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Meta-silencer with designable timbre

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Abstract
Timbre, as one of the essential elements of sound, plays an important role in determining sound properties, whereas its manipulation has been remaining challenging for passive mechanical systems due to the intrinsic dispersion nature of resonances. Here, we present a meta-silencer supporting intensive mode density as well as highly tunable intrinsic loss and offering a fresh pathway for designable timbre in broadband. Strong global coupling is induced by intensive mode density and delicately modulated with the guidance of the theoretical model, which efficiently suppresses the resonance dispersion and provides desirable frequency-selective wave-manipulation capacity for timbre tuning. As proof-of-concept demonstrations for our design concepts, we propose three meta-silencers with the designing targets of high-efficiency broadband sound attenuation, efficiency-controlled sound attenuation and designable timbre, respectively. The proposed meta-silencers all operate in a broadband frequency range from 500 to 3200 Hz and feature deep-subwavelength sizes around 50 mm. Our work opens up a fundamental avenue to manipulate the timbre with passive resonances-controlled acoustic metamaterials and may inspire the development of novel multifunctional devices in noise-control engineering, impedance engineering, and architectural acoustics.

Keywords: designable timbre, acoustic metamaterial, multi-functional sound silencer, resonance modulation

1. Introduction
Timbre is a foundational aspect of sound, which is naturally a conception with close ties to broadband sound and is commonly regarded as determined by the inherent characteristic of passive mechanical systems [1–3]. This is because timbre manipulation has been remaining challenging due to the dispersion nature of resonances that hinders broadband wave modulations. The recent advance in acoustic metamaterials [4–8] greatly enriches the wave-manipulation approaches via enhanced performances and compressed sizes, and owing to the development of additive manufacturing technology [9–11], the construction of high-performance metamaterial-based acoustic devices are more feasible, leading to versatile functionalities such as negative refraction [12–14], super-resolution imaging [15, 16] and deep-subwavelength absorption [17–20]. Although the majority of acoustic metamaterials are still restricted by strong resonance dispersion and designed to deal with wave manipulation in a single frequency or a relatively narrow frequency band [21–30], broadband sound-absorbing metamaterials shed new light on the understanding of the control of resonances and pave a way for timbre manipulation in broadband [31–34].
With typical strongly enhanced acoustic fields and subwavelength structural scale, the initial designs of sound-absorbing metamaterials are remarkably efficient for narrow-band performances [17–20, 35–39]. In pursuit of broadband sound absorption/attenuation, an efficient approach is to combine a series of component acoustic absorbers operating at different frequencies [31, 34, 40–52]. Despite the fascinating achievements of high-efficiency broadband sound absorption/attenuation, it still remains a significant gap in the realization of timbre manipulation since timbre manipulation puts higher demands on the frequency-selective and delicate modulation of resonances in broadband.

In this work, we investigate the fundamental physical mechanism of resonances’ interactions and reveal that strong and well-controlled global coupling offers a feasible pathway for designable timbre. We present a sound meta-silencer composed of perforated plates, metal foams and neck-embedded Helmholtz resonators (NEHRs) to demonstrate our concept. The meta-silencer can support intensive mode density and meanwhile provide efficient modulation of intrinsic loss, which can induce strong global coupling and allows delicate modulation of resonances against their strong dispersion nature. With the theoretical model established here by employing the coupled mode theory, we clearly demonstrate the underlying physical picture of the global coupling in timbre modulation.

Guided by the theoretical model, we designed three meta-silencers to comprehensively manifest how we control the resonances to achieve a designable timbre. The first meta-silencer is designed with the target of efficient sound attenuation under grazing incidence. This meta-silencer achieves an average transmission loss (TL) of 28.4 dB in 500–3200 Hz with a deep sub-wavelength thickness of 53.5 mm. The second meta-silencer is designed with a more strict target that puts demands on a controlled attenuation efficiency rather than only pursuing roughly high efficiency. With remarkable fulfillment to the target, the second designed meta-silencer, with a thickness of 43.4 mm, achieves relatively uniform sound-attenuation efficiency of 12 dB within 500–3200 Hz. At last, we set a more rigorous target for constructing designable timbre for the third meta-silencer, which aims for moderate sound attenuation (18 dB) from 500 Hz to 930 Hz, mild sound attenuation (3 dB) within 930–1720 Hz, and intense sound attenuation (35 dB) within 1720–3200 Hz. As a result, the third designed meta-silencer (thickness 44.4 mm) successfully adjusts the timbre by alleviating the fundamental-frequency sound, relatively highlighting the first overtone and muting the second overtone.

2. Experimental setup and theoretical models

In this section, we will demonstrate the configuration and manufacturing of the meta-silencer and show the experimental setup. Then, we will introduce the theoretical model established in this work and reveal the requirements for achieving a designable timbre.

