Stochastic Geometric Analysis of the Terahertz (THz)-mmWave Hybrid Network With Spatial Dependence

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ABSTRACT The Terahertz (THz) band (0.1–10 THz) contains abundant spectrum resources that can offer ultra-high data rates. Despite these potential benefits, the adoption of THz communication has been stagnant until very recently due to the poor penetrability and limited coverage of the THz links. To overcome the aforementioned obstacles and take full advantage of the THz band, we introduced a hybrid network consisting of THz and millimeter-wave (mmWave) nodes deployed within a finite area. Furthermore, the mmWave nodes are spatially distributed by a Poisson Point Process (PPP), whereas the THz nodes are clustered around the mmWave nodes, forming a Poisson Cluster Process (PCP) with the parent process of mmWave tier. We derive the Laplace transform of the interference in a closed form and evaluate the coverage probability (CP) based on the maximum biased power (Max-BRP) association strategy. The proposed framework provides insights into how the spatial dependence between THz and mmWave tier and clustering setting affects network performance. We quantitatively reveal its impact on the network performance. It is revealed that this inter-tier spatial dependence introduces the flexibility of nodes deployment by tuning the scattering variance and the number of nodes per cluster. Furthermore, we use numerical simulation to demonstrate the significant impact of bias ratio, density, and blockage on the CP, indicating the importance of choosing the optimal combination of the parameters.

INDEX TERMS Terahertz communication, stochastic geometry, Poisson cluster process, THz-mmWave hybrid network.
meteorological factors [3], e.g., atmospheric gases and fog, etc. Third, directional antennas are utilized due to the power constraint, generating highly directive THz transmission. Given the above, THz communication has the characteristics of high data rate, strong directivity but low penetrability, and limited coverage. A promising solution to overcome these drawbacks is densely deploying many THz small cells with highly directional antennas.

Despite the limited coverage, THz communications are envisioned as a promising application for future wireless networks. Nevertheless, the stand-alone deployment of THz base stations (TBSs) is not sufficient to achieve ubiquitous coverage. A compromised but reliable strategy is to deploy TBSs as part of a THz-mmWave hybrid network. A hybrid network can adaptively select between TBSs and mmWave base stations (MBSs) based on their link quality and association strategy, resulting in a better link quality for the deployed network. Specifically, a UE is associated with TBS if the THz links outperform the mmWave links. Alternatively, the UE is associated with MBS if the mmWave channel provides a better link quality than THz. Due to the limited transmission range of the THz band, the traditional PPP-based model is not realistic. Therefore, we assume that TBSs are clustered around MBSs, resulting in an in-tier spatial dependence between TBSs and MBSs. The proposed network model aligns well with the cluster configuration introduced in the 3rd Generation Partnership Project (3GPP) simulation model [4], thus providing more practical significance to future clustered and hybrid networks. Moreover, indoor communication plays an important role in THz blockage model and THz networks [5], [6], [7]. Due to the clustering setting, the deployment area will affect the network performance. The impact of the finite spatial constraints in PCP network is also investigated.

We aim to model and analyze the hybrid networks consisting of TBSs and MBSs in a finite area using stochastic geometry. To our best knowledge, this is the first paper to analyze the clustering effect and spatial dependence of THz-enabled HetNet. The main contribution is summarized below.

- We demonstrated that the inter-tier spatial dependence introduced by the proposed PCP-based framework will achieve a higher CP and provides flexibility by tuning scattering variance $\sigma^2$, mean number of nodes per cluster $\bar{c}_T$ and bias ratio to combat the blockage effect. The results show that the proposed framework will outperform the independent PPP-based network with suitable parameters. The gain from the inter-tier spatial dependence is particularly significant when the deployment area is limited. However, the CP will gradually decrease to that of PPP-based network as the deployment area increases. Moreover, the results also show that the proposed framework has a better capacity of combating blockage than the independent PPP-based framework.

The rest of this paper is organized as follows. In Sec II, we will review the previous works about THz networks and hybrid networks. In Sec III, the network topology and system model will be elaborated. Sec IV will introduce a framework to evaluate the CP. In Sec V, we present the numerical results and then conclude the paper in Sec VI.

II. RELATED WORKS

The THz band is envisioned as the key enabler of the next-generation wireless communication system. Various previous works have investigated the THz propagation characteristics and THz network performance. In [2], the authors performed a comprehensive study on the THz channel model with the numerical investigation of the propagation characteristics, i.e., the molecular absorption loss, severe path loss, and molecular absorption noise. The mathematical framework of the THz channel model was proposed, and the channel capacity was evaluated for different medium compositions. The results revealed a significant impact of molecular absorption loss on link quality by water molecules, demonstrating the importance of incorporating absorption loss in THz channel modeling. Apart from the severe path loss, blockage also has a significantly detrimental impact on THz communication. The blockage function and the blockage effect in the PPP-based network have been investigated in [6], [7], and [8]. To combat the severe blockage effect and propagation loss of THz communication, the THz nodes have to be deployed densely in previous works in order to shorten the distance between typical UE to tagged BS. In this work, we want to combat the blockage at the nodes deployment level by taking the advantage of the clustering setting of THz nodes.

Stochastic geometry is widely utilized to evaluate the performance of THz cellular networks and THz-supported networks. In [9], the THz networks were modeled by the PPP, and the aggregated interference and SINR were evaluated. The proposed model considered the THz channel characteristics, e.g., the molecular absorption loss, severe path loss of THz, molecular absorption noise, and the Johnson-Nyquist (JN) noise with simplified directional antenna pattern and blockage probability. In [10], the authors adopted the channel model in [2] and focused on the interference power and outage probability of THz networks. However, there is no
investigation of the PCP-based THz network to the best of our knowledge. It is pointed out in [11] that the conventional PPP-based network without spatial correlation will lead to a deviation from practical results. Moreover, the PPP-based model cannot represent the THz network which has the nature of clustering trend [12]. It is also pointed out that in the multi-tier networks, the locations of ultra-cell small base stations (BS) present a spatial clustering profile, which requires a more realistic stochastic geometric model.

