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Dynamic Evolution Analysis of Metro Network Connectivity and Bottleneck Identification: From the Perspective of Individual Cognition

YUXIN HE¹, ZHONGZHI XU¹, YANG ZHAO¹,², AND KWOK-LEUNG TSUI¹,²

¹School of Data Science, City University of Hong Kong, Hong Kong 999077
²Centre for Systems Informatics Engineering, City University of Hong Kong, Hong Kong 999077

Corresponding author: Yang Zhao (yang.zhao@my.cityu.edu.hk)

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ABSTRACT Metro network connectivity is crucial for ensuring reliable operation of metro systems. Despite the rich literature on the connectivity analysis of transportation network, very little attention has been paid to passengers’ heterogeneous cognition toward congestion and connectivity incorporating subjective judgment. In this paper, we develop a data-driven framework to analyze metro network connectivity evolution involving individual cognition by characterizing it as a transit percolation process. The concept of individual tolerance index of congestion and a measure named network friendliness are proposed. By comparing individual tolerance index and friendliness of metro network, metro network connectivity with regard to different passengers can be depicted quantitatively. The evolution of network connectivity can be monitored both as individual tolerance changes and as time goes on. We also demonstrate how global transit breaks down when the identified bottlenecks are congested from the perspective of the passengers’ cognition. The proposed method is validated using a real-world case of the Shenzhen Metro in China. Results show that the proposed method is effective in capturing the dynamic evolution of the Shenzhen metro network connectivity and enable effective identification of transit bottlenecks. The network connectivity and friendliness are found to be significantly increased through a small improvement of the bottlenecks pinpointed.

INDEX TERMS Bottleneck identification, dynamic evolution, metro network connectivity, network friendliness, percolation, tolerance index.

I. INTRODUCTION

Connectivity is a critical performance assessment of transportation network by measuring how well different locations connect to one another via traffic flow [1]. High connectivity in transportation network implies low isolation and high accessibility, which is a fundamental basis for the reliable operation of transportation systems. A good performance of connectivity is particularly important to metro network as metro network usually serves the foundation for relieving congestion in today’s transportation environment, especially in high-income cities [2].

The connectivity of transportation network is primarily determined by two types of factors, engineering factors and operational factors. The relationship between engineering factors and connectivity has been well studied since engineering factors such as equipment fault, technical fault, road construction, network structure, etc., are objective and clearly defined. While operational factors are fuzzy and complex as an involvement of human factors, such as passengers’ tolerance of congestion (the focus of this paper). For road network, the connectivity between two locations is greatly dependent on engineering factors. Nevertheless, for metro network, operational factors is more dominant in influencing connectivity. In fact, for passengers whose tolerance of congestion is low, stations connected with metro links under crowded condition are not accessible although physically connected. This makes the passengers’ individual cognition non-negligible to fully understand the connectivity of metro network. Despite the rich literature on connectivity of transportation networks (including metro network) [3]–[7], research on the connectivity of metro network with individual tolerance of congestion being involved is yet to come.

Identifying critical links where the traffic cannot pass easily in transportation network, namely bottlenecks,
is important for maintaining connectivity effectively in transportation network. Most research works on traffic bottlenecks identification lied in the field of physics and transportation operations [8]. In general, the methods for bottlenecks identification in physics domain are constructed from the perspective of traffic engineering, such as detecting and controlling bottlenecks based on allocating sensors and traffic signal devices [9], [10]. In the field of transportation operations, representative works focused on identifying bottlenecks induced by congestion by considering the interaction between bottlenecks and traffic flow [11]–[17]. However, these studies only focused on the association between bottlenecks and traffic flow without considering network structure. To study the effects of network structure, Li et al. [18] (2015) developed a novel method to detect recurrent bottlenecks by incorporating the interaction between roads’ network structure and flow based on a concept in complex network field named percolation theory, and they focused on the road network connectivity and traffic dynamics from the perspective of system. Li et al. [18] (2015)’s research opens up new perspectives that fully justify the pursuit of this research. For metro transportation network, how to identify transit bottlenecks which takes into account both network structure and passenger flows by incorporating individual cognition is still a challenging problem.