2.1. Meta-silencer configuration and manufacturing

The configuration of the presented meta-silencer is illustrated in Figure 1(a). The bottom part of the meta-silencer is composed of eight identical units in y direction. The unit of the meta-silencer consists of 16 NEHRs with a unanimous thickness of $H_s$. The right subfigure of Figure 1(a) illustrates the details of the NEHR, which contains a rectangular cross-sectional cavity (length $a$ and width $b$) and an embedded square cross-sectional neck (depth $l_p$ and side length $d_e$). Above the NEHRs, the metal foam is located between the resonator array and the perforated plate for providing effective modulation on the intrinsic loss of the meta-silencer [53]. The metal foams are with the flow resistance of 1500 Pa × s m$^{-2}$ (thickness $H_f$, length $A_f$ and width $B_f$), which are separated by shells (wall thickness $t_f$) having a height of $H_f$, a length of $A$ and a width $B$ for each unit cell. The top part of the meta-silencer is a perforated plate with a thickness of $t_p$ and an aperture diameter of $d_p$, which can effectively protect the internal materials and prevent the generation of sub-noise on the surface of the meta-silencer [45, 53]. The nickel metal foam was manufactured via electroplating sintering technology, which possesses the combinations of outstanding acoustic and mechanical properties such as effective sound dissipation and good strength and stiffness [54]. At the same time, compared with traditional sponges, metal foam can also be used in extreme environments such as low temperature and high temperature. The result parts of the meta-silencer were fabricated via additive manufacturing technology (three-dimensional printing) using laser stereolithography (140 μm) with a photosensitive resin (UV curable resin), exhibiting a manufacturing precision of 0.1 mm. The density, sound velocity and Young’s modulus of the manufacturing material are $\rho_{pp} = 1300$ kg m$^{-3}$, $E = 2600$ MPa and $c_{pp} = 2700$ m s$^{-1}$, respectively. The ratio of the characteristic acoustic impedance between the manufacturing material of the meta-silencers and the air is $R \approx 8457$, which shows a rather contrasting impedance mismatch and allows the samples’ walls of 1 mm to be approximately treated as ‘acoustically hard boundaries’. Here, the thickness of the manufacturing materials (meta-silencers’ walls) will not significantly affect the performance unless the thickness is very thin or very thick. However, when the meta-silencers’ walls are too thin to support the effect of ‘acoustically hard boundaries’ or they are very thick greatly reducing the available cross-sectional space for sound attenuation, the meta-silencers’ working performance will decrease significantly.

2.2. Experimental setup

Figure 1(b) shows the setup in the acoustic duct experiments. The duct has a primary waveguide with a square cross-section whose side length $W_{tube}$ is 51 mm. The digital signal from
an NI PX3-4461 AO 0 channel is sent to a power amplifier and then drives the upstream sound source consisting of 4 BMS 5599 loudspeakers. The test meta-silencer is mounted on one side of the duct wall and located in the middle of the microphones. Both ends of the duct are equipped with mufflers to reduce external and measurement noise. There are 4 1/4-inch microphones of GRAS type 46BD mounted on the designated positions of the duct wall to measure the information of the incident, reflected, and transmitted waves regarding their amplitudes and phases and accordingly the TL of the tested meta-silencer can be calculated.

2.3. Theoretical model

Designable timbre fundamentally requires extraordinary modulation of resonances. To pave a way for this goal, a theoretical model is established utilizing the coupled mode theory [55, 56]. This theoretical model comprehensively describes the interactions among resonances by means of the fundamental qualities of radiation and thermal-viscous dissipation of each resonance, which therefore reveals the underlying physical picture of global coupling and facilitates the modulation of global coupling for timbre modulation.

Suppose a resonant system supports $N$ coherently coupled modes, the amplitude of the $n$th mode/resonance can be expressed as [57–59]

$$\frac{d}{dt} \tilde{a}_n = (j\omega_n - \gamma_n - \Gamma_n) \tilde{a}_n + j\sqrt{\gamma_a} \tilde{S}_l + j\sqrt{\gamma_s} \left( \sum_{m \neq n} i \sqrt{\gamma_m} \tilde{a}_m \right),$$

(1)

where $\tilde{a}_n = a_n e^{j\omega t}$ represents the amplitude of the $n$th mode, $j$ is the unit imaginary number, $\omega_n$ is the resonance frequency of the $n$th mode, $\gamma_n$ is the corresponding radiative decay rate, and $\Gamma_n$ represents the dissipative decay rate related to the intrinsic loss, $\tilde{S}_l = S_l e^{j\omega t}$ represents the incident waves. The last term on the right-hand side of equation (1) indicates the interactions of the $n$th mode with the other modes, i.e. the global coupling effect. These $N$ modes amplitudes $\tilde{A} = (\tilde{a}_1, \ldots, \tilde{a}_N)^T$ evolve in time as $-j\frac{d}{dt} \tilde{A} = HA + \tilde{}$ with Hamiltonian

$$H = \begin{pmatrix} \omega_1 + j\gamma_1 + j\Gamma_1 & \cdots & j\sqrt{\gamma_1 \gamma_n} & \cdots & j\sqrt{\gamma_1 \gamma_N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ j\sqrt{\gamma_n \gamma_1} & \cdots & \omega_n + j\gamma_n + j\Gamma_n & \cdots & j\sqrt{\gamma_n \gamma_N} \\ \vdots & \cdots & \ddots & \ddots & \vdots \\ j\sqrt{\gamma_N \gamma_1} & \cdots & j\sqrt{\gamma_N \gamma_n} & \cdots & \omega_N + j\gamma_N + j\Gamma_N \end{pmatrix},$$

(2)
Figure 2. (a) Schematic diagram of transmission loss (TL) property as a function of mode density and frequency. \( \rho_{\text{mode}} \) varies from 2.4 to 8.8, \( \gamma_n \) and \( \Gamma_n \) are set to 0.03\( \omega \) and 0.06\( \omega \), respectively. The orange, dark cyan and green dashed lines represent typical conditions of sparse mode density, moderate mode density and intensive density, respectively, and their TL curves are shown in (b) in the corresponding colors.

