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An Economical Optimization Model of Non-Periodic Maintenance Decision for Deteriorating System

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\section*{ABSTRACT} Condition-based maintenance is an effective method for deciding when to maintain deteriorating system. In this paper, a non-periodic maintenance model with adaptive inspection intervals is proposed by the consideration of system stability and deterioration. Different maintenance plans and monitoring strategies are adopted in distinct stages of life cycle of the deteriorating system to reduce costs, which is a complex multiple optimization parameter problem. And particle swarm optimization algorithm combining heuristic rules is designed to solve this multi-objective problem. Finally, a numerical example is implemented to illustrate the effectiveness and rationality of the proposed model. The comparison between the proposed model and other maintenance models illustrates the economy of the proposed model and the importance of considering system stability. Sensitivity analysis is also performed to investigate the effect of seven cost factors.

\section*{INDEX TERMS} Condition based maintenance, deteriorating system, imperfect inspection, reliability, system stability.

\section{I. INTRODUCTION}
A manufacturing system consists of many components. Any failure of these components might lead to a stop or performance degradation of the system. Therefore, maintenance strategy is required to be established to keep such the manufacturing system functioning well. Meanwhile, due to the performance degradation of the system with accumulated time in service, the maintenance strategy should vary for different stages of system lifetime.

Since the earliest work of Eicke \cite{1}, plenty of maintenance models have been proposed by scholars. Maintenance strategies adopted in the maintenance models are mostly time-based maintenance (TBM) in which maintenance schedules are developed based on elapsed time \cite{2}. With the development of sensor techniques and system state analysis method, it is much easier and more precise to estimate the system state and predict the remaining useful life \cite{3}–\cite{6}. Under this circumstance, condition based maintenance (CBM) has received more and more attention.

In CBM models, maintenance strategy is generally classified into three levels: minimal repair, imperfect maintenance (IM) and major repair (MR). Preventive maintenance (PM) is one kind of IM implemented at the scheduled time, which can restore the system between “as good as new” and “as bad as old” \cite{1}. Compared to PM, corrective maintenance (CM) proposed by Mamer \cite{7} is another kind of IM passively implemented when quality shift, defined as partial failure state, occurs. After implementing IM, the expected running time between partial failure and failure usually is greatly reduced \cite{8}. In \cite{9}, IM is implemented when soft components fail and MR is implemented when hard components fail. Except for these basic maintenance methods, factors such as inspection quality, inspection frequency, optimization criteria, and optimal design are required to be considered in CBM optimization models.

Most of CBM models assume that the inspections are perfect \cite{10}–\cite{13}. In practice, however, it is difficult to satisfy this assumption. So it is acceptable to assume that the
inspections are imperfect. In recent, some scholars have proposed some improved CBM models. In [14], an optimal inspection and preventive maintenance model was proposed, which considered revealed and unrevealed failures. In [15], inspection policies were divided into 3 types: partial, perfect and imperfect. In this paper, we consider imperfect inspections with type I (false positive) and type II (false negative) error to make proposed model more realistic.

As to inspection frequency, continuous monitoring, periodic inspection and non-periodic inspection are often used in CBM models. Each inspection policy has its advantages and disadvantages. Continuous monitoring lets professionals know the real time conditions of the system but needs high inspection cost and may lead to too much false positive error. Periodic inspection is much more cost saving than continuous monitoring, but this inspection policy might result in higher false negative error and maintenance cost. Compared to periodic inspection, non-periodic inspection is most cost effective with better performance in false negative error rate. But the plan-making process is much more complex compared to both continuous monitoring and periodic inspection. All three types of inspection frequency have been used in CBM models. In [16]–[19], proposed models were built to implement continuous monitoring in oil, gas industrial, and nuclear power plant. In [20], CBM models with periodic inspection were proposed. Compared to continuous monitoring and periodic inspection, non-periodic inspection model is rare. In [21], an optimal non-periodic inspection for multivariate degradation model was proposed. In [22], a CBM model with non-periodic maintenance for two-unit series system was proposed, which designed adaptive inspection for deteriorating system. Under this inspection policy, inspection frequency is adjusted according to system states and maintenance cost.

Besides inspection strategies, maintenance strategies is another important factor to be scheduled [23]. Traditional PM is based on the elapsed time, and CBM is based on the system state. PM model may gain experience from historical failure date and CBM model may be used to estimate system state based on collected sensor date. In this paper, we take advantage of two kinds of models. On the one hand, a non-periodic PM strategy is scheduled according to system failure probability. On the other hand, non-periodic inspection is implemented to assess the system state.