To the best of our knowledge, two fundamental questions are neglected in related works and yet to be answered: a) How to evaluate metro network connectivity considering network structure (engineering factors), as well as individual tolerance of congestion (operational factors) of metro system? b) How to quantitatively describe the relationship between metro network connectivity and passengers with heterogeneous tolerance of congestion?

To answer the first question, we develop a measure, named network friendliness, that can well describe the connectivity of a metro network considering both network structure and individual tolerance of congestion of metro system. To answer the second question, we propose an index, named tolerance index, to describe one’s tolerance of congestion. By comparing individual tolerance index and friendliness of metro network, we can quantitatively depict metro network connectivity with regard to different passengers. By harnessing actual travel demand data collected from Shenzhen metro Automated Fare Collection (AFC) system and geographic information of Shenzhen metro network, we simulate the dynamic evolution procedure of metro network. Answering these questions with real-world metro network and travel demand data can inform effective data-driven control and management of metro system. To the extent of our knowledge, this is the first work that shows how to quantitatively study the dynamic evolution of metro network connectivity (friendliness) with an individual perspective being involved. Here we characterize this dynamic process as “transit percolation”, and present how global transit breaks down when bottleneck links are congested from the perspective of passengers’ cognition.

The remainder of this paper is organized as follows. Section II deals with the problem statement and proposes a general data-driven framework for the dynamic evolution analysis of metro network connectivity and bottleneck identification, and the data description is also provided. Section III conducts the case study with a real-world dataset of Shenzhen Metro, and the main findings of the dynamic evolution of metro network connectivity are presented. Then bottleneck identification as an effective measure to improve network friendliness is conducted in Section IV. Section V presents the concluding remarks of our study.

II. MATERIAL AND METHODOLOGY
A. NOTATIONS AND PRELIMINARIES
A metro network can be modeled as a directed graph \( D(V, E) \), where \( V \) denotes a set of vertices. \( E \supseteq e_{ij} \) denotes a set of edges from vertex \( i \) to \( j \). In the context of metro network, a vertex represents a metro station, and edges represent rails connecting two corresponding stations, i.e., \( e_{ij} \) denotes the metro link from \( i \) to \( j \). Considering transferring factors, effective routes and even network complexity, real-word passenger demand are mapped onto metro network using a proper model in conjunction with specific cases for transit assignment. Thus, we can get the passenger volume on metro link \( e_{ij} \) within the \( t \)-th time slot (denoted as \( f^{(t)}_{ij} \)). \( f^{(t)}_{ij} \) refers to the limited maximal passenger volume (can be deemed as the capacity from the perspective of operational conditions) on \( e_{ij} \), and it can be obtained from the statistical analysis of the data related to passenger volume. \( r_{ij}^{(t)} \) is the ratio between the current passenger volume and the limited maximal volume on \( e_{ij} \) within the \( t \)-th time slot (can be regarded as Volume Capacity ratio (V/C) ratio from the perspective of operational conditions), and

\[
I_{ij}^{(t)} = \frac{f_{ij}^{(t)}}{f_{ij}^{L}}
\]

\( I_{ij} \) denotes an availability indicator of the metro link \( e_{ij} \), which is a binary value with 1 indicating available and 0 indicating unavailable. Let \( q \) be a given constant in the percolation process, and it is defined to quantify the passengers’ tolerance index of congestion in the context of this paper. \( q^{(t)} \) is the critical threshold of the percolation network during the \( t \)-th time slot, and it is described as the friendliness of metro network to passengers. \( B^{(t)} \) denotes the set of identified bottlenecks during the \( t \)-th time slot.

In view of the graph generated by real-time metro network being directed graph, the components connected in metro network are defined as “strongly connected components” in which all pairs of vertices are mutually reachable from each other along a directed path [18]–[20]. A network is connected as there is a path between every pair of vertices. Thus, the failure of some edges in a network will lead to loss of the paths connecting different clusters (cluster is a connected set of vertices within which there is a path between any pair of vertices) and decomposition of large strongly connected clusters. \( G \) represents the size of the largest
Since they cannot tolerate the congestion at that time. People may regard the metro trip as unavailable and disconnected to the operational states of the network components (edges or vertices).