The sound-attenuation performances of meta-silencers, evaluated by the TL in this study, can be calculated by

\[
\rho_{\text{mode}} = \frac{\sum_{n} \frac{1}{\log_{2}(\omega_{n,\text{max}}/\omega_{n,\text{min}})}}{N}
\]

where \( \omega_{n,\text{max}} \) and \( \omega_{n,\text{min}} \) are the maximum and minimum values of the resonant frequencies of these \( N \) modes, and \( \sigma = \frac{4\pi(\gamma_n + \Gamma_n)}{\omega} \).

Firstly, the effect of mode density on the global coupling is investigated in appendix B and the results show that the more intensive mode density can induce the stronger global coupling. Therefore, although the dispersion nature of resonances is inevitable, when the global coupling is strong enough, it may greatly suppress the dispersion of individual resonances and control the overall acoustic properties of the whole couple system. Following this revelation, we further investigate the effect of mode density on the TL.

Without loss of generality, we first theoretically consider a system with 16 coherently coupled modes, where the resonance frequency of the eighth mode is \( \omega_8 = 2\pi \times 400 \text{ Hz} \), the ratio of the resonance frequencies of two adjacent modes is \( \sigma \), i.e., \( \sigma = \frac{\omega_{n+1}}{\omega_n} \) \( (n = 1, 2, \ldots, 15) \). Figure 2(a) depicts the evolution of the TL curve as the variation of mode density, where the frequency band is selected between 200 and 800 Hz, \( \gamma_8 \) is 0.03\( \omega_8 \), \( \Gamma_8 \) is 2\( \gamma_8 \), and \( \sigma \) varies from 1.1 to 1.4, i.e., \( \rho_{\text{mode}} = \frac{1}{N} \log_{2}(\sigma) \) varies from 2.4 to 8.8, where \( \sigma = \frac{4\pi(\gamma_n + \Gamma_n)}{\omega} \) and \( N = 16 \). With the increase of mode density, the TL gradually increases, and the fluctuation of the curve decreases significantly. We select three typical values of mode densities to indicate the conditions of sparse (\( \rho_{\text{mode}} = 2.5 \)), moderate (\( \rho_{\text{mode}} = 3.9 \)) and intensive (\( \rho_{\text{mode}} = 7.4 \)) mode density, which correspond to the orange, dark cyan, and green dashed lines in figure 2(a). It is intuitive to see that when the system has a sparse mode density, the TL is low and its curve has large oscillations due to the dispersion nature of resonances (figure 2(b)). Nevertheless, when the system has a more intensive mode density, a higher and smoother TL curve can be achieved. These results manifest that the intensive mode density can offer new opportunities for manipulating sound in broadband against resonance dispersion. It is noteworthy that there exists a trade-off between the TL performance and the working frequency bandwidth regarding the mode density, which is, although the larger mode density consistently produces a higher level of TL performance, the working frequency band will become narrower when the mode density is at high values. Therefore, the mode density should be modulated considering both the suppression of resonance dispersion and working bandwidth. And in practice, the number of the
resonators cannot be infinitely increased due to the existence of resonators’ wall thicknesses, and this restricts the upper bound of the mode density supported by parallelly coupled resonators. As another effective technique for tuning mode density, adding porous materials such as metal foam can also enhance the operating frequency bandwidth of each resonance and consequently increase the mode density and suppress the resonance dispersion (see appendix C for details). In the following, we will judiciously modulate the global coupling of the meta-silencers to realize the designing targets of high-efficiency broadband sound attenuation, efficiency-controlled sound attenuation and designable timbre.

3. Results and discussion

In this section, three meta-silencers with different effects are provided to verify the theoretical model and demonstrate our design concept. The designing targets of the three meta-silencer are increasingly vigorous, and finally, a designable timbre is achieved by efficiently modulating the frequency-selective distribution of the mode density.

3.1. Broadband sound attenuation via meta-silencer

With the guidance of the design concept proposed above, a broadband sound-attenuation meta-silencer was designed with the goal of the possible highest TL within the target band. We set the target frequency band at 500–3200 Hz and restricted the thickness to a deep subwavelength scale, and then optimized the structural parameters to achieve the goal of making the average TL as large as possible, where the anticipated minimum TL is set to be greater than 10 dB. As shown in figure 3(a), the meta-silencer exhibits excellent attenuation performance from 500 Hz to 3200 Hz with an average TL of 28.4 dB. The maximum and minimum values are 85.7 dB at 1920 Hz and 10.0 dB at 500 Hz, respectively. The experimental results agree with the theoretical results in most trends, but some deviations are also observed, which may result from the fact that the high TLs are too easily affected by the background noise, the manufacturing errors, and the possible imperfect sealing condition of the experimental setup. In addition, figure 3(b) illustrates that most energy of the grazing acoustic waves is absorbed by the meta-silencer, and only a small part is reflected.

The overall thickness of the meta-silencer is 53.5 mm, which is only 1/13th of the wavelength at 500 Hz. It is noteworthy that the thickness of the meta-silencer is possible to be further reduced, such as by using thinner constructing walls (but enough to support an approximately acoustic solid boundary) and by reducing the excessive response outside the target frequency band [33]. The thicknesses of the perforated plate, the metal foam and coupled resonator array are 1 mm, 26 mm and 26.5 mm, respectively, and the perforated plate with a perforation rate of 28%, and a perforation diameter of 1 mm. The coupled resonator array is composed of 16 NEHRs in a 4 × 4 arrangement (as shown in figure 1(a)). The NEHRs share the same consistent thickness of 26.5 mm, while the other geometric parameters of each NEHR are allowed to be adjusted under a compact shape of the overall NEHR array to fulfill the design target. The detailed geometric parameters of NEHRs are demonstrated in appendix D. The acoustic resistance and reactivity of the meta-silencer are around 1 and 0 (figure 3(c)), respectively. It can be observed that there are only alleviated oscillations, indicating that the meta-silencer has significantly suppressed the resonance dispersion. Besides, in order to more intuitively show the attenuation capability of the meta-silencer, the distributions of sound pressure amplitudes |p| in the duct at four selected frequency points (the magenta star of figure 3(a)) are illustrated in figure 3(d). Theoretical results clearly demonstrate that the sound energy is strongly dissipated during the transmission along the meta-silencer.

