased value of the gas velocity due to the reduced distribution parameter is estimated to be 0.64 (=0.08 × 8) at $\langle j^+ \rangle = 8$. Thus, the decrease in the distribution parameter is not the sole reason to explain the result in Fig. 10. The drift velocity approaches zero as the void fraction approaches unity. The distribution parameter for dispersed two-phase flow approaches zero as the void fraction approaches unity.

Fig. 10 shows an excellent agreement between the new drift-flux correlation and the measured gas velocity. As discussed in the validation using the ORNL database, the deviation between the calculated and measured gas velocities is approximately 1.08, calculated for the pressure of 3, 7, and 12 MPa, respectively. Fig. 11 (b) and 11(d) are the drift-flux plots for pressure of 0.25, 0.41, 2.0, 5.0, 7.0, and 7.5 MPa, respectively. The collected data are

<table>
<thead>
<tr>
<th>Database Source</th>
<th>Void Fraction</th>
<th>Gas Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m_t$ [-]</td>
<td>$s_t$ [-]</td>
</tr>
<tr>
<td>NUPEC</td>
<td>0.0401</td>
<td>0.0244</td>
</tr>
<tr>
<td>ORNL</td>
<td>0.132</td>
<td>0.0654</td>
</tr>
<tr>
<td>JAERI</td>
<td>0.144</td>
<td>0.104</td>
</tr>
<tr>
<td>CRIEPI</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Purdue University</td>
<td>0.0961</td>
<td>0.0820</td>
</tr>
<tr>
<td>Average All Data</td>
<td>0.123</td>
<td>0.0822</td>
</tr>
</tbody>
</table>

Fig. 10. Drift-flux plots for ORNL data. (a) 4.0 MPa and (b) 8.0 MPa.

For $\langle \alpha \rangle > 0.7$, the new drift-flux correlation with $w$ tends to overestimate the gas velocity due to the transition from dispersed two-phase flow to separated two-phase flow (or annular flow).

Validation using the JAERI database
Fig. 11 compares the new drift-flux correlation of Eqs. (23) and (31) with the JAERI data. Fig. 11(a), 11(c), and 11(e) are the drift-flux plots for pressure of 3, 7, and 12 MPa, respectively. Fig. 11(b) and 11(d) are expanded scale plots for Fig. 11(a) and 11(c), which focus on a low mixture volumetric flux.

Fig. 11 shows an excellent agreement between the new drift-flux correlation with $w$ and the data for $0 \leq \langle j^+ \rangle \leq 1.5$, where the flow regime transitions from bubbly to beyond-bubbly flows. The new drift-flux correlation agrees well with the data for the condition $\langle \alpha \rangle \leq 0.7$. For $\langle \alpha \rangle > 0.7$, the new drift-flux correlation with $w$ overestimates the gas velocity. As discussed in the validation using the ORNL database, the deviation between the calculated and measured gas velocities is explained as the transition from dispersed two-phase flow to separated two-phase flow (or annular flow).

Validation using the CRIEPI database.
Fig. 12 compares the new drift-flux correlation of Eqs. (23) and (31) with the CRIEPI data. Fig. 12(a), 12(b), 12(c), 12(d), 12(e), and 12(f) are the drift-flux plots for pressure of 0.25, 0.41, 2.0, 5.0, 7.0, and 7.5 MPa, respectively. The collected data are $\langle j^+ \rangle$ and $\langle \langle \nu_g^+ \rangle \rangle$ data, and do not...
include \( \alpha \) data. Fig. 12 presents an excellent agreement between the new drift-flux correlation with \( w \) and the data for all pressure conditions. Some data scattering around the new drift-flux correlation is observed for the pressure of 2.0 MPa. Since scattering is not observed for the other pressure conditions, it may be due to measurement error.

Fig. 12 indicates that the drift-flux correlation with \( D_H \) considerably deviates from the data, demonstrating that the appropriate length scale for a rod bundle is not the subchannel hydraulic equivalent diameter. Although the CRIEPI data are not adequate for describing the transition characteristics of the drift velocity between bubbly to beyond-bubbly flows, the data are suitable for validating Eq. (23).

Table 4 indicates that the new drift-flux correlation (Eqs. (23) and (31) with \( w \)) reproduces the data with some bias and reasonable random uncertainty. Table 5 shows that the drift-flux correlation based on hydraulic equivalent diameter (Eqs. (23) and (31) with \( D_H \)) significantly overestimates the void fraction due to the underestimated drift velocity. The comparison between the new drift-flux correlation and the CRIEPI data leads to the following essential conclusions.

- The distribution parameter for a rod bundle is represented by Eq. (23).
- The appropriate length scale for a rod bundle is the outer casing width of a rod bundle.

Validation using the Purdue University database.

Fig. 13 compares the new drift-flux correlation of Eqs. (23) and (31)
with the Purdue University data. Fig. 13(a) and 13(c) are the drift-flux plots using the characteristic length of \( w \) for pressure of 0.1 and 0.3 MPa, respectively. Fig. 13(b) and 13(d) are the drift-flux plots using the characteristic length of \( D_H \) for pressure of 0.1 and 0.3 MPa, respectively. Although the data are scattered, the new drift-flux correlation with \( w \) shows a better prediction than the drift-flux correlation with \( D_H \).