The optimization target in CBM models mainly includes cost [20], [22], age [24], [25], reliability [23], [26], availability, and multi-object. In [27], a multi-criteria decision model was proposed, which optimized conflicts among total cost, environment, and quality objective. In [28], a multi-criteria decision model was defined, which optimized conflicts between cost and down time. It can be seen that cost is the most concerned object when building a maintenance model. Other objects such as reliability and lifetime are also needed to be constrained. So a balance among them is very important. In this paper, we designed the inspection interval and maintenance strategy to get an economic optimum.

Except for inspection quality, inspection interval, and maintenance strategy, system stability are also very important to a production system. However, there are very few CBM models considering system stability. In [29], a decision-making model for power transform was proposed in which stability is considered. Instability may lead to more and more glitches as time goes by. The maintenance of a few glitches may only lead to negligible cost. But when glitches occur frequently, the cost can be very high. So considering system stability is really critical.

In this paper, we developed a novelty economical optimization model of non-periodic maintenance decision for deteriorating system. In this model, we considered the complexity of a manufacturing system at first. Four kind of maintenance methods are implemented according to different types of failure. Then, we considered the deterioration of the system over time and developed a non-periodic maintenance strategy according to the degree of system degradation. As the system state deteriorates, we optimized the inspection interval for each maintenance period adaptively. Except for those aspect, system stability is considered and estimated by the frequency of glitches.

The remainder of this paper is organized as follows: Section II introduces the problem descriptions of the system. Section III develops an economical optimization model with adaptive inspection intervals and illustrates the calculation process of this model. Section IV designs a PSO algorithm for this multivariable problem. Section V gives an illustrative example to explain the advantages of this model. Sensitivity analysis is conducted to investigate the parameters that affect the optimal maintenance strategies. Finally, a conclusion of this paper is drawn in Section VI.

II. PROBLEM STATEMENT
The maintenance framework is shown in FIGURE 1. As the system deteriorates, the initial state of \((i + 1)^{th}\) maintenance period (state \(0_{i+1}\)) is worse than the initial state of \(i^{th}\) maintenance period (state \(0_i\)). So a non-periodic maintenance strategy is designed. The scheduled time for PM is generally become shorter and shorter from one maintenance period to next. In order to prevent the system from partial failure (state \(1_i\)) and failure (state \(2_i\)) before preventive maintenance, we monitored the system status through scheduled inspections. Due to the deterioration of the system, the probability of the system failure is increasing, and the inspection interval is adjusted adaptively. System failure is catastrophic and costly. In order to avoid system failure, we restore the system and reduce the risk of failure through MR.

A system scenario for non-periodic maintenance decision of a deteriorating system is described in detail. The deteriorating system includes three states as follows:

- **State 0**: The system is in good state during \(i^{th}\) maintenance period.
- **State 1**: The system is in partial failure state during \(i^{th}\) maintenance period. This state is a transitional state between
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FIGURE 1. System scenario for non-periodic maintenance decision of a deteriorating system.

good state and failure state. At this state, the system produces more unqualified products.

State 2: The system is in the failure state. At this state, the system stops unexpected and causes great lost.

Meanwhile, inspection is implemented to monitor production process of the system. The inspection interval is $h_i$ unit time and monitors whether the system is in state 0 or state 1. The inspection is imperfect and has a rate of false positive $\alpha$ (type I error) and a rate of false negative $\beta$.

In the non-periodic maintenance decision process, each maintenance period begins at state 0 and ends with an IM including CM and PM except the last one. IM restores the system to state 0 but the system cannot be restored as good as new. After $M$ maintenance periods, a MR is scheduled to renew the system instead of IM. For $i^{th}$ maintenance period after $(i-1)^{th}$ IM, the system begins at state 0 and implements inspections every $h_i$ time unit. Once an alarm is generated, a comprehensive check will be conducted immediately to confirm its truthfulness. If it is true, a CM will be conducted to avoid more quality lost. If not, the system will continue operating until next inspection and the inspection times $k = k + 1$. In consideration of the system deterioration, PM is scheduled after $(N_i - 1)$ times of inspection if no alarm is generated or alarms are false. In addition, there is a slight risk that the system transfer to state 1 but the inspection fails to detect it and the system transfers to state 2. In this condition, a MR is conducted to restore the system to as good as new.