3.2. Efficiency-controlled sound attenuation

In this subsection, we set a more strict design target that demands efficiency-controlled sound attenuation. This target is achieved via a meta-silencer with moderate mode density of the NEHRs but the strength of the global coupling of the overall system is enhanced by the metal foam. In other words, in this design, the metal foam plays a crucial role in controlling the sound-attenuation efficiency in broadband. As shown in figure 4(a), when without the metal foam, the TL curve exhibits intense fluctuations (gray line), which results from the moderate mode density of the NEHRs (16 NEHR modes in this broadband frequency range and ρ_mode = 3.4). As mentioned above, this moderate mode density is unable to support strong global coupling to suppress the dispersion of the resonances. Here, utilize the metal foam to ideally enhance the intrinsic loss (Γ_p) to majorly modulate the global coupling for suppressing the oscillations (can also see the demonstration in appendix C). After introducing the metal foam, the fluctuation of the TL curve becomes much more smooth and the sound-attenuation efficiency within 500–3200 Hz is considerably controlled at around 12 dB (green solid line in figure 4(a)). The total thickness of the meta-silencer is 43.4 mm (the inset of figure 4(a)), and the thicknesses of the perforated plate, metal foam and coupled resonator array thickness are 1 mm, 15 mm and 27.4 mm, respectively. The perforated plate has a perforation diameter of 1 mm and a perforation rate of 19.6%. Compared to a conventional silencer, the presented meta-silencer manifests several remarkable advantages, including more broadband working bandwidth with consistently high working efficiency, more controllable working efficiency, a more compact structure, and a higher degree of freedom of acoustic properties in broadband. These advantages would enable the presented meta-silencer to achieve improved sound-attenuation and wave-modulation performances when dealing with broadband sound.

Besides, as shown in figure 4(b), the impedance profile of this efficiency-controlled meta-silencer is distinct from the meta-silencer aiming for the possible highest TL shown in figure 3(c). And the much less fluctuating curves after introducing the metal foam also confirm the reduced dispersion.
Figure 3. Meta-silencer with high-efficiency attenuation performance. (a) Theoretical (lines) and experimental (circles) TL of the meta-silencer in 500–3200 Hz. (b) Theoretical (lines) and experimental (circles) transmission coefficients (in bright green), absorption coefficients (in red) and reflection coefficients (in blue) of the meta-silencer under grazing incidence of acoustic waves. The inset shows the picture of the experimental sample. The total thickness of the sample was $H_{tot} = 53.5$ mm, the thickness of the coupling resonator array $H_s = 26.5$ mm, and the thickness of the metal foam $H_f = 26$ mm. (c) Theoretical (lines) and experimental (circles) acoustic resistance (in orange) and reactance (in blue) of the meta-silencer. (d) The distribution of sound pressure amplitudes $|p|$ of the meta-silencer at each frequency point is marked by the magenta pentagrams in (a). The sound pressure amplitude data are calculated based on the theoretical model in appendix A.

of resonances. As demonstrated in the theoretical analysis and the two abovementioned proof-of-concept designs, both the approaches of increasing the number of resonators and employing suitable metal foam can effectively modulate the interactions of resonances. Comparing the two approaches, increasing the number of resonators has the major advantage of the high degree of freedom of resonance modulation but will suffer from a more complicated configuration with the increase of resonators. And employing the metal foam will not significantly complicate the configuration of the meta-silencer but its capability of modulating the resonant frequency is restricted. When facing a rather strict design target, the appropriate combination of the two approaches will greatly facilitate the designs of meta-silencers and improve the working performance.

3.3. Designable timbre

The two designs above validate our concept of utilizing intensive mode density to induce strong global coupling and then suppress the resonance dispersion. In this subsection, our goal is to manipulate sound, including alleviation, highlight and mute in different overtones for broadband, i.e. a designable timbre. To this end, the third meta-silencer is designed aiming to achieve 18 dB, 3 dB and 35 dB of TL at 500–930 Hz (the fundamental frequency), 930–1720 Hz (the first overtone) and 1720–3200 Hz (the second overtone), respectively. This target demands not only strong but also tunable global coupling over different overtone ranges.