Considering the current wireless network infrastructure, it is feasible that the THz networks will be overlaid on top of existing mmWave networks. Additionally, the hybrid deployment can complement the THz communication and enhance the overall network coverage. The THz-mmWave hybrid network is a two-tier heterogeneous network (HetNet) with zero inter-tier interference across THz and mmWave band. Several works considered a hybrid network of mmWave and radio frequency (RF) bands. In [13], a hybrid network of mmWave and sub-6 GHz was investigated, where both the mmWave and sub-6 GHz tiers were modeled as PPP networks. Decoupled association strategy was adopted in [13], where a UE will independently connect to a mmWave BS or a sub-6 GHz BS on both the uplink and downlink. The authors numerically analyzed the association strategies, SINR CP, and rate. In [14], a THz-sub-6 GHz PPP-based HetNet was investigated to characterize the overall handoff probability. A THz-mmWave PPP-based HetNet is investigated in [15] with the simplified LOS ball blockage model which can not capture the clustering trend of THz nodes. A similar framework was considered in [16] where mmWave and sub-6 GHz tier were modeled as a PCP and a PPP, respectively. Compared to other prior works that modeled the spatial distribution of UE as PPP, the authors of [16] assumed independent PCP for the UEs distribution. In [17], the authors analyzed the hybrid network from a general perspective and proposed the rate coverage expression. Furthermore, the authors justified that this type of HetNet can be modeled with clustered point processes such as the Neyman-Scott process, where the sub-6 GHz BSs will be located at the center of clusters while the mmWave BSs will be deployed around the central sub-6 GHz BSs. In [18], the heterogeneous cellular networks consisting of multi-tier networks with flexible cell association sharing the same spectral band were investigated. Unlike the proposed THz-mmWave hybrid networks, inter-tier interference has to be considered in this heterogeneous network. A similar hybrid network composed of THz and RF networks was proposed in [19], which utilized the THz propagation characteristics and simplified directional antenna patterns. A closed-form expression of aggregate interference was derived to evaluate the CP of THz networks. The opportunistic and hybrid association strategies were investigated in [19]. A UE adopting the opportunistic association scheme connects to the BS with maximum biased received signal power. In contrast, a UE adopting the hybrid association scheme can maintain a connection to both RF and THz BS.

In [20], the authors investigated a cache-enabled two-tier THz HetNet, where both tiers are modeled as independent PPP for the sake of tractability. A HetNet consisting of THz, mmWave and sub-6GHz tier is investigated in [21]. Although all the tiers are modeled by independent PPPs, the typical UE is assumed to be served by a cluster of BSs. However, this user-centric dynamic BS clustering design can not capture the inter-tier attraction, which is more realistic for THz nodes. The THz-enabled unmanned aerial vehicles (UAVs) HetNet is investigated in [22], where all the tiers are modeled as independent PPPs. In general, all the previous works adopted the independent framework for the sake of tractability while ignoring the dependence between the THz tier and the low-frequency tier.

To summarize, evaluating the performance of the PCP-based THz network is still an open problem. Furthermore, to the best of our knowledge, there is so far no investigation on the THz-mmWave hybrid networks considering inter-tier dependence, which is of more practical significance than the independent PPP-based framework. In this paper, we aim to reveal the impact of different parameters on the performance of PCP-based hybrid networks and their capacity to combat the blockage effect. The closed-form expression of the Laplace transform of interference in the inhomogeneous HetNet will be derived. Finally, the CP will be analyzed to evaluate the network’s performance.

### III. SYSTEM MODEL

#### A. NEYMAN-SCOTT CLUSTER PROCESS

Neyman-Scott cluster process [23] is a type of Poisson cluster process that is formed by applying homogeneous independent clustering to a stationary PPP, namely the parent point process. Specifically, we define a PPP $\Phi_p$ of density $\lambda_p$ as the parent process, denoted as $\Phi_p = \Phi(\lambda_p)$. Then, the clusters can be expressed as $N^c = N + c_i$ for $c_i \in \Phi_p$, where the $N$ denotes a finite set of independent and identically distributed (i.i.d.) offspring points with probability density function (PDF) $f(x)$, and $+c_i$ means shifting the cluster center to $c_i$. The number of offspring points in $N^c$ is a Poisson random variable (r.v.) with mean $\tilde{c}$. Therefore, the Neyman-Scott cluster process is the union of offspring point process translated by $c_i$ for $c_i \in \Phi_p$. Note that the cluster centers $c_i$ are excluded from the cluster process. Using the notations from [24], we define the Neyman-Scott cluster process as

$$\Phi(\lambda_p, f(x), \tilde{c}) = \bigcup_{c_i \in \Phi_p} N^c_i. \quad (1)$$

Two special cases of Neyman-Scott cluster process are considered; namely, Matern cluster process (MCP) and Thomas cluster process (TCP). For the MCP, the offspring points are uniformly distributed in a ball of radius $R$ around the origin.

$$f(x) = \begin{cases} \frac{1}{\pi R^2} & \text{if } ||x|| \leq R \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$
TABLE 1. Notation and simulation parameters.

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<td>0.06, 0.01</td>
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</table>

For the TCP, the offspring points are distributed around the origin by a symmetric normal distribution with variance $\sigma^2$:

$$f(x) = \frac{1}{2\pi \sigma^2} \exp\left(-\frac{||x||^2}{2\sigma^2}\right),$$

(3)

where $|| \cdot ||$ denotes the Euclidean norm.

B. NETWORK TOPOLOGY

This work considers a two-tier hybrid network consisting of TBSs and MBSs in a finite area with radius $R_a$, communicating to UEs over downlink channels. To achieve flexible and extended coverage, we assume TBSs are scattered around MBSs, as illustrated in Fig. 1. Specifically, we assume MBSs are modeled as a PPP as $\Phi_m = \Phi(\lambda_m)$ and the TBSs are modeled as a Neyman-Scott cluster process with parent process of $\Phi_m$ and $\lambda_T = \lambda_m \cdot \tilde{c}_T$, where $\tilde{c}_T$ is the average number of nodes per cluster for THz networks, i.e., $\Phi_T = \Phi(\lambda_m, f(x), \tilde{c}_T)$. We assume the UEs are distributed as a two-dimensional independent PPP $\Phi_u$ with density $\lambda_u$. Without loss of generality, the typical UE is assumed to be located at the origin. To combat the severe pathloss, we assume all the TBSs and MBSs are equipped with highly directional antennas with $N_T$ and $N_m$ elements. In contrast, the UEs are equipped with the omnidirectional antenna with antenna gain $G_u = 1$.