During the lifetime described above, the state space of the deteriorating system is $\{0, 1, 2; i = 1, 2, \ldots, M\}$, where $\{0, 1, 2; i = 1, 2, \ldots, M\}$ is the set of unobservable production states and (2) is the failure state. The probability that the system transfers from state 0 to state 1 follows exponential distribution with parameter $\lambda^0_i$. The probability that the system transfers from state 1 to state 2 follows exponential distribution with parameter $\lambda^1_i$ where $\lambda^0_i < \lambda^1_i$. $1/\lambda^0_i$ and $1/\lambda^1_i$ denote the average time the system stays under state 0 and state 1, respectively. There are sequences $\lambda^0_1 < \ldots < \lambda^0_i < \ldots < \lambda^0_M$ and $\lambda^1_1 < \ldots < \lambda^1_i < \ldots < \lambda^1_M$. The average time of $i^{th}$ maintenance period for each state is reduced compared to $(i-1)^{th}$ maintenance periodic with $\lambda^0_i = \xi \lambda^0_{i-1}$, where $\xi$ is the reduction factor and $\xi > 1$. In addition, except for the maintenance methods to control the production process, MMR is prepared for those minor problems which are easy to be solved with maintenance time ignored. The times of conducting MMR during a maintenance period follows non-homogeneous Poisson process (NHPP) with parameter described by Weibull law intensity.

According to the description above, there are some additional statements below:

The reliability of the system is modeled by exponential distribution $f_c(t) = f(t|\lambda^c_i) = \lambda^c_i e^{-\lambda^c_i t}$, $c = 0, 1$, where $c$ denotes system state and $1/\lambda^c_i$ denotes the mean time of the system which stays under $c$ state during $i^{th}$ period.
The frequency of MMR is modeled by NHPP. The parameter \( \lambda_i(t) \) of NHPP is given by Weibull law intensity function \( \lambda_i(t) = abt^{b-1} \), where \( a \) is the scale parameter and \( b \) is the deteriorating rate. The advantage and applicability of this assumption has been investigated in [23] and [27]–[31].

The MMR implementing times is a reflection of system stability. The parameter of NHPP is a sequence of \( \{\lambda_i(t) ; i = 1, 2, \ldots, M\} \) where \( \lambda_i(t) = \xi \lambda_{i-1}(t) \) and \( \xi > 1 \).

Inspection monitors the production process at every time interval \( h \) with \( \alpha \) rate of type I error and \( \beta \) rate of type II error.

PM is scheduled after \( N_i \) times of inspection for \( i^{th} \) maintenance period if the inspection doesn’t generate any correct alarm.

The system can only survive \( M \) maintenance periods and a MR is taken at the end of \( M^{th} \) maintenance period.

The inspection interval \( h_i \) is different from each other owing to the different situation of each maintenance period.

### III. ECONOMICAL OPTIMIZATION MODEL WITH ADAPTIVE INSPECTION INTERVALS

According to the descriptions and assumptions above, an economical optimization model of non-periodic maintenance with adaptive inspection intervals for the deteriorating system is built. Its goal is to minimize the total expected cost per unit time, denoted as ECPUT, which can be presented by a ratio of total cost \( (C_{total}) \) to total time \( (T_{total}) \). And, the optimization parameters include maximum number of maintenance period \( M \), maximum inspection times of each maintenance period \( N_i (i = 1, \ldots, M) \) and inspection interval for each maintenance period \( h_i (i = 1, \ldots, M) \). Herein, the unit time could be an hour, a month or thousands of hours, so the parameters in this paper are dimensionless.

As mentioned, the ECPUT is expressed as:

\[
\text{min } ECPUT(M, N_1, \ldots, N_M, h_1, \ldots, h_M) = \frac{C_{total}}{T_{total}}
\]

subject to:

\[
M, N_1, \ldots, N_M, \geq 1
\]

\[
h_1, \ldots, h_M > 0
\]

\[
M, N \in \text{Integer}
\]

The system may fail at \( i^{th} \) \( (i = 1, \ldots, M) \) maintenance period or not fail during whole system lifetime. So, there is \((M + 1)\) MR implementing situations, denoted \( S_1 \sim S_{M+1} \). For situations \( S_1 \sim S_{M+1} \), failure happens during \( i^{th} \) maintenance period with probability \( PS_i \) and MR is taken timely. The \((M + 1)^{th}\) situation indicates that failure does not happen during whole system lifetime and MR is implemented at the end of \( M^{th} \) maintenance period.

In order to calculate ECPUT, expected cost per unit time of each situation, denoted as \( ECPUT_i \), is deduced as follows:

\[
ECPUT_i = \frac{\sum_{j=1}^{i-1} EPC_j + EPC_i}{\sum_{j=1}^{i-1} EPT_j + EPT_i}
\]

where \( EPC_j \) and \( EPT_j \) are expected cost and expected running time of the maintenance period in which failure happens; \( EPC_j \) and \( PT_j \) are the expected cost and expected running time of the maintenance periods during which IMs are implemented.

b) For the situation \( S_{M+1} \):

\[
ECPUT_{M+1} = \frac{\sum_{j=1}^{M-1} EPC_j + EPC_{M+1}}{\sum_{j=1}^{M} EPT_j}
\]

Considering all the situations and its probability of occurrence, the ECPUT can be obtained as follows:

\[
ECPUT(M, N_1, \ldots, N_M, h_1, \ldots, h_M) = \frac{C_{total}}{T_{total}} = \sum_{i=1}^{M} PS_i \cdot ECPUT_i + PS_{M+1} \cdot ECPUT_{M+1}
\]

Where \( PS_i \) is the probability that \( S_i \) happens.