Owing to the outstanding mode density modulation capacity of the presented meta-silencer, intensive and sparse mode densities supported by the NEHRs are designed for the fundamental frequency range and the first overtone range, respectively. For the second overtone range with the significantly high goal of sound attenuation, the strong co-work of the NEHRs, perforated plate and the metal foam provides an efficient solution for this challenging subject. As shown in figure 5(a), the theoretical and experimental results of TL fulfill the targets in different overtones with remarkable accordance. It is noteworthy that the target of the second overtone (35 dB of TL) practically requires a transmission coefficient as low as around 0.003, so a relatively slight variation of the transmission coefficient will lead to significant fluctuation of the corresponding TL curve, as shown in figure 5(a). Besides, the deviation between the experiment and theory in the high-frequency range shown in figure 5(a) may result from two factors. Firstly, the experiments were not conducted in an ideal anechoic room, and therefore parts of the emitted wave will be reflected and increase the background noise, causing significant influence on the measured data with smaller pressure amplitudes within...
Figure 4. Meta-silencer with efficiency-controlled sound attenuation. (a) The green line and the circles represent the theoretical and experimental TL of the meta-silencer, respectively. The dark grey solid line indicates the theoretical TL of the silencer without metal foam. The total thickness of the sample is $H_{\text{tot}} = 43.4$ mm (the inset picture), the thickness of the coupling resonator array $H_s = 27.4$ mm, and the thickness of the metal foam $H_f = 15$ mm. (b) Theoretical (lines) and experimental (circles) acoustic resistance (in orange) and reactance (in blue) of the meta-silencer. The dark grey solid line and the light grey solid line indicate the theoretical resistance and reactance of the meta-silencer without metal foam, respectively.

the high-frequency range. Secondly, the manufacturing errors and possible defects of the experimental setup will lead to relatively more obvious deviations for the high TL curve. Nevertheless, the aim of the ‘mute’ function can still be well achieved in the second overtone.

The overall thickness of the multifunctional meta-silencer is 44.4 mm, which consists of a 29.4 mm coupled resonator array, a 14 mm metal foam and a 1 mm perforated plate with a perforation rate of 8%. Here, the reason we call the meta-silencer ‘multifunctional’ is that this design could serve more than one functionality in different application environments, such as a traditional acoustic silencer for attenuating duct noise in noise-control engineering, a passive timbre-modulation device for orchestral instruments, and a reflection-type meta-silencer for adjusting the timbre of early reflected and reverberation sound (see the sample demonstration in appendix E).

The transmission curves of the components of the meta-silencer present an intuitive view of our design concept mentioned above. As shown in figure 5(b), it can be observed that the fundamental frequency band (500–930 Hz) has a very intensive mode density of NEHRs ($\rho_{\text{mode}} = 8.2$), and here the perforated plate and metal foam provide only trivial effect. The function of the first overtone band (930–1720 Hz) is set to be relatively highlighted with slight modulation, and therefore a void area of mode distribution ($\rho_{\text{mode}} = 0$) is designed to lift major restrictions of transmission of sound in this band. Within the second overtone band (1720–3200 Hz), the metal foam and perforated plates play a dominant role in the strong dissipation, and meanwhile, a few NEHR resonances contribute to the more sharp transformation of transmission property in the vicinity of the initial boundary in this target frequency band as well as enhance the energy dissipation.

In addition, the impedance profile of the meta-silencer also validates the extraordinary manipulation of resonances, where the intensive mode density enables the flat impedance curves and the void area of mode distribution arouses a giant antiresonance-induced fluctuation (figure 5(c)). Figure 5(d) intuitively exemplifies the effect of a designable timbre by presenting the pressure distribution at four selected frequencies. When acoustic waves pass through the meta-silencer, the timbre of the transmitted waves is tuned as our anticipations with the desirable effects of alleviation (600 Hz), highlight (1200 Hz) and mute (2400 Hz) at corresponding overtones. On the whole, guided by our design concept, the presented multifunctional meta-silencer efficiently modulates the global coupling and finally leads to the achievement of a designable timbre.
Figure 5. Designable timbre via a multifunctional meta-silencer. (a) Theoretical (lines) and experimental (circles) TL of the meta-silencer. The TL target (dashed line) of the meta-silencer in 500–930 Hz, 930–1720 Hz and 1720–3200 Hz are 18 dB, 3 dB and 35 dB, respectively, where the corresponding transmission coefficients are 0.016, 0.5 and 0.003 (black dashed line in (b)), respectively. The total thickness of the sample is $H_{\text{tot}} = 44.4$ mm (The inset picture). The thickness of the coupling resonator array is $H_s = 29.4$ mm. (b) Theoretical (line) and experimental (circles) transmission coefficients of the meta-silencer under grazing incidence of acoustic waves. The colorful dashed lines represent the transmission coefficients of the NEHRs. The dashed dot line (in grey) represents the transmission coefficients of the individual perforated plate backed with a metal foam depth of $H_f = 14$ mm. (c) Theoretical (lines) and experimental (circles) acoustic resistance (in orange) and reactance (in blue) of the meta-silencer. (d) The distribution of pressure amplitudes $|p|$ of the meta-silencer at each frequency point is marked by the magenta pentagrams in (a).

4. Conclusion

To summarize, we theoretically and experimentally investigate the physical mechanism of the global coupling in timbre modulation and present a multifunctional acoustic meta-silencer that can support intensive mode density as well as highly tunable intrinsic loss to validate our theoretical results and achieve the final goal of a designable timbre. The fundamental radiation and dissipation qualities of resonance are essentially linked with acoustic impedance as well as TLs by employing the coupled mode theory. The established theory reveals the effect of the distribution of the mode density on the strength of global coupling and offers a platform to efficiently design frequency-selective sound-attenuation performance in broadband. Three designs are presented to comprehensively manifest the merits of the delicate modulation of global coupling in designing a multifunctional meta-silencer. The first meta-silencer achieves an average TL of 28.4 dB in 500–3200 Hz with a deep sub-wavelength thickness of 53.5 mm. The second meta-silencer further achieves efficiency-controlled sound attenuation in 500–3200 Hz with a thickness of 43.4 mm. At last, the third designed meta-silencer, with a thickness of 44.4 mm, fulfills the rigorous target of a designable timbre in 500–3200 Hz by exerting alleviation, highlight and mute functionalities within the fundamental frequency band and overtone bands with controlled sound-attenuation efficiency.