C. DIRECTIONAL ANTENNA PATTERN

Highly directional antennas are indispensable for both mmWave and THz links to combat severe propagation loss. When considering the analog beamforming and uniform linear array antenna [25], [26], the commonly adopted antenna pattern model for analysis is the simplified flat-top antenna model, which binarized the antenna gain into two levels to improve tractability but at the cost of a low approximation accuracy and lack of insights to reveal the diverse impact of the directional antenna pattern [27]. To resolve this issue, we utilized the MLFT antenna pattern, which was proposed in [28] and demonstrated to achieve a close approximation to the actual antenna pattern while maintaining a similar tractability level as the simplified flat-top model. The antenna gain function $G(\varphi)$ of the MLFT model with $N$ elements is given by

$$G(\varphi) = \begin{cases} G_k, & \text{if } \varphi_k - \frac{\psi}{2} \leq |\varphi| < \varphi_k + \frac{\psi}{2}, \\ 0, & \text{otherwise}, \end{cases}$$

(4)

where $K = \left\lfloor \frac{N}{2} \right\rfloor$, $1 \leq k \leq K$, $\varphi$ is the cosine direction of the angle of departure (AoD), $\varphi_k = \frac{2k-1}{2N}$, $\varphi_1 = \frac{\psi}{2}$, $G_1 = N$, and $\psi$ is the half-power beamwidth (HPBW) defined by the actual antenna pattern $G_{\text{act}}(\varphi)$ as follows

$$G_{\text{act}}(\varphi) = \frac{\sin^2(\pi N \varphi)}{N \sin^2(\pi \varphi)}, \quad G_{\text{act}}(\psi) = \frac{N}{2},$$

(5)

and $G_k = G_{\text{act}}(\varphi_k)$ for $k \geq 2$. We adopt the MLFT pattern to both TBSs and MBSs, whose antenna gains are $G_T$ and $G_m$, respectively. It is assumed that the BSs serve the associated UE with perfect beam alignment, which is a commonly
adopted assumption in [6], [7], [10], and [27]. Note that the beam steering algorithm plays a vital role in high-frequency communication, especially in THz communication, where a fast beam alignment is required to reduce the communication delay. This is left for future work.

D. BLOCKAGE MODEL

Unlike lower frequency signals, mmWave and THz links are susceptible to obstacles like human bodies or walls. Therefore, a communication link can be either a LOS link or a non-LOS (NLOS) link due to the severe penetration loss. Notably, for both mmWave and THz signals, the user itself will block the BSs within a cone with angle $\omega_s$. This blockage effect is referred to as self-blockage [8], [29], [30]. Formally, assuming the orientation of users is uniformly distributed in $[0, 2\pi]$, the probability that a BS is blocked due to self-blockage is $\frac{\omega_s}{2\pi}$. Here we define $p_r = 1 - \frac{\omega_s}{2\pi}$. We assume that self-blockage will completely block the signals, i.e., causing infinite penetration loss. Note that it is validated in [29] that the infinite penetration loss assumption can accurately model the severe attenuation loss in terms of CP.

Additionally, the signals will be blocked by other static obstacles, like walls, buildings, and human bodies. For mmWave signals, we adopt the commonly used exponential blockage model proposed in [31]. Specifically, a mmWave link with length $r$ is a LOS link with a probability of $e^{-\beta_m r}$, where $\frac{1}{\beta_m}$ is the average LOS distance which is determined by the density and size of blockers. Naturally, the NLOS probability of a mmWave link is given as $1 - e^{-\beta_m r}$. Furthermore, the static blockage has a more detrimental effect on THz communication. The THz signals will be attenuated severely passing through obstacles. The impact of the static blockers is investigated in several works [6], [7], [8], [32]. In this paper, we consider a finite indoor scenario where the blockers is investigated in several works [6], [7], [8], [32]. We assume that self-blockage and static blockage, the LOS probability function of blockers and walls. Therefore, combining the self-blockage with parameter $\kappa$ and static blockage, the LOS probability function of THz links is given as $1 - e^{-\beta_m r}$, where $\beta_T$ is a constant determined by the density and size of blockers. Naturally, the NLOS probability function is $p_{\alpha_T}^N(r) = 1 - p_{\alpha_T}^L(r)$. Note that only the LOS THz links are considered due to their poor penetrability.

E. PROPAGATION MODEL

For a mmWave link, the received signal power at the typical UE from an MBS at location $x_m$ can be expressed as $P_mG_m(\phi_m|h_mL_m(x_m)$, where $P_m$ is the transmitting power, $G_m(\phi_m)$ is the antenna gain of MBS with AoD $\phi_m$. We adopt Nakagami-m fading with parameter $\kappa$ to model the small scale fading of mmWave links, where $\kappa \in \{L, N\}$ indicating LOS or NLOS link, i.e., $h_m \sim \text{Gamma}(\kappa, 1/M)$. The pathloss term $L_v(x_m) = (c/4\pi f_m)^2 x_m^{-\alpha_v}$ where $\alpha_v$ denote the pathloss exponent for LOS and NLOS links.

Similarly, THz propagation undergoes severe attenuation, including propagation loss, absorption loss, molecular absorption loss, and Johnson-Nyquist noise [2], [7], [9]. We denote the propagation loss over the THz link as $L_P(x_T) = (\frac{4\pi f_T}{c})^2 x_T^{-\alpha_T}$, the molecular absorption loss as $L_A(x_T) = e^{-k_d(x_T)}$, the absorption noise as $N_A = PTGL_P(x_T)(1 - L_A(x_T))$, and the JN noise as $N_{JN} = \frac{2k_B}{\hbar\nu} \ln(1 + \frac{\nu}{\nu_T})$, where $f_T$ is the operating frequency of the THz link, $\alpha_T$ is the pathloss exponent, $h$ denotes the Planck’s constant, $k_B$ is the Boltzmann constant and $T$ represents the temperature in Kelvin. The JN noise remains constant until 0.1 THz, then declines non-linearly. We utilize the simplified absorption model [33] to represent the absorption loss $L_A(x_T)$ for the frequency range of 275-400 GHz.

The received power at the typical UE from a TBS at location $x_T$ is evaluated as $S = PTGL_T(x_T)L_P(x_T)h_T$, where $G_T(\phi_T)$ is the antenna gain with AoD $\phi_T$, $h_T$ denotes the small scale fading. In [6], [7], [35], and [36], Nakagami-m fading has been demonstrated as a tentative model to approximate the small-scale fading of THz link. We follow this approach and model the power fading coefficient $h_T$ as a Gamma r.v. with parameter $M_T$, i.e., $h_T \sim \text{Gamma}(M_T, 1/M_T)$.

Therefore, denoting the associated BS as $x_0$, the received SINR at the typical UE in mmWave links is given by

$$\text{SINR}_m(x_0) = \frac{P_mNmh_m \left( \frac{c}{4\pi f_m} \right)^2 x_0^{-\alpha_L}}{\sigma_m^2 + I_m},$$

where $\sigma_m^2$ refers to the noise power of mmWave band, $I_m = \sum_{x \in \Phi_m} P_mG_m(\phi_i|h_iL_s(x))$ is the interference. Here, $\Phi_m$ and $\Phi_m'$ denote the point process formed by interfering LOS MBSs and NLOS MBSs. Similarly, the received SINR at the typical user in THz links is given by

$$\text{SINR}_T(x_0) = \frac{S}{P_N + I_T} = \frac{PTNL_TE^{-k_d(x_T)}h_T \left( \frac{c}{4\pi f_T} \right)^2 x_T^{-\alpha_T}}{P_N + I_T},$$

where $P_N$ is the noise power of THz band and $I_T$ is the aggregated interference, which can be expressed as below.

$$I_T = \sum_{x \in \Phi_T} G_T(\phi_T)PTL_A(x)LP(x)h_T$$

$$P_N = N_{JN} + \sum_{x \in \Phi_T} G_T(\phi_T)PT(1 - L_A(x))LP(x)h_T$$

IV. NETWORK ANALYSIS

In this section, we elaborate on the mathematical analysis of the proposed hybrid network from the perspective of distance distribution, association probability, and CP.