In percolation theory, the failure of a vertex/edge of network is modeled by vertex/edge removal [29], which refers to the operational states of the network components (edges or vertices). The framework only considers the case of bond percolation [30]. In the simplest case, edges could be removed from a network purely at random. Here we construct the percolation network assuming that the probability of link failure has a strong relationship with individual cognition towards congestion. In practice, large passenger volume causes overcrowded conditions in metro, and some people may regard the metro trip as unavailable and disconnected since they cannot tolerate the congestion at that time.

For each metro link, passenger volume \( f_{ij}^{(t)} \) varies during a day, and \( t \) denotes the aggregated time slot. For each link \( e_{ij} \), it is not appropriate to define its traffic states based on the absolute value of passenger volume, so we use the ratio \( r_{ij}^{(t)} \) of the current passenger volume \( f_{ij}^{(t)} \) to the limited maximal volume \( f_{ij}^{L} \) of each link to quantify its relative passenger volume (as in (1)). For a given constant \( q \), the link \( e_{ij} \) can be classified into two categories: available when \( r_{ij}^{(t)} \leq q \), and unavailable when \( r_{ij}^{(t)} > q \). Therefore, the removal scheme of edges can be expressed by (2).

\[
I_{ij} = \begin{cases} 
1, & r_{ij}^{(t)} \leq q \\ 
0, & r_{ij}^{(t)} > q 
\end{cases} \quad (1)
\]

For the unavailable link, some passengers are not willing to get on board as the crowdedness has exceeded their tolerance, thus the link is unavailable for them. The constant \( q \) is defined as the passengers’ tolerance index of congestion. This indicator can explain the relationship between individual cognition and the current congestion state. In this way, an available metro network can be constructed for a given \( q \), which becomes more diluted as \( q \) decreases [18].

**B. OVERALL FRAMEWORK FOR DYNAMIC EVOLUTION ANALYSIS AND BOTTLENECK IDENTIFICATION**

In this paper, we develop a data-driven framework based on percolation theory by using AFC data. This framework aims to monitor dynamic evolution of metro network connectivity and detect recurrent bottlenecks effectively by considering individual cognition towards congestion. The main idea of the proposed framework is to present connectivity evolution in a metro network by characterizing it as a percolation process. The breakdown of global transit can be presented as identified bottlenecks are congested from the perspective of individual cognition. Real-world data of geographical information of metro network and passenger volume are needed for implementing the proposed framework. The developed quantified framework contains 4 main steps. Fig. 1 elaborates the overall procedures of the proposed framework.

The 4 main steps are detailed as follows. 

**Step 1: OD demand generation**

1. Construct trip data profiles from the raw data of AFC records. Recognize tap-in and tap-out records and sort the trips (start from tapping-in and end at tapping-out) according to the identifier of a smart card’s holder.

2. Divide the trip data of a whole day by a specific time segment, and aggregate the trip data according to starting station (Origin) and ending station (Destination) during each time segment. Note that if a trip covers two time segments, the trip will be aggregated into the OD demand in the segment of which the trip covers a higher proportion. For example,
as shown in Fig. 1 Step 1, two trips are detected covering two periods: the trip from 7:50 station C to 8:25 station D is defined in period II, and the trip from 8:30 station E to 9:10 station F is also defined in period II.

Step 2: Transit assignment. There is a significant difference between traffic assignment of road networks and transit assignment of metro networks. Since transit assignment in metro networks is concerned with multiple complicated factors, such as time, transferring, ticket prices, safety, comfort and service quality, which can influence the conditions of traffic distribution in networks [31], so we need to consider generalized travel cost function to conduct transit assignment, e.g., a transfer cost based logit model (details see SI.1). As shown in Fig. 1 Step 2, a set of effective routes for one OD pair is presented, and the transferring times on the effective routes, riding time, travel cost, and proportion of transit assignment can be obtained. Thus, we assign the generated OD demands to metro links according to the proportion, and then the passenger volume \( f_{ij}^{(t)} \) of each metro link within the \( t \)-th time slot can be obtained. More details about transit assignment method used in this paper are presented in SI.1. Note that it is necessary to consider the metro network complexity when choosing a transit assignment model for a specific case. For example, according to reference [31], [32], All-Or-Nothing (AON) model is applicable to those metro networks with simple structure and relatively small size, since passengers of simple metro networks usually judge and select the route by the network sketch map, so factors affecting their judgment are the line distance, number of stations and number of transfer stations in the map, and there is not so many effective routes for them to choose.