### A. PROBABILITY CALCULATION

For every maintenance period, there are four scenarios to describe the maintenance decision process as shown in FIGURE 2.

In \( MS_1 \sim MS_2 \), PM is implemented at the scheduled end of the maintenance period as the system does not transfer to state 1 in \( MS_1 \) or the system transfers to state 1 but the inspection failed to detect it in \( MS_2 \). In \( MS_3 \), CM is implemented immediately when the inspection gives an alarm after the system transfers to state1. If \( MS_1 \sim MS_3 \) happens at \( M^{th} \) maintenance period, MR is implemented instead of PM and CM.

The probability of each maintenance scenario is computed as follows:

For the scenarios \( MS_1 \) and \( MS_2 \), the probability of implementing PM during \( i^{th} \) maintenance period, denoted as \( P^i_{PM} \), is expressed as

\[
P^i_{PM} = \int_0^{h_i} \left( F^i_1(N_i \cdot h_i - s) \cdot (1 - F^i_1(N_i \cdot h_i - s)) \right) \beta N_i - \left[ \frac{s}{h_i} \right] ds + (1 - F^0_1(N_i \cdot h_i))
\]

\[
= \sum_{m=1}^{N_i} \left[ \beta N_i - m \int_{(m-1)h_i}^{mh_i} f^0_1(s) \cdot (1 - F^1_1(N_i \cdot h_i - s)) ds \right] + (1 - F^0_1(N_i \cdot h_i))
\]

\[
= \sum_{m=1}^{N_i} \left[ \beta N_i - m + 1 \right] \left( e^{(\lambda_0^{N_i} - \lambda^i_0)h_i} - 1 \right) + e^{-\lambda_0^{N_i}h_i}
\]

where \( \left[ . \right] \) is the operator of round down.

For the scenario \( MS_3 \), assuming that the system goes to partial failure state at time \( s \) and the inspection give a correct alarm at \( k^{th} \) \( (s/h_i) < k \leq N_i \) inspection, the probability...
of conducting CM during \(i\)th maintenance period, denoted as \(P_{CM}^i(k)\), is defined as

\[
P_{CM}^i(k) = \int_0^{h_i} f_i^0(s) \cdot (1 - F_i^1(k \cdot h_i - s)) \cdot \beta^{k-1} \left(\frac{t_i}{\beta} \right)^{k-1} ds
\]

\[
= \sum_{m=1}^{k} \left( \beta^{k-m}(1 - \beta)^{\frac{\lambda_i^0}{\lambda_i^0 - \lambda_i^1}} \right) (e^{(\frac{\lambda_i^0}{\lambda_i^0 - \lambda_i^1})h_i} - 1) \times e^{-\left(\frac{\lambda_i^0}{\lambda_i^0 - \lambda_i^1}\right)m \cdot h_i - \lambda_i \cdot k \cdot h_i}
\]

(6)

For scenario \(MS4\), assuming that the system transfers to state 1\(_i\) at time \(t\) and transfers to state 2\(_i\) at time \(x(x > s)\) and inspection fail to detect the state 1 situation between time interval \([s, x]\), the probability of conducting MR during \(i\)th maintenance period, denoted as \(P_{MR}^i\), is written as

\[
P_{MR}^i = \int_0^{h_i} \int_s^x f_i^0(s) f_i^1(x - s) \beta^{n-1} \left[ \frac{t_i}{\beta} \right] ds dx
\]

\[
= \sum_{n=1}^{N_i} \int_{(n-1)h_i}^{nh_i} \int_0^x f_i^0(s) f_i^1(x - s) \beta^{n-1} \left[ \frac{t_i}{\beta} \right] ds dx
\]

(7)

This formulation can be divided into two parts. One is that the system fails before inspect, when the system transfers to state 1\(_i\). Denote this part as \(A1\):

\[
A1 = \sum_{n=1}^{N_i} \int_{(n-1)h_i}^{nh_i} \int_0^x f_i^0(s) f_i^1(x - s) \beta^{n-1} \left[ \frac{t_i}{\beta} \right] ds dx
\]

The other is that the inspection fails to detect the partial failure state. Denote this part as \(A2\):

\[
A2 = \sum_{n=1}^{N_i} \int_{(n-1)h_i}^{nh_i} \int_0^x f_i^0(s) f_i^1(x - s) \beta^{n-1} \left[ \frac{t_i}{\beta} \right] ds dx
\]