1We refer the interested readers to the HTRAN database [34] for a more accurate model of the molecular absorption loss $L_A(x_T)$.
A. DISTANCE DISTRIBUTION

Due to the blockage effect, the point process of each tier is divided into LOS part \( \Phi^L \) and NLOS part \( \Phi^N \), i.e., \( \Phi_\kappa = \{ \Phi^L_\kappa, \Phi^N_\kappa \} \) where \( \kappa \in [T, m] \). Denote \( d_m, d_T \) as the distance of the closest LOS MBS and closest LOS TBS, respectively. A vital step is to derive the distribution of \( d_m, d_T \). Since MBS are distributed following homogeneous PPP, taking into account the self-blockage and static blockage, the LOS MBS \( \Phi^L_m \) form an inhomogeneous PPP with density of \( \lambda^L_m(r) = \lambda_m p_r e^{-\beta_m r} \). Therefore, the distribution of \( d_m \) is given in [17] and [25] as

\[
\text{PDF: } f_{d_m}(r) = 2\kappa_m \pi \lambda_m p_r r \exp(-\beta_m r - 2\pi \lambda_m U(r)),
\]

\[
\text{CDF: } F_{d_m}(r) = \kappa_m (-\exp(-2\pi \lambda_m U(r))),
\]

where \( r < R_a, U(r) = (1 - e^{-\beta_m r}(1 + \beta_m r))p_s/\mu_0^2 \) and \( \kappa_m = (1 - \exp(-2\pi \lambda_m U(R_a)))^{-1} \) which is a coefficient introduced by the limited deployment area. Note that we have \( \kappa_m \approx 1 \) for networks with large density. It is validated in [37, corollary 4.13] that conditioned on the parent process, a PCP \( \Phi^L_m, f(x, \tilde{c}) \) is an inhomogeneous PPP with intensity function \( \bar{c}_T \sum_{z \in \Phi} f(x(z)) \). Combining the blockage function, conditioned on the parent point process \( \Phi_P \), we have that the LOS TBS network is an inhomogeneous PPP with intensity

\[
\lambda^P_{f}(x) = \bar{c}_T p_f(x) \sum_{z \in \Phi} f(x(z)).
\]

Based on the contact distance distribution of PCP networks investigated in [24], [38], [39], and [40], the PDF of \( d_T \) conditioned on the parent process \( \Phi_P \) can be given as follows.

\[
F_{d_T}(r|\Phi_P) = 1 - \prod_{z \in \Phi} \exp\left(-\bar{c}_T \int_0^r f_s(x(z)) dx \right),
\]

\[
f_{d_T}(r|\Phi_P) = \bar{c}_T \sum_{z \in \Phi} f_s(r(z)) \prod_{z \in \Phi} \exp(-\bar{c}_T F_s(r(z))),
\]

where \( F_s(r|z) \) and \( f_s(r|z) \) are the distribution function conditioned on cluster center \( z \), which are expressed as

\[
F_s(r|z) = \int_0^r \int_{2\pi} \rho^L(r) f\left(\sqrt{r^2 + z^2 - 2rz \cos \theta}\right) rd\theta d\rho,
\]

\[
f_s(r|z) = \int_0^{2\pi} \rho^L(r) f\left(\sqrt{r^2 + z^2 - 2rz \cos \theta}\right) rd\theta.
\]

The detailed proof can be derived from the contact distance distribution of PCP [24], [40]. Assume \( \rho^L_T(r) = 1 \), two special cases of \( f_s(r|z) \) for MCP and TCP are given in [24] and [40].

Example 1 (MCP): For an MCP with offsporing points scattering uniformly in a disk with radius \( R \), the conditional distance distribution given the cluster center \( z \) can be expressed as

\[
f^{\text{MCP}}(r|z) = \begin{cases} 2r & 0 \leq r \leq R - z, 0 \leq z \leq R \\ 2r \frac{2r cos^{-1}\left(\frac{r^2 + z^2 - R^2}{2rz}\right)}{\pi R^2}, & \|R - z\| < r \leq R + z. \end{cases}
\]

Example 2 (TCP): For a TCP with variance \( \sigma^2 \), the conditional distance distribution given the cluster center \( z \) is given by

\[
f^{\text{TCP}}(r|z) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + z^2}{2\sigma^2}\right) I_0\left(\frac{rz}{\sigma^2}\right),
\]

where \( I_0(\cdot) \) is the modified Bessel function of the first kind with order zero.

If \( f_s(r) \) is given, the exact expression of contact distance distribution can be derived by averaging over the parent process and using the sum-product functional.

Definition 1: (PGFL) Given \( \mu(x) : \mathbb{R}^2 \rightarrow [0, 1] \) is measurable, the PGFL of a PPP with intensity \( \lambda(x) \) is given by

\[
\mathbb{E}\left[\prod_{x \in \Phi} \mu(x)\right] = \exp\left(-\int_{\mathbb{R}^2} (1 - \mu(x)) \lambda(x) dx\right).
\]

The PGFL for a Neyman-Scott cluster process \( \Phi(\lambda, f_s, \tilde{c}) \) is given as follows

\[
\mathbb{E}\left[\prod_{x \in \Phi} \mu(x)\right] = \exp\left(-\lambda \int_{\mathbb{R}^2} (1 - \exp(-\tilde{c}) \left(1 - \int_{\mathbb{R}^2} \mu(x) f_s(x(z)) dz\right)) dz\right).
\]

Definition 2: (Sum-Product Functional) Given that \( v(x) \) and \( \mu(y) \) are measurable, where \( v(x) : \mathbb{R}^2 \rightarrow \mathbb{R}^+ \) and \( \mu(y) : \mathbb{R}^2 \rightarrow [0, 1] \), the sum-product functional of a homogeneous PPP \( \Phi(\lambda) \) is given as follows

\[
\mathbb{E} \left[ \sum_{x \in \Phi} v(x) \prod_{y \in \Phi} \mu(y) \right] = \lambda \int_{\mathbb{R}^2} v(x) \mu(y) dx \times \exp\left(-\lambda \int_{\mathbb{R}^2} (1 - \mu(y)) dy\right).
\]

By utilizing Definition (1) and Definition (2), we can derive the contact distance distribution as below.