This work offers fresh insights into the modulation of global coupling for suppressing resonance dispersion, and presents versatile and efficient ways to manipulate the mode density distribution, coupling effects and intrinsic loss via an acoustic meta-silencer. The presented design concept can also be employed to construct reflection-type meta-structures for a wider range of applications in room acoustics. These results would benefit the development of multifunctional and efficient acoustic silencers for aero-engine and ventilation systems, and open up an avenue for the study of designable timbre.

Appendix A. The mode matching method analyzes the TL of the impedance wall.

Without the meta-silencer, there are four sides of rigid walls, it is the square waveguide. The frequencies that we discuss here are under the cut-off frequency ((00) order). In other words, there is only the plane wave in the waveguide. The acoustic pressure is $p = Ae^{-j(kx + \gamma)} + Be^{-jk - \gamma}$ (neglecting $e^{j\omega t}$) with the dispersion relation $k = \omega/c_0$. $\omega$ is the circular frequency, $c_0$ is...
the speed of sound in the air. Whereas, when a test silencer is installed at one side wall of the sample part of the duct, the specific impedance at one side of the wall induces higher orders of z direction (perpendicular to impedance wall) wave modes, throughout the entire sample part and the neighbor of the joint between the rigid and impedance wall. Now the acoustic pressure in the part of the sample, front and back can be respectively expressed as

\[
p_{\text{sample}} = \sum_{m=1}^{\infty} \cos(k_m z) \left( A_m e^{-j k_m y} + B_m e^{j k_m y} \right) \quad (0 < y < L_{\text{sample}}),
\]
\[
p_{\text{front}} = \sum_{n} \cos(k_n z) \left( C_n e^{-j k_n y} + D_n e^{j k_n y} \right) \quad (y < 0),
\]
\[
p_{\text{back}} = \sum_{n} \cos(k_n z) \left( E_n e^{-j k_n y} + F_n e^{j k_n y} \right) \quad (y > L_{\text{sample}}).
\]

\(L_{\text{sample}} = 400 \text{ mm}\) is the length of sample part. With dispersion relation of \(k^2_{\text{cm}(n)} + k^2_{\text{ym}(n)} = k^2\). Wave number of z-direction \(k_{zm} = n\pi / W_{\text{tube}}\), \(n = 0, 1, 2, \ldots\). While in the sample part, it follows the impedance boundary condition \(p / v_c = Z\) (the detailed calculation has been mentioned in the previous article [53]) at the surface of impedance wall, \(v_i = -1 / \rho \frac{\partial p}{\partial x_i}, x_j = x, y, z\). The impedance boundary condition deduces a transcendental equation which shows the relation of \(k_{zm}\) and \(k_{ym}\), from which only the numerical solutions can be obtained. Now we respectively truncated the order of wave modes at \(n_{\text{Max}} = N, m_{\text{Max}} = M\) in order to conduct numerical calculation with acceptable precision. According to the continuity of sound pressure and normal velocity at \(y = 0\)

\[
p_{\text{front}} = p_{\text{sample}},
\]
\[
v_{c, \text{front}} = v_{c, \text{sample}}.
\]

Two sides of equation (A4) are multiplied by each function of the series \(\{\cos(k_{zm} z)\}\), and later integrated over the square cross-section or only its y-direction width. Ultimately, we exchange the order of integral and sum operations and obtain

\[
\sum_n \left( C_n + D_n \right) \left( \int_0^{W_{\text{tube}}} \cos(k_n z) \cos(k_{zm} z) \, dz \right)
\]
\[
= \sum_m \left( A_m + B_m \right) \left( \int_0^{W_{\text{tube}}} \cos(k_m z) \cos(k_{zm} z) \, dz \right).
\]

For simplicity, we define

\[
C = [C_0, C_1, \ldots, C_N]^T,
\]
\[
D = [D_0, D_1, \ldots, D_N]^T,
\]
\[
A = [A_0, A_1, \ldots, A_m, \ldots, A_M]^T,
\]
\[
B = [B_0, B_1, \ldots, B_m, \ldots, B_M]^T.
\]

Thus, we get a matrix equation

\[
P_{\text{IL}} (\mathbf{C} + \mathbf{D}) = P_{\text{IR}} (\mathbf{A} + \mathbf{B}). \tag{A9}
\]

Also, we multiply equation (A5) by \(\{\cos(k_{zm} z)\}\) \(k_{ zm} = n\pi / W_{\text{tube}}\) and repeat the same operations to get

\[
V_{\text{IL}} (\mathbf{C} - \mathbf{D}) = V_{\text{IR}} (\mathbf{A} - \mathbf{B}), \tag{A10}
\]

with

\[
P_{\text{IL}} = \begin{bmatrix}
P_{\text{IL}}(j\mu_0,\lambda_0) & \cdots & P_{\text{IL}}(j\mu_0,\lambda_N) \\
\vdots & \ddots & \vdots \\
P_{\text{IL}}(j\mu_N,\lambda_0) & \cdots & P_{\text{IL}}(j\mu_N,\lambda_N)
\end{bmatrix},
\]
\[
P_{\text{IR}} = \begin{bmatrix}
P_{\text{IR}}(j\mu_0,\lambda_0) & \cdots & P_{\text{IR}}(j\mu_0,\lambda_N) \\
\vdots & \ddots & \vdots \\
P_{\text{IR}}(j\mu_N,\lambda_0) & \cdots & P_{\text{IR}}(j\mu_N,\lambda_N)
\end{bmatrix}
\]