(8)

The probability that the system transfers to state 2\(_i\) during \(i\)th maintenance period is denote \(PS_i\). It means that the system does not transfer to state 2\(_i\) during \((1 \sim i - 1)\)th maintenance periods. \(PS_{M+1}\) denotes the probability that the system does not transfer to state \(PS_{M+1}\) until the end of system lifetime.

\[
PS_1 = P_{MR}^i
\]

\[
PS_i = \prod_{j=1}^{i-1} (1 - P_{MR}^j) P_{MR}^i
\]

(10)
**B. COST CALCULATION**

The model proposed in this paper includes four maintenance costs: cost of CM (CMC), cost of PM (PMC), cost of MR (MRC), and cost of MMR (MMRC) and other three types of costs: check cost after inspection (CheckC), quality loss cost under partial failure state (QLC) and inspection cost (InspC).

As to the situations $S_t \sim S_m$, assuming that the system transfers to state $j_t$ at time $s$ and fails at time $x_t$ during $i_{th}$ maintenance period, $j_t (j < i_t)$ is denoted as the maintenance period in front of $i_{th}$ period. For each $j_{th}$ maintenance period, the expected cost, denote $EPC_{j}$, includes two parts: the expected cost of PM denoted as $EPC_{PM}^{j}$ and the expected cost of CM denoted $EPC_{CM}^{j}$.

The $EPC_{PM}^{j}$ is the cost of S1 and S2, which is presented as:

$$EPC_{PM}^{j} = \text{CheckC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ QLC \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{PMC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$EPC_{CM}^{j} = \text{CheckC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{0}^{N_{jhj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

Then, if combining Eq. (11) and Eq. (12), $EPC_{j}$ is derived as:

$$EPC_{j} = \frac{EPC_{PM}^{j} + EPC_{CM}^{j}}{P_{PM}^{j} + P_{CM}^{j}}$$

In S4, the expected cost, denoted as $EPC_{s}$, includes MR cost, quality lost cost, check cost and MMR cost. The $EPC_{s}$ is calculated as:

$$EPC_{s} = \text{CheckC} \int_{x=0}^{N_{hj}} f_j^0(s)(1 - F_j^1(N_{hj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{x=0}^{N_{hj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{x=0}^{N_{hj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$

$$+ \text{InspC} \int_{x=0}^{N_{hj}} a_j b_j h_{b-1} \alpha \text{and} \text{PMC} \int_{0}^{N_{jhj}} f_j^0(s)(1 - F_j^1(N_{jhj} - s)) \beta N_j - \left[ \frac{s}{h_j} \right] ds$$
\[
\text{calculated as the whole system lifetime. Either a correct alarm is gen-
}\]

\[
\text{In the (MRC + MMRC \cdot a_i(EPT^R))^+}
\]

\[
\text{And the EPT}_i \text{ when MR is implemented is:}
\]

\[
EPT_i = \frac{1}{P_{\text{MR}}^i} \int_0^{N_i-h_i} \int_0^x f_i^0(s) f_i^1(x-s) \beta \left[ \tilde{p}_i \right] - \left[ \hat{p}_i \right] x ds dx
\]

\[
= \frac{1}{P_{\text{MR}}^i} \sum_{n=1}^{N_i} \lambda_i^0 - \lambda_i^1 (1 - e^{(\lambda_i^0 - \lambda_i^1)h_i})
\]

\[
\times e^{-\lambda_i^0 - \lambda_i^1} [n(h_i - (n-1)h_i) e^\lambda_i^1] + \sum_{n=1}^{N_i} \left( \frac{1}{\lambda_i^1 - \lambda_i^0} (1 - e^{(\lambda_i^0 - \lambda_i^1)h_i}) \right)
\]

\[
\times (1 - e^{\lambda_i^1 h_i}) + \frac{1}{N_i} \sum_{n=1}^{N_i} \left( \frac{\lambda_i^0}{\lambda_i^0 - \lambda_i^1} \right) e^{-\lambda_i^0 (n-1)h_i - \lambda_i^1 h_i} (1 - e^{-\lambda_i^0 h_i})
\]

\[
+ \frac{1}{N_i} \sum_{n=1}^{N_i} \left( \frac{\lambda_i^0}{\lambda_i^0 - \lambda_i^1} \right) e^{-\lambda_i^0 (n-1)h_i - \lambda_i^1 h_i} (1 - e^{-\lambda_i^0 h_i})
\]

\[
+ \frac{1}{N_i} \sum_{n=1}^{N_i} \left( \frac{\lambda_i^0}{\lambda_i^0 - \lambda_i^1} \right) e^{-\lambda_i^0 (n-1)h_i - \lambda_i^1 h_i} (1 - e^{-\lambda_i^0 h_i})
\]

\[
(15)
\]