Lemma 1: The contact distance distribution of LOS TBS with parent process of \( \Phi_m \) is given as

\[
\text{CDF: } F_{d_T}(r) = \kappa_T (1 - G(r))
\]

\[
\text{PDF: } f_{d_T}(r) = \xi G(r) \int_0^{R_a} f_s(r(z)) \exp(-\tilde{c}_T F_s(r(z))) zdz,
\]

where \( \xi = 2\pi \kappa_T \lambda_m \tilde{c}_T, \kappa_T = (1 - G(R_a))^{-1} \) and

\[
G(r) \triangleq \exp\left(-2\pi \lambda_m \int_0^R [1 - \exp(-\tilde{c}_T F_s(r(z)))] zdz\right).
\]

Proof: Firstly, assuming \( r_T \) and \( r_m \) are the contact distance for PCP and PPP networks without boundary. Therefore, we have

\[
F_{d_T}(r) = \mathbb{P}(r_T < r) r_T < R_a, r_m < R_a)
\]
from TBS and MBS, which are given by
\[
\begin{align*}
\mathbb{P}(r_T < r_T < R_0 | r_m < R_0) & = \mathbb{P}(r_T < R_0 | r_m < R_0) \\
\mathbb{P}(r_T < R_0 | r_m < R_0) & = \mathbb{P}(r_T < r_m < R_0), \quad (22)
\end{align*}
\]
where \( \kappa_T = \left[ \mathbb{P}(r_T < R_0 | r_m < R_0) \right]^{-1} = \mathcal{G}(R_0) \). To derive the distribution of \( d_T \), we have to decondition \( \Phi_P \) in the conditional distribution expressed in (11) and (12), which can be resolved by utilizing the PGFL and sum-product functional of PPP given in (16) and (18) as follows.

\[
F_{d_T}(r) = \kappa_T \mathbb{E}_{\Phi_P} \left[ F_{d_T}(r | \Phi_P) \right] = \kappa_T \left( 1 - \mathbb{E}_{\Phi_P} \left[ \prod_{z \in \Phi_P} \exp \left( -\tilde{c}_T \int_0^r f_s(x) dx \right) \right] \right)
\]
\[
= \kappa_T \left( 1 - \mathcal{G}(r) \right). \quad (23)
\]
This completes the proof. \( \Box \)

With Lemma 1, we can get the contact distance distribution for TCP and MCP models. It has been demonstrated that the TCP and MCP model results in a similar analysis \[24\]. Moreover, compared to MCP, which has a sharp boundary, the TCP model is more realistic to model the clustering setting \[42\]. Therefore, this paper will utilize the TCP model to analyze the results. Note that the analysis with the MCP model can be readily achieved with our proposed framework.

**B. ASSOCIATION PROBABILITY AND DISTANCE DISTRIBUTION**

We adopt the Max-BRP \[13\] strategy, where typical UE is associated with the strongest BS in terms of long-term averaged biased received power. We assume that the link between typical UE and associated BS must be a LOS link to guarantee the quality of service. Formally, the associated BS can be expressed as follows

\[
x_0 = \arg \max_{x \in \Phi_{T} \cup \Phi_m} P_{r,x}, \quad \kappa \in \{T, m\}. \quad (24)
\]
The term \( P_{r,x} \) denotes the intended biased received power from TBS and MBS, which are given by

\[
P_{r,T} = N_T P_T B_T \left( \frac{c}{4 \pi f_T} \right)^2 e^{-k_0(f_T) x_T - \sigma_T}
\]
\[
P_{r,m} = N_m P_m B_m \left( \frac{c}{4 \pi f_m} \right)^2 x_m^{-\sigma_m}, \quad (25)
\]
where \( B_T \) and \( B_m \) are the biased factors for THz and mmWave tier, respectively. Based on the association strategy, we define the association probability as \( \mathcal{A}_\kappa \), where the subscript \( \kappa \in \{T, m\} \) denotes the THz and mmWave tier.

**Lemma 2:** The association probability to the mmWave tier, denoted as \( \mathcal{A}_m \), is given by

\[
\mathcal{A}_m = \int_0^{R_m} f_{d_m}(r) \left[ 1 - \kappa_T \left( 1 - \mathcal{G}(v(r)) \right) \right] dr,
\]
\[
\nu(x) \equiv \frac{\alpha_T}{\kappa_T} W \left( k_0(f_T) \left( \frac{x}{\kappa_T} \right) \right), \quad (26)
\]
where \( \epsilon = \frac{B_m P_m N_m}{B_T P_T N_T} (\frac{f_T}{f_m})^2 \), \( W(x) \) is the Lambert \( W \) function, and \( \mathcal{G}(r) \) is defined in (21).

**Proof:** Based on the Max-BRP association policy, the association probability can be evaluated as follows,

\[
\mathcal{A}_m = \mathbb{E}_{x_m} \left[ P(r_T < r_{m}) \right] = \mathbb{E}_{x_m} \left[ P(x_T > v(x_m)) \right] = \mathbb{E}_{x_m} \left[ 1 - F_{d_T}(v(x_m)) \right] = \int_0^{R_m} \left[ 1 - F_{d_T}(v(r)) \right] f_{d_m}(r) dr. \quad (27)
\]
Therefore, the association probability to the THz tier is given by \( \mathcal{A}_T = 1 - \mathcal{A}_m \), Note that \( \mathcal{A}_T \) can also be derived similarly by taking the expectation over \( x_T \) which is omitted here. This completes the proof. \( \Box \)

Given that the typical UE is associated with the \( \kappa \)-th tier, \( \kappa \in \{T, m\} \), the distance distribution \( f_{x_\kappa}(r) \) between the typical UE and the associated BS can be derived as follows.

**Lemma 3:** The distance distribution of a typical UE asso-

\[
f_{x_m}(r) = \frac{1}{\mathcal{A}_m} f_{d_m}(r) \left[ 1 - \kappa_T \left( 1 - \mathcal{G}(v(r)) \right) \right],
\]
\[
f_{x_T}(r) = \frac{1}{\mathcal{A}_T} f_{d_T}(r) \left[ 1 - F_{d_m}(v(r)) \right], \quad (28)
\]
where \( r_\kappa(r) = \left( \frac{\epsilon r^2 e^{k_0(f_T)r}}{\kappa_T} \right)^{\frac{1}{\sigma_T}} \).

**Proof:** See Appendix A. \( \Box \)

**C. COVERAGE PROBABILITY**

This subsection will elaborate on the analytical framework to evaluate the CP, defined as the probability of SINR achieving the desired threshold. The CP of the hybrid networks can be expressed as

\[
\mathcal{P}_C(\tau) = \mathcal{A}_T \mathcal{P}_C(\tau) + \mathcal{A}_m \mathcal{P}_C_m(\tau), \quad (29)
\]
where \( \tau \) is the desired threshold, \( \mathcal{A}_T \) and \( \mathcal{A}_m \) are the association probability given in Lemma 2, and \( \mathcal{P}_C(\tau) \) and \( \mathcal{P}_C_m(\tau) \) are the coverage probabilities given that typical UE is associated with TBS and MBS, respectively. To enhance tractability, we used Alzer’s inequality which is widely adopted in many papers and validated to obtain a tight bound of the CP.