\[
V_{\text{IL}} = \begin{bmatrix}
V_{\text{IL}}(j\mu_0,\lambda_0) & \cdots & V_{\text{IL}}(j\mu_0,\lambda_N) \\
\vdots & \ddots & \vdots \\
V_{\text{IL}}(j\mu_N,\lambda_0) & \cdots & V_{\text{IL}}(j\mu_N,\lambda_N)
\end{bmatrix},
\]
\[
V_{\text{IR}} = \begin{bmatrix}
V_{\text{IR}}(j\mu_0,\lambda_0) & \cdots & V_{\text{IR}}(j\mu_0,\lambda_N) \\
\vdots & \ddots & \vdots \\
V_{\text{IR}}(j\mu_N,\lambda_0) & \cdots & V_{\text{IR}}(j\mu_N,\lambda_N)
\end{bmatrix}
\]

and

\[
V_{\text{IL}}(j\mu,\lambda) = \int_0^{W_{\text{tube}}} k_0 \cos(k_0 y) \cos(k_{zm} z) \, dz
\]
\[
V_{\text{IR}}(j\mu,\lambda) = \int_0^{W_{\text{tube}}} k_0 \cos(k_0 y) \cos(k_{zm} z) \, dz.
\]

Similarly, with the acoustical continuity conditions at \(y = L_{\text{sample}}\)

\[
p_{\text{sample}} = p_{\text{back}},
\]
\[
v_{y, \text{sample}} = v_{y, \text{back}}.
\]

another two matrix equations are got

\[
P_{\text{IR}} (\mathbf{M} + \mathbf{M} - \mathbf{B}) = P_{\text{IL}} (\mathbf{N} + \mathbf{E} + \mathbf{N} - \mathbf{F}),
\]
\[
V_{\text{IR}} (\mathbf{M} + \mathbf{M} - \mathbf{B}) = V_{\text{IL}} (\mathbf{N} + \mathbf{E} + \mathbf{N} - \mathbf{F}).
\]
with
\[
E = [E_0, E_1, \ldots, E_n, \ldots, E_N]^T
\]
\[
F = [F_0, F_1, \ldots, F_n, \ldots, F_N]^T,
\]
\[
P_{2R} = P_{1R} P_{2L} = P_{1L}
\]
\[
V_{2R} = V_{1R} V_{2L} = V_{1L}
\]
\[
M_{\pm} = \text{diag}(e^{\mp \beta_m N \omega_m}, e^{\mp \beta_m N \omega_m}, \ldots, e^{\mp \beta_m N \omega_m})
\]
\[
N_{\pm} = \text{diag}(e^{\mp \beta_m N \omega_m}, e^{\mp \beta_m N \omega_m}, \ldots, e^{\mp \beta_m N \omega_m}).
\]
\[\text{A14}\]
\[\text{A15}\]

Now, we have a larger matrix equation by changing a little the equations (A9), (A10) as well as (A13) and combining them together
\[
P_{1R} A + P_{1R} B - P_{1L} D = P_{1L} C
\]
\[\text{A16}\]

With zero reflection of duct end or known reflection \( F = \text{RE} \) (\( R \) may be \( O \)), equation (A16) is rewritten in the form of matrix equation
\[
\begin{bmatrix}
P_{1R} & P_{1R} & -P_{1L} & O  \\
V_{1R} & -V_{1R} & V_{1L} & O  \\
P_{2R} M_+ & P_{2R} M_- & O & -P_{2L} (N_+ + N_- R)  \\
V_{2R} M_+ & -V_{2R} M_- & O & -V_{2L} (N_+ - N_- R)
\end{bmatrix}
\times
\begin{bmatrix}
A  \\
B  \\
C  \\
D  \\
E
\end{bmatrix}
= \begin{bmatrix}
P_{1L} C  \\
V_{1L} C  \\
O  \\
O
\end{bmatrix}.
\]
\[\text{A17}\]

In experiment, the reflection of end \( R \) is very close to zero on account of the well-designed mufflers, and it can also be got by conducting two different end conditions in case of lacking of muffler.

The amplitudes of each order \((A, B, D, E)\) can be calculated by solving equation (A17), with a given incident acoustic pressure \( C \).

The acoustic energy flow is defined as \( \frac{1}{2} \text{Re}(p \cdot v^*) \), and a trick for calculating it is to sum the energy flow of all orders. So far, we can obtain the incident, reflection and transmission energy flow \((I_{in}, I_{re}, I_{tr})\) as well as the coefficient of reflection, transmission and absorption and the TL \((r, t, \alpha, TL)\)
\[
r = I_{re}/I_{in},
\]
\[
t = I_{tr}/I_{in},
\]
\[
\alpha = 1 - r - t,
\]
\[
TL = 10 \log \left( \frac{1}{t} \right).
\]
\[\text{A18}\]
perfect acoustic energy dissipation, conversely, by constructing a very sparse mode density for almost lossless passage of acoustic energy. In this manner, the dissipation and propagation of sound waves can be effectively adjusted, thus enabling arbitrary designable timbres.