In the (M + 1)th situation, failure doesn’t happen through the whole system lifetime. Either a correct alarm is generated or the scheduled end of maintenance period comes, a MR is taken instead of IM in case of further deterioration. The expected cost of this situation is denoted as EPC_{i+1} and calculated as

\[
EPC_{M+1} = EPC_{PM}^M + EPC_{CM}^M - P_{PM}^M \cdot P_{CM}^M - CMC
\]

\[
\times \sum_{k=1}^{N} \sum_{k=1}^{N} \left( \frac{P_{PM}^M}{P_{CM}^M} \right) \text{PRM}(k)
\]

\[
\sum_{k=1}^{N} \sum_{k=1}^{N} \left( \frac{P_{CM}^M}{P_{CM}^M} \right) \text{PRM}(k)
\]

\[
(16)
\]

In addition, the EPT_j mentioned in Eq. (2) when IM is implemented is:

\[
EPT_j = \frac{1}{P_{PM}^j} \sum_{k=1}^{N_j} (P_{PM}^j N_j h_j + \sum_{k=1}^{N_j} P_{CM}^j(k) h_j)
\]

\[
(17)
\]

IV. SOLUTION ALGORITHM DESIGN BASED ON PSO COMBINING HEURISTIC RULES

As mentioned earlier, the proposed economical optimization model have (1 + 2M) variables to be identified, those are, M, h_1, h_2, \ldots, h_M and N_1, N_2, \ldots, N_M. After trying some optimization algorithms such as genetic algorithm, PSO algorithm is chosen to solve this multi-objective problems owing to its fast convergence rate. At the same time, heuristic rules are designed to decrease the computation time as the calculation process is complex and time-consuming. The solution space is too large to search them all. From the structural of the solution space, we can find that the solution space and number of optimization variables are growing along with M. It is not suitable to use the PSO algorithm to calculate M. The other 2M variables are integers. PSO can be used to calculate integer optimization problems efficiently. So it is reasonable to use PSO algorithm for each M. The PSO based solution algorithm for the economical optimization model includes the following steps:

Step I: Set M = 1 and use PSO algorithm to calculate the corresponding N_j and h_j. The result is saved as a candidate for the optimal solution. According to the principle of PSO,
TABLE 2. Optimization result.

<table>
<thead>
<tr>
<th>M</th>
<th>N₁, N₂, ..., N₉</th>
<th>h₁, h₂, ..., h₉</th>
<th>ECPU/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.35, 39, 36, 4</td>
<td>11.39, 11.60, 15.11, 14.58, 1</td>
<td>13.5470</td>
</tr>
</tbody>
</table>

FIGURE 3. Convergence speed of the algorithm.

FIGURE 4. Optimal solution of each M.

the processes are expressed as:

\[
\begin{align*}
    v_{i,j}(t+1) &= w \ast v_{i,j}(t) + c₁r₁[p_{i,j} - N_{i,j}(t)] \\
    &\quad + c₂r₂[p_{g,j} - N_{i,j}(t)] \\
    N_{i,j}(t+1) &= N_{i,j}(t) + v_{i,j}(t+1), \quad j = 1, 2, \ldots, M; \quad i = 1, 2, \ldots, P
\end{align*}
\]

(18)

where P is the number of particles, \( r₁ \) and \( r₂ \) are random numbers from 0 to 1, w is inertial weights which controls exploration and exploitation, c₁ and c₂ are acceleration coefficients which determine whether particles tend to move closer to \( p_{i,j} \) or \( p_{g,j} \). Each particle \( N_{i,j} \) has velocity \( v_{i,j} \) [26].

Step 1.1: Set the initial parameters of the PSO algorithm. Number of particles P is set to 40 with consideration of the convergence rate after calculation test. The other parameters are set \( c₁ = c₂ = 2 \), \( w = 0.5 \).

Step 1.2: Generate P illegal particles by heuristic rules. Each particle has 2M elements including \( N_{i,1}, N_{i,2}, \ldots, N_{i,M} \) and \( h_{1,1}, h_{1,2}, \ldots, h_{1,M} \). The initial value range of \( N_{i,j} \) is set to \([1, \lfloor 1/λ_j,0 + 1/λ_j,1 \rfloor / h_{i,j}]\), which means that the system is not allowed to operate longer than the average running length from state 0 to state 2. The initial value range of \( h_{i,j} \) is set to \((0, 50)\).

Step 1.3: Randomly generate P integer particles with 2M elements as velocity values \( v_{i,j} \). The maximum initial velocity is set to 15.