1) **MmWave TIER CP**

The conditional CP of the mmWave tier is derived as

\[
\mathcal{P}_C_m(\tau) = \mathbb{P}[\text{SINR}_m(x_0) > \tau]
\]
which can be simplified by using the following notations

\[
\frac{P_m N_m m_l}{\sigma_m^2 + L_m} \chi_{0}^{-\alpha_l} > \tau
\]

\[
= \mathbb{P} \left[ \frac{P_m N_m m_l}{\sigma_m^2 + L_m} \chi_{0}^{-\alpha_l} > \tau \right]
\]

\[
= \mathbb{P} \left[ \hat{I}_m > \tau \chi_{0}^{-\alpha_l}(\hat{\sigma}_m^2 + \hat{L}_m) \right]
\]

\[
\approx 1 - e^{-\alpha_l \tau \chi_{0}^{-\alpha_l}(\hat{\sigma}_m^2 + \hat{L}_m)}
\]

\[
= \sum_{n=1}^{M_L} \left( \frac{M_L}{n} \right) (-1)^{n+1} \mathbb{E} \left[ e^{-P_m(\chi_{0})}(\hat{\sigma}_m^2 + \hat{L}_m) \right]
\]

\[
= \sum_{n=1}^{M_L} \left( \frac{M_L}{n} \right) (-1)^{n+1} \int_0^{R_a} f(x_m)(r)m(r)dr, \quad (30b)
\]

where we used the Alzer’s inequality [43] in (30a) and utilized binomial expansion with the following notations in (30b).

\[
\zeta_m(r) = e^{-P_m(r)\hat{\sigma}_m^2} L_m(P_m(r)), \quad P_m(x) = n a_m x^\alpha_L
\]

\[
\check{I}_m = \sum_{\nu \in \{L, N\}} \sum_{x \in \Phi_m} \hat{G}_m h_v x^{-\alpha_{\nu}}, \quad a_m = M_L(M_L)^{-\frac{1}{M_L}},
\]

\[
\hat{\sigma}_m^2 = \frac{\sigma_m^2}{P_m N_m} M_L = \frac{\sigma_m^2}{N_m}. \quad (31)
\]

The term \( L_m(s) \) denotes the Laplace transform of \( \check{I}_m \), which is given in the following Lemma.

**Lemma 4:** The Laplace transform of interference in mmWave networks is given as follows,

\[
L_m(s) = \prod_{\nu} \exp \left[ 4\pi \lambda_m \psi_m \int_0^{R_a} \chi_m(s)p_m(x)dx \right], \quad (32)
\]

where \( \nu \in \{L, N\} \) and \( \chi_{m}(s, x) \) is given as

\[
\chi_m(s, x) = \sum_{k=1}^{K_m} \left[ 1 + \frac{s\hat{G}_m k x^{-\alpha_{\nu}}}{M_k} \right] M_k.
\]

**Proof:** See Appendix B.

**2) THz TIER CP**

The conditional CP of THz tier is defined as

\[
\mathcal{P}_{C_T}(\tau) = \mathbb{P}[\text{SINR}_{T}(x_0) > \tau]
\]

\[
= \mathbb{P} \left[ \frac{P_T N_T e^{-\lambda_0} h_T \chi_{0}^{-\alpha_T}}{P_N + I_T} > \tau \right], \quad (34)
\]

which can be simplified by using the following notations

\[
\zeta_T(r) = e^{-\hat{Q}_n(r)\hat{P}_N} L_T(O_n(r)), \quad \alpha_T = M_T(1)^{-1},
\]

\[
Q_n(x) = n a_T \tau e^{\lambda_0} x^\alpha_T, \quad \hat{P}_N = \frac{N_N P_T N_T}{4\pi \lambda_T c}, \quad \hat{G}_T = \frac{G_T(\Phi_T)}{N_T}, \quad \hat{I}_T = \sum_{x \in \Phi_T} \hat{G}_T h_T x^{-\alpha_T}. \quad (35)
\]

Then, \( \mathcal{P}_{C_T}(\tau) \) can be derived as

\[
\mathcal{P}_{C_T}(\tau) = \mathbb{P} \left[ h_T > \tau e^{k_0(\tau)_0} x_0^{\alpha_T} \hat{P}_N + \hat{I}_T \right]
\]

\[
\approx 1 - e^{-\alpha_T \tau \chi_{0}^{-\alpha_T}(\hat{\sigma}_m^2 + \hat{L}_m)}
\]

\[
= \sum_{n=1}^{M_T} \left( \frac{M_T}{n} \right) (-1)^{n+1} \mathbb{E} \left[ e^{-Q_n(x)}(\hat{P}_N + \hat{I}_T) \right]
\]

\[
= \sum_{n=1}^{M_T} \left( \frac{M_T}{n} \right) (-1)^{n+1} \int_0^{R_a} f(x_T(r))\zeta_T(r)dr. \quad (36)
\]

The Laplace transform of \( \hat{I}_T \) is calculated by taking the expectation over the PCP network and is given below.

**Lemma 5:** The Laplace transform of interference in THz networks is given as

\[
\mathcal{L}_I_T(s) = \exp \left[ -2\pi \lambda_m \sum_{k=1}^{K_T} \frac{2\pi \psi_T}{(1 + \frac{s\hat{G}_m x^{-\alpha_T}}{M_k}) M_k} \right],
\]

\[
\hat{\chi}_T(s, x) = \frac{2\pi \psi_T}{\sum_{k=1}^{K_T} (1 + \frac{s\hat{G}_m x^{-\alpha_T}}{M_k}) M_k}, \quad (37)
\]

where \( v(r) \) is defined in Lemma 3.

**Proof:** See appendix C.

While the expression of the coverage probability contains several integrals, due to the finite upper limit \( R_a \), the analytical result can be numerically computed more flexibly and conveniently when investigating a larger deployment region.

**V. NUMERICAL RESULTS**

This section will demonstrate the accuracy of the theoretical framework for the proposed hybrid networks and analyze
the impact of the clustering profile of the THz tier. The analytical results will be illustrated and validated by Monte Carlo simulation over $10^5$ realizations. All the simulation parameters are listed in Table 1 unless stated otherwise.

### A. ACCURACY OF FRAMEWORK

In this subsection, we will validate the accuracy of the proposed framework in terms of association probability and CP. In Fig. 2, the analytical and simulated association probability of the THz tier $A_T$ are plotted by dotted and solid curves for varying bias value $B_T$ and $\bar{c}_T$. We observed that the simulated results are well-matched to the analytical results with a negligible gap, which verifies the accuracy of the Lemma 2. We noticed that both $\bar{c}_T$ and $B_T$ enhance $A_T$.