**Appendix C. Modulation of intrinsic loss to the TL curve**

In this appendix we demonstrate the modulation of intrinsic loss on the TL of a system with moderate mode density to demonstrate the necessity of introducing metal foam into our configuration. As shown in figure C1, we select the frequency band between 200 and 800 Hz, \( \gamma_n = 0.03\omega_n \) and the moderate mode density \( \rho_{\text{mode}} = 3.9 \). By adjusting the magnitude of intrinsic loss to observe the variation of the TL curve. It can be clearly seen that when the intrinsic loss increases, the oscillation of the TL curve is greatly reduced, and the trough gradually disappears. However, to achieve flexible control of intrinsic loss, a better approach is to introduce resistant materials such as metal foam into our configuration. This proves that employing metal foam to provide additional intrinsic loss is a very effective way to pursue a flat TL curve. At the same time, due to the requirement for mode density being reduced, we can utilize fewer coupled resonators to achieve broadband sound attenuation, thereby reducing the complexity of the structure. In addition, the metal foam has excellent mechanical properties and good stability in high and low temperature environments, which is difficult for traditional sponges to achieve. This also makes the meta-silencer more robust in a variety of extreme environments.

**Appendix D. Geometric parameters of coupled resonator array in meta-silencer**

In this study, the coupled resonator arrays of the presented meta-silencers are constructed by coplanar and compact NEHRs. The geometric parameters of the coupled resonator arrays (16 NEHRs) corresponding to the meta-silencers presented in figures 3(b), 4(a) and 5(a) will be elaborated in this appendix. Here, the wall thickness of all the samples is \( t_s = 1 \) mm. The walls are considered acoustically rigid boundaries in theory. For the first presented design in figure 3(b), it has \( d_c = [1.7, 2.4, 2.2, 2.3, 5.8, 2.6, 1.8, 2.1, 5.9, 4.1, 5.9, 5.9, 6.2, 2.2, 1.4, 2.3] \text{ mm} \), \( l_c = [11.3, 17.7, 15.7, 10.2, 1.0, 8.6, 3.3, 8.8, 1.0, 4.8, 1.0, 1.0, 5.0, 8.4, 4.9, 1.1] \text{ mm} \), \( a = [9.2, 9.2, 9.2, 9.9, 9.9, 9.9, 7.9, 7.9, 7.9, 17.9, 17.9, 17.9, 17.9, 17.9, 17.9] \text{ mm} \), \( b = [11.1, 13.4, 6.2, 14.4, 8.1, 13.3, 4.1, 19.6, 7.9, 12.6, 7.9, 16.7, 19.1, 6.6, 5.8, 13.6] \text{ mm} \). In addition, for the second presented design in figure 4(a), it has \( d_c = [2.9, 3.9, 3.0, 2.1, 2.3, 2.2, 3.1, 3.0, 1.7, 1.1, 2.0, 5.3, 2.4, 3.5, 3.1, 4.0] \text{ mm} \), \( l_c = [6.7, 18.6, 7.6, 1.7, 2.7, 2.4, 17.2, 16.4, 2.2, 1.1, 9.0, 16.7, 7.9, 8.0, 9.1, 13.1] \text{ mm} \), \( a = [6.1, 6.1, 6.1, 6.1, 6.4, 6.4, 6.4, 13.0, 13.0, 13.0, 13.0, 19.5, 19.5, 19.5, 19.5, 19.5] \text{ mm} \), \( b = [6.5, 5.9, 8.0, 24.5, 5.3, 4.2, 21.7, 13.9, 3.7, 3.3, 14.4, 23.5, 4.4, 5.5, 18.0, 17.0] \text{ mm} \). And the geometric parameters of the NEHRs of the third design in figure 5(a) are \( d_c = [4.5, 3.9, 1.0, 2.1, 1.3, 2.6, 1.1, 3.0, 1.9, 1.1, 2.5, 5.3, 2.6, 2.7, 3.1, 3.5] \text{ mm} \), \( l_c = [1.0, 1.5, 1.9, 9.4, 4.8, 2.8, 1.5, 12.5, 6.8, 6.8, 1.9, 6.3, 18.0, 7.3, 9.8, 5.2, 13.3] \text{ mm} \), \( a = [6.5, 6.5, 6.5, 6.5, 6.3, 6.3, 6.3, 12.9, 12.9, 12.9, 12.9, 19.2, 19.2, 19.2, 19.2, 22.4, 5.9, 4.6, 20.8, 13.6, 4.8, 17.3, 13.9, 21.7, 5.1, 5.3, 19.2, 15.5] \text{ mm} \).

**Appendix E. Reflection-type meta-silencer**

The design concepts and techniques presented in this work can be utilized to achieve a designable timbre based on reflection. To demonstrate this, we have theoretically designed a reflection-type meta-silencer with the aim of absorption coefficients of 0.6, 0.2 and 0.9 in the range of 500–930 Hz, 930–1720 Hz and 1720–3200 Hz, respectively (figure E1). The thickness of the perforated plate, metal foam and coupled resonator array is \( t_p = 1 \) mm, \( H_1 = 10 \) mm and \( H_2 = 24.8 \) mm, respectively. The perforated plate has a perforation diameter of 1 mm and a perforation rate of 2%. The geometric parameters of each NEHRs are as follows: \( d_c = [4.5, 3.9, 1.0, 2.1, 1.3, 

---

**Figure C1.** TL curves when the intrinsic loss \( \Gamma_n \) is equal to \( \gamma_n \) (in orange), \( 3\gamma_n \) (in dark cyan) and 5\( \gamma_n \) (in green), respectively, where the frequency band is selected at 200–800 Hz, the radiation loss \( \gamma_n = 0.03\omega_n \), and the mode density \( \rho_{\text{mode}} = 3.9 \).

**Figure E1.** Theoretical absorption coefficients of the reflection-type meta-silencer with thicknesses of 35.8 mm.
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