Table 4 indicates that the new drift-flux correlation (Eqs. (23) and (31) with \( w \)) reproduces the data with some bias and reasonable random uncertainty. Table 5 shows that the drift-flux correlation based on hydraulic equivalent diameter (Eqs. (23) and (31) with \( D_H \)) significantly overestimates the void fraction due to the underestimated drift velocity. The comparison between the new drift-flux correlation and Purdue University data supports the essential conclusions obtained in the validation using the other databases.

As shown in Table 4, the values of the statistical indices for the new drift-flux correlation (Eqs. (23) and (31) with \( w \)) are \( m_{v_\text{rel}} = -0.00811 \), \( s_{v_\text{rel}} = 0.0571 \), and \( m_{v_\text{rel},\text{ab}} = 16.1 \% \) for void fraction. The values of the statistical indices for gas velocity are \( m_{u_\text{rel}} = -0.389 \text{ m/s} \), \( s_{u_\text{rel}} = 2.21 \text{ m/s} \), and \( m_{u_\text{rel},\text{ab}} = 15.4 \% \). The new drift-flux correlation (Eqs. (23) and (31) with \( w \)) reproduces the void fraction with negligible bias (-0.00811 in an absolute measure and -0.881 % in a relative measure) and reasonable random uncertainty (0.0571 in an absolute measure and 16.1 % in a relative measure). The new drift-flux correlation reproduces the gas velocity with some bias (-0.767 % in a relative measure) and reasonable random uncertainty (15.4 % in a relative measure).
As shown in Table 5, the values of the statistical indices for the drift-flux correlation based on hydraulic equivalent diameter (Eqs. (23) and (31) with $D_{H}$) are $m_{d} = 0.123$, $s_{d} = 0.0822$, $m_{rel} = 28.1 \%$, and $s_{rel, ab} = 23.4 \%$ for void fraction. The values of the statistical indices for gas velocity are $m_{d} = -2.43$ m/s, $s_{d} = 2.28$ m/s, $m_{rel} = -21.3 \%$, and $s_{rel} = 18.2 \%$. The drift-flux correlation based on hydraulic equivalent diameter (Eqs. (23) and (31) with $D_{H}$) overestimates the void fraction with significant bias (0.123 in an absolute measure and 28.1 \% in a relative measure).

In summary, the comparisons between the new drift-flux correlation and the collected data validate Eq. (31) with $w$ and support the following essential conclusions.

- The distribution parameter for a rod bundle is represented by Eq. (23).
- The appropriate length scale for a rod bundle is the outer casing width of a rod bundle.
- The behavior of the drift velocity at the transition between bubbly and beyond-bubbly flows is represented by Eq. (31).
- For $\langle \alpha \rangle > 0.7$, the new drift-flux correlation with $w$ tends to overestimate the gas velocity due to the transition from dispersed two-phase flow to separated two-phase flow (or annular flow).

5. Conclusion

The present study aimed at understanding the appropriate geometrical length scale for one-dimensional heat and mass transfer analyses in a vertical rod bundle and comprehending the drift velocity characteristics at the transition between bubbly and beyond-bubbly flows. The new drift-flux correlation considering the drift velocity’s dynamic change at the transition between bubbly and beyond-bubbly flows was extensively evaluated using 317 data collected from five different sources. The fluid systems were air-water and steam-water systems. The outer casing width of rod bundles ranged from 68.0 to 140 mm. The rod diameter-to-pitch ratio ranged from 0.644 to 0.769. The subchannel hydraulic equivalent diameter ranged from 11.5 to 21.3 mm. The mass velocity was changed from 0.0 to 2000 kg/m$^2$s. The operating pressure ranged from 0.1 to 12 MPa.

The extensive experimental assessment demonstrated that the appropriate geometrical length scale for a vertical rod bundle was the outer casing width of a rod bundle. The effect of the geometrical scale on the predictions of gas velocity and void fraction was more pronounced at low mixture volumetric flux conditions. Using the hydraulic equivalent diameter as a characteristic length in the drift velocity calculation caused significant underestimations of gas velocity and overestimations of void fraction.

The values of the statistical indices for void fraction were $m_{d} = -0.00811$, $s_{d} = 0.0571$, $m_{rel} = -0.881 \%$, and $s_{rel} = 16.1 \%$. The prediction bias of the new drift-flux correlation was negligibly small, and the random error was 0.0571 in an absolute measure and 16.1 \% in a relative measure. The values of the statistical indices for gas velocity were $m_{d} = -0.389$ m/s, $s_{d} = 2.21$ m/s, $m_{rel} = -0.767 \%$, and $s_{rel} = 15.4 \%$. The prediction bias of the new drift-flux correlation was negligibly small, and the random error was 15.4 \% in a relative value measure. It should be noted here that the above statistical analyses do not consider measurement errors. The actual random error for void fraction prediction increases to 16.8 \% and 18.9 \% if 5 and 10 \% measurement errors are considered.

The above extensive evaluation results demonstrated the validity of the new drift-flux correlation using the outer casing width as a rod bundle characteristic length for a wide range of test conditions.
including pressure from 0.1 to 12 MPa and outer casing width of a rod bundle up to 140 mm. This drift-flux correlation is explicit for operating conditions, such as \( j_g \) and \( j_l \). Additionally, it is simple enough to be implemented into existing one-dimensional computational codes without any substantial code architecture change. The drift-flux correlation used in the codes is simply replaced with the new drift-flux correlation without algorithm change.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Data availability**

The authors do not have permission to share data.

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**References**