In Fig. 3, we illustrate the analytical and simulated CP $P_C$ of mmWave and THz tier with different $\bar{c}_T$. The agreement of analytical and simulated results validates the accuracy of the proposed framework. Due to the accuracy of the proposed framework, we will just plot the theoretical results in the following figures. It is notable that for the THz tier when the SINR requirement is relatively low, large $\bar{c}_T$ achieves a better coverage. However, the system with a smaller $\bar{c}_T$ will outperform when the SINR requirement is high. The reason is that when the SINR threshold is high, interference will become the major limiting factor. Therefore, according to the service requirement, $\bar{c}_T$ can be adjusted accordingly for better network performance. In Fig. 4, we compare the CP of the THz-only network modeled with PCP and the proposed THz-mmWave HetNet with the same density. As depicted in the figure, an additional mmWave tier can significantly boost the network performance. This is because the THz-only network suffers from its poor penetrability and extremely short transmission range. Moreover, the clustering of THz nodes also aggravates the detrimental effect of interference. Note that comparing to the additional mmWave tier, the beneficial effect of the additional sub-6 GHz tier is not that evident due to the limited rate it can support, especially when a higher SINR is required.

### B. EFFECT OF BLOCKAGE

The blockage effect significantly impacts high-frequency communication, especially THz communication. The blockage effect on mmWave communication has been thoroughly investigated in previous works [29], [31], and thus we omit...
it here. Since the adopted blockage model will transform the THz nodes into a non-homogeneous PCP. We want to investigate how the clustering setting combats the blockage. Here, we utilize the coefficient $\beta_T$ to present the severity of the blockage, i.e., a large $\beta_T$ indicating server blockage effect.

Fig. 5 plots the association probability to THz tier $A_T$ and CP $P_C$ with different $\bar{c}_T$ and $\sigma$ for varying $\beta_T$. First, we can observe that as the blockage severity increases, $A_T$ and $P_C$ have different patterns. $A_T$ will be monotonously decreased as $\beta_T$ increases. This is because large $\beta_T$ will make it difficult for typical UE to establish connections with LOS THz nodes. Therefore, to combat the blockage effect, a larger $\bar{c}_T$ will improve the association probability. It is also observed that a larger $\sigma$ will decrease $A_T$. This can be explained by that a larger $\sigma$ will make THz nodes less concentrated resulting longer distance between typical UE and tagged TBS, thus resulting in a lower $A_T$.

In contrast, as the severity of blockage increases, $P_C$ will increase to a peak and become saturated. This is due to the high density of THz nodes introducing greater interference. When $\beta_T$ is relatively low, the interference is the dominant limiting factor, and severe blockage will reduce the interference, thus resulting in a better CP. However, as $\beta_T$ increases, the blockage effect will be more significant and make the network performance saturated. Likewise, a larger $\bar{c}_T$ will also have an adverse impact on the interference-limited-regime, while in the blockage-limited-regime, increasing $\bar{c}_T$ will combat the blockage and provide a better coverage. On the contrary to the impact of $\sigma$ to $A_T$, a larger $\sigma$ is beneficial to network performance in the interference-limited-regime. A larger $\sigma$ indicates less interference, thus resulting in a better CP. When in the blockage-limited-regime, a smaller $\sigma$ is preferred to improve the CP.

**C. EFFECT OF BIAS VALUES AND DENSITY**

The association policy and node density are two critical factors affecting network performance. We adopted the Max-BRP association policy to achieve a more flexible association for the UEs. The impact of bias on association probability has been investigated in the previous subsection. Now, we will elaborate on the impact of bias values and density on the CP $P_C$.

Fig. 6 shows the impact of bias ratio and density on the CP for varying $\beta_T$. Since THz communication is extremely susceptible to obstacles, $\beta_T$ will vary in different scenarios. As illustrated in Fig. 6(a), the impact of bias ratio for various $\beta_T$ are disparate due to different $\sigma$. First, it is observed that the CP of the proposed PCP-based network is higher than that of the PPP-based network when $B_T/B_m > 1$. Since THz nodes need to be densely deployed to overcome the severe propagation loss. Therefore, when $\beta_T$ is small, the interference is the dominant limiting factor. In the interference-limited regime, a larger bias ratio is not beneficial to the network performance. While in the blockage-limited regime, a larger bias ratio will improve the coverage, hence achieving a better performance. However, the CP will be reduced when the bias ratio is too large since the interference will degrade the network performance, especially when the blockage effect is not very severe. In the figure, we observed in the curve for $\beta_T = 0.02$ that the coverage performance deteriorates with a higher bias ratio when $\sigma$ is relatively low, $e.g., \sigma = 10$ and 20. This is due to the elevated interference level across THz and mmWave links when $\beta_T$ is small. It is observed that when $\beta_T = 0.02$, the CP is not monotonously varying with $\sigma$. The smaller $\sigma$ indicates a smaller distance from typical UE to TBS but greater interference. This result reveals that due to the dense deployment of THz networks and the susceptibility of
THz links, the network performance tuning strategy should be adjusted according to the environment.

Another density-related factor is the mean number of THz nodes \( \bar{c}_T \) of each cluster. Due to the spatial dependence of the mmWave tier and THz tier. The density of THz nodes can be maintained constant while tuning the value of \( \bar{c}_T \) and \( \lambda_m \). In Fig. 6(b), we compare the CP for varying \( \bar{c}_T \) with fixed \( \lambda_m = 0.001 \text{ m}^{-2} \) (red line) and fixed \( \lambda_T = 0.01 \text{ m}^{-2} \) (gray line) when \( R_a = \infty \).