Step 1.4: Evaluate the fitness of each particle and record the best solution of ith particle as \( p_{y,i} \) and corresponding \( N_{i,1}, N_{i,2}, \ldots, N_{i,M}, h_{1,1}, h_{1,2}, \ldots, h_{1,M} \) as \( p_{i,1}, \ldots, p_{i,2M} \). Record the best solution of all particles as local optimal solution \( g_y \) and corresponding \( N_{1,1}, N_{2,1}, \ldots, N_{M,1}, h_{1,1}, h_{1,2}, \ldots, h_{1,M} \) as \( g_{1,1}, \ldots, g_{2M} \).

Step 1.5: Renew \( N_{i,j} \) and \( v_{i,j} \) according to Eq. (18) and calculate the new fitness of each particle. Comparing the new fitness with \( p_{y,i} \) of each particle, renew \( p_{y,i} \) with new one if it is better.

Step 1.6: Compare \( p_{y,i} \) with \( g_y \) and renew \( g_y \) if \( p_{y,i} \) is better.
Step 1.7: Repeat step 1.4 to step 1.6 100 times and record $g_y$ and corresponding $g_1, \ldots, g_{2M}$ as a candidate optimal solution.

Step 2: Set $M = M + 1$. Repeat the step 1.1 to step 1.7 and get the corresponding $g_1, \ldots, g_{2M}$. By comparing the new $g_y$ with $g_y$, we have got and renew it with new one if it is better.

Step 3: Repeat step 2 until an appropriate $M$ value. The system cannot survive too many times of maintenance. In this paper, the maximum $M$ is set 15. From FIGURE 4, we can find that it is enough to get the optimal $M$.

Step 4: Record $g_y$ and the corresponding $g_1, \ldots, g_{2M}$ as the optimal solution.

V. ILLUSTRATIVE EXAMPLE

In this section, a numerical example is presented to illustrate the effectiveness and rationality of the proposed economical optimization model. Suppose that the parameters of probability density function are given as $\lambda_0 = 1/800$ and $\lambda_1 = 1/100$ respectively. The reduction factor is set as $\xi = 1.05$. According to the renew theory, the parameters of MMR are given as $b = 1.7$ and $a = 0.001$. The type I error rate and
type II error rate of the inspection are respectively estimated as $\alpha = 0.1$ and $\beta = 0.1$. These parameters can be estimated by a lot of methods such as maximum likelihood estimation, data mining and least square method using history data or test data from accelerated life test technique. The remaining parameters are given in TABLE 1.

All the algorithm and solution methods are implemented in Matlab 2014(a), and the numerical example is performed on normal PC with 4 Intel cores.

FIGURE 3 shows the iteration process of each M. From FIGURE 3, we can find that 150 times of iteration is sufficient to get an acceptable result.

### TABLE 6. Experiments of 7 costs.

<table>
<thead>
<tr>
<th>factor</th>
<th>level</th>
<th>value</th>
<th>$M$</th>
<th>$N_1, N_2, \ldots, N_M$</th>
<th>$h_1, h_2, \ldots, h_M$</th>
<th>ECPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>high</td>
<td>50</td>
<td>8</td>
<td>86,50,46,34,36,26,26,26</td>
<td>10.59,13.93,14.04,15.57,13.40,16.78,15.10,14.43</td>
<td>13.7172</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMC</td>
<td>low</td>
<td>800</td>
<td>8</td>
<td>80,57,39,36,40,35,28,27</td>
<td>11.39,11.60,15.11,14.58,11.71,12.38,14.43,13.60</td>
<td>13.5470</td>
</tr>
<tr>
<td></td>
<td>high</td>
<td>1000</td>
<td>8</td>
<td>81,59,47,42,40,32,28,33</td>
<td>11.25,12.30,13.72,13.97,13.21,15.26,15.42,12.02</td>
<td>13.7150</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMC</td>
<td>low</td>
<td>1000</td>
<td>8</td>
<td>80,57,39,36,40,35,28,27</td>
<td>11.39,11.60,15.11,14.58,11.71,12.38,14.43,13.60</td>
<td>13.5470</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRC</td>
<td>low</td>
<td>5000</td>
<td>8</td>
<td>80,57,39,36,40,35,28,27</td>
<td>11.39,11.60,15.11,14.58,11.71,12.38,14.43,13.60</td>
<td>13.5470</td>
</tr>
<tr>
<td></td>
<td>high</td>
<td>300</td>
<td>8</td>
<td>97,63,51,52,42,40,33,32</td>
<td>9.37,10.66,11.47,10.13,11.39,10.94,1.87,11.88</td>
<td>14.5337</td>
</tr>
<tr>
<td></td>
<td>high</td>
<td>50</td>
<td>8</td>
<td>63,44,34,29,25,25,19,21</td>
<td>14.52,16.39,18.07,18.25,18.57,17.06,20.17,18.64</td>
<td>14.8879</td>
</tr>
<tr>
<td></td>
<td>high</td>
<td>70</td>
<td>9</td>
<td>67,45,33,27,28,26,26,21,17</td>
<td>10.91,11.15,14.22,15.43,14.07,13.38,12.76,13.91,17.05</td>
<td>15.1543</td>
</tr>
</tbody>
</table>