D. IMPACT OF CLUSTERING EFFECT AND ABSORPTION COEFFICIENT

Since the THz nodes are scattered around the MBS forming an inter-tier dependence. Previous work [24] has pointed out that the CP of PCP network will gradually be increased and approach to that of a PPP network as \( \sigma \) increases. However, when \( \sigma \) is comparable with \( R_a \), the clustering setting will have a more beneficial effect achieving a higher CP as illustrated in Fig. 7(a). Here, we define the CP of PPP-based network as \( P_B \). It is revealed that \( P_C \) is a concave function of \( R_a \), while \( P_B \) monotonously drops as \( R_a \) increases. Moreover, we notice that \( P_C \) is always higher than \( P_B \). However, \( P_C \) will gradually drop and finally approach \( P_B \) as \( R_a \) increases.
When $R_a$ is 10 times larger than $\sigma$, the CP of PCP and PPP network are almost the same. This is because when $R_a$ and $\sigma$ is comparable, there will be only several clusters within the finite area. Hence, due to the inter-tier dependence, the typical UE will have a higher chance to associate with LOS BS. As $R_a$ further increases, the dense deployment of TBS will introduce greater interference. To further investigate the impact of the clustering effect, we define the clustering gain (CG) as the ratio of $P_C$ and $P_B$, i.e., $CG = \frac{P_C}{P_B}$. It is illustrated in Fig. 7(b) that the CG is a concave function of $a$ with a peak value larger than 1. The results show that there is always existing a suitable $\sigma$ for the proposed PCP network that can outperform the baseline PPP network. Moreover, a larger $\tilde{c}_T$ will achieve a higher CG. However, when $\sigma$ is sufficiently large, the baseline PPP will outperform the PCP-based HetNet. This is because when $\sigma$ is large enough, the mean number of TBSs in the finite area will be decreased and $P_C$ will also be gradually decreased. Another distinct feature of THz communication is molecular absorption which is determined by the environment and the frequency. In Fig. 8, we illustrated the CP of THz tier, mmWave tier, and the overall HetNet for varying absorption coefficient $\alpha$. It is observed that as $K_\alpha(f)$ increases, the CP of THz tier will drop and CP for the mmWave tier will be improved. However, the overall performance maintains almost the same when $K_\alpha(f)$ is small and decreases slightly when $K_\alpha(f)$ approaches to 1. It is also revealed that increasing $\sigma$ can improve the network performance when $K_\alpha(f)$ is relatively small. This is because a larger $K_\alpha(f)$ will introduce greater absorption noise. A larger $\sigma$ means longer communication distance and smaller absorption noise, thus improving the CP of the THz tier.

VI. CONCLUSION
In this work, we investigated a hybrid network of THz and mmWave in a finite area, where the locations of TBSs are modeled by a PCP model with the parent process of MBSs. Combining the clustering setting of THz nodes and a hybrid deployment of HetNet, we want the proposed framework to reveal more practical significance of scattering variance and inter-tier dependence. Considering the severe spreading loss and blockage effect, the LOS mmWave and THz nodes follow inhomogeneous point processes. Using conditional distance distribution and Laplace transform of the interference, we evaluated the SINR CP with the Max-BRP association strategy. The proposed framework quantitatively assesses the impact of the blockage effect of THz communication. It is revealed that the impact of bias ratio is significantly disparate under different blockage coefficients $\beta_T$. Especially in a low-$\beta_T$-regime, the interference will be the dominant limiting factor, where a larger bias ratio will have an adverse effect. While in the interference-limited-regime, severer blockage in dense THz networks will reduce the interference and improve the CP when the bias ratio is large. Compared with the traditional PPP-based HetNet, we demonstrate that the inter-tier dependence can flexibly improve the network performance by tuning $\sigma$ and $\tilde{c}_T$ according to the severity of blockage and area of deployed region. The proposed analytical framework can be applied in the future network deployment strategy so that the network performance can be optimized by tuning different parameters accordingly. This work can be further extended to investigate the UE-BS coupling, where the UEs are also modeled as a PCP with the same parent process as that of TBSs. Moreover, a more practical and sophisticated blockage model and power constraint can also be adopted to make the analysis more of practical significance. A possible solution for a more realistic blockage model is by invoking the Boolean model to analyze the multi-path effect of THz nodes. Furthermore, a tailored modulation scheme for THz communication can be considered by combining the clustering setting of THz nodes. This is left for future work.

APPENDIX A

PROOF OF THE LEMMA 3
Given the association event with mmWave tier, we derive $P[X_m > r]$ as below

$$P[X_m > r] = P[X_m > r | \kappa = m]$$

$$= \frac{P[X_m > r, \kappa = m]}{P[\kappa = m]}$$

$$= \frac{P[X_m > r, \kappa = m]}{A_m}$$

$$= \frac{1}{A_m} \int_R P(r_m > P_r, T) f_{dx_m}(x_m) dx_m.$$  (38)

$f_{dx_m}(r)$ can be derived by differentiating $1 - P[X_m > r]$ with respect to $r$. The derivation steps are similar to Lemma 2.

For the conditional distance distribution given the typical UE is associated with THz tier is derived similarly as below.

$$P[X_T > r] = \frac{1}{A_T} \int_R f_{dx_T} P(r_m < P_{r,T}) dx_T.$$
\[
\mathcal{L}_{I_m}^c(s) = \mathbb{E}\left[\exp\left(-s \sum_{x \in \Phi_m^c} \hat{G}_m^c h_{mL} x^{-\alpha L}\right)\right]
\]
\[= \mathbb{E}\left[\prod_{x \in \Phi_m^c} \exp\left(-s \hat{G}_m^c h_{mL} x^{-\alpha L}\right)\right]^{(a)} = \mathbb{E}\left[\prod_{x \in \Phi_m^c} \frac{1}{1 + \frac{s\hat{G}_m^c h_{mL} x^{-\alpha L}}{M_L}}\right]^{(b)} \Phi_m^c \prod_{x \in \Phi_m^c} \left(1 - 2K_m \psi_m\right) + \frac{2\psi_m}{M_L} \sum_{k=1}^{K_m} \frac{s\hat{G}_m^c h_{mL} x^{-\alpha L}}{M_L}^{(c)} = \exp\left(4\pi \lambda_m \psi_m \int_{r}^{R_m} \frac{\hat{L}_m^c(s, x) \psi_m L_{m}(x) x dx}{r}\right),
\]

where (a) is calculated by the moment generating function of Gamma random variable, (b) is by computing the mean of antenna gain, (c) follows the PGFL of the PPP and \(K_m = \left\lceil \frac{N_m}{T} \right\rceil\). \(\psi_m\) is the half-power beamwidth for mmWave nodes, and \(\chi^c_{s, x}(s, x)\) is given in Lemma 4. The Laplace transform of \(I_m^c\) can be derived following similar steps except the lower bound of the integral is 0 due to the assumption that only LOS links are considered to support the service. This completes the proof.

APPENDIX C

PROOF OF THE LEMMA 5

The Laplace transform of interference in THz networks can be derived as below.

\[
\mathcal{L}_{I_T}^c(s) = \mathbb{E}\left[\exp\left(-s \sum_{x \in \Phi_T^c} \hat{G}_T h_T x^{-\alpha T}\right)\right]
\]

\[
= \frac{\int_{r}^{R_T} f_{d_T}(x_T)\mathbb{E}\left(x_m > \left(\varepsilon x_T^{2\alpha T} e^{\kappa_d(x_T) x_T} x_T^{\frac{1}{2}}\right)\right) dx_T}{A_T}
\]
\[= \frac{1}{A_T} \int_{r}^{R_T} f_{d_T}(x_T) \left[1 - F_{d_m}(r_T(x_T))\right] dx_T,
\]

where \(r_T(r) = \left(\varepsilon e^{-\kappa_d(x_T) x_T} x_T^{\frac{1}{2}}\right)^{\frac{1}{2}}\). The final result is derived by differentiating \(1 - F[x_T > r]\) with respect of \(r\). This completes the proof.

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