### A. OPTIMIZATION RESULT

We get the optimal solutions as follows:

**FIGURE 4** shows the evolution of $ECPUT$ alone with $M$. From **FIGURE 4**, it is concluded that as the maintenance times $M$ increase, the $ECPUT$ decreases quickly at first and slowly increases after. The optimal $M$ is obtained at 8.

**TABLE 3** lists the parameters related to the optimal result. They reflect the evolution process of the parameters during system lifetime. It’s obvious that the scheduled maintenance time is shorter and shorter along with the mean time of state 0. Further, we analyze the varied trend of each maintenance method in each maintenance period and depict it.
in **FIGURE 5**. From **FIGURE 5**, we can conclude that the probability of implementing PM is growing and the probability of implementing CM is decreasing along with the IM maintenance times. The probability of implementing MR is controlled at a very low level at each maintenance period. Through **FIGURE 5**, we divide the system lifetime into 3 stages:

\[ M = 1, 2, 3. \]

This time period reflects the first stage of the system lifetime. During this period, the system is new and stable. The scheduled PM time is close or even longer than the mean time of which the system transfers to state 1. This is to say, the system has a high possibility of being at state 1 when it comes to PM time. But when we turn to inspection intervals corresponding to these periods, we can find that the inspection interval is small. So the maintenance strategy at this period is controlling the process by inspecting intensively so that the state 1 condition can be detected and CM is the main maintenance method.

\[ M = 4, 5, 6. \]

During this period, the probability of PM and CM is close to each other and the scheduled maintenance time is close to the meantime of state 0. The maintenance strategy during this stage is controlling the production process by CM and controlling the costs by PM.

\[ M = 7, 8. \]

During this period, the probability of implementing PM is higher than CM. The scheduled maintenance time is much shorter than the mean time of state 0 and it is of low probability to transfer to state 1 when it comes to schedule maintenance time. In this situation, the stability of the system plays an important role. In order to control the costs, PM is scheduled at a relatively early time at this situation.

### B. COMPARISON ANALYSIS

Many maintenance models can be compared to this model as the deteriorating system has been studied by many scholars. However, under the assumptions in this paper, some of them are just a local optimization solution. For example, we can fix the inspection interval to turn this model into a non-periodic maintenance model. Furthermore, we can fix the length of each maintenance period to turn it into a periodic maintenance model. The comparison of different models is shown in **TABLE 4**. So this kind of comparison model will not be done. We want to compare is the non-periodic model without consideration of stability which means the parameter b is 1. The result is shown in **TABLE 5**.

Comparing **TABLE 5** and **FIGURE 6** with **TABLE 3** and **FIGURE 5**, we can find that the probability of each maintenance method is close to the first situation mentioned above. But the ECPUT is much lower, so the stable factor has a great influence in cost. In reality, it is common to see systems go through 3 situations through its lifetime. This result illustrates the importance and reasonability of considering stability factor.

### C. SENSITIVE ANALYSIS

All parameters in **TABLE 1** may have effects on ECPUT value of the maintenance strategy. A sensitive analysis is completed on these 7 factors. The result is listed in **TABLE 6** and the effect of these parameters on ECPUT is depicted in **FIGURE 7**.

From **TABLE 6** and **FIGURE 7**, we can conclude that all these 7 costs have a positive effect on ECPUT. But there are

![FIGURE 7. Mean effect of 7 factors on ECPUT.](image-url)
VI. CONCLUSIONS

This paper develops a non-periodic maintenance model with adaptive inspection intervals for deteriorating system. Four kind of maintenance methods are considered to deal with complex failure scenarios. The lengths and inspection intervals of each maintenance period are designed targeted as equipment health status change along with system deteriorating. System stability is also estimated by frequency of MMR. Even though so many factors are considered, this model has very good flexibility and a wide range of applicability as it is easy to be adjusted to satisfy different manufacturing situations. There is still room for improvement in this research. We look forward to introducing new technologies, such as big data, machine learning and so on, to monitor the system state with better precision. At the same time, the reliability of the system and reduction factor $\xi$ is able to be obtained by these new technologies to make this model more practical.

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