Step 3: Percolation network construction.
1. Generate the directed graph \( D(V, E) \) of a metro network. Vertices denote metro stations, and edges denote metro links.
2. Calculate the relative passenger volume ratio \( r_{ij}^{(t)} \) of each metro link as (1).
3. Classify the metro links into two categories (available when \( I_{ij} = 1 \) and unavailable when \( I_{ij} = 0 \)) as (2).
4. Tune the value of \( q \) lower slightly from 1 to 0, and remove the unavailable metro links \( e_{ij} \) gradually when \( r_{ij}^{(t)} > q \).
5. Evaluate the network connectivity and generate the connected components of the network.

Step 4: Bottleneck identification.
1. As \( q \) decreases, observe the variation of the largest connected cluster (G) and the second largest connected cluster (SG), and identify critical threshold \( q_{c}^{(t)} \) when the size of SG reaches the maximum.
2. Identify the bottlenecks of the metro network by comparing the available network just above and immediately below the critical threshold.

Algorithm 1 presents the pseudo-code for dynamic evolution analysis and bottleneck identification of metro network from the perspective of individual cognition.

### Algorithm 1 Dynamic Evolution Analysis and Bottleneck Identification of Metro Network

**Input:** \( D(V, E) \), the directed graph generated from a metro network;
Raw data of AFC records of a whole day;
\( t \), time slot index during a day.

**Output:** \( q_{c}^{(t)} \), percolation critical threshold (friendliness)
within the \( t \)-th time slot;
\( B^{(t)} \), identified bottleneck within the \( t \)-th time slot.

//Step 1: OD demand generation.//
1: Construct trip data profiles from the raw data of AFC records;
2: Divide the trip data of a whole day by a specific time segment;
3: Obtain OD demand within the \( t \)-th time slot by aggregating the trip data according to starting station (Origin) and ending station (Destination), notated as \( OD^{(t)} \);
//Step 2: Transit assignment using the OD demand \( OD^{(t)} \) generated from step 1.//
4: Assign OD demand \( OD^{(t)} \) to each metro link \( e_{ij} \) according to a specific transit assignment model, and calculate the passenger volume \( f_{ij}^{(t)} \) of each metro link \( e_{ij} \) within the \( t \)-th time slot;
//Step 3: Percolation network construction.//
5: Calculate the relative passenger volume ratio \( r_{ij}^{(t)} \) of each metro link \( e_{ij} \) according to (1);
6: Classify the metro links into two categories according to (2);
7: \( q = 1 \);
8: for \( q \geq 0 \) do
9: \( q = q - 0.01 \);
10: Evaluate the network connectivity and generate the connected components of the network;
11: \( q = q - 0.01 \);
12: end for
//Step 4: Bottleneck identification.//
13: \( q = 1 \);
14: for \( q \geq 0 \) do
15: Compute the sizes of the largest connected cluster (G) and the second largest connected cluster (SG);
16: Identify critical threshold \( q_{c}^{(t)} \) when the size of SG reaches the maximum;
17: Identify the bottlenecks \( B^{(t)} \) within the \( t \)-th time slot by comparing the available network just above and immediately below \( q_{c}^{(t)} \);
18: \( q = q - 0.01 \);
19: end for
20: return \( q_{c}^{(t)} \), \( B^{(t)} \)

### C. DESCRIPTION OF SHENZHEN METRO AFC DATA

The Shenzhen metro AFC data used in the research are provided by Shenzhen Metro Corporation in China. The dataset covers 7 days from October 14 (Mon) to 20 (Sun) in 2013,
consisting of the complete information about entry-exit smart card records. Table 1 lists the Shenzhen Metro AFC data fields used in this paper.

**TABLE 1. List of AFC data fields.**

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction_id</td>
<td>Identifying a transaction</td>
</tr>
<tr>
<td>Card_id</td>
<td>Identifying a passenger</td>
</tr>
<tr>
<td>Line_id</td>
<td>Identifying a metro line</td>
</tr>
<tr>
<td>Station_name</td>
<td>Name of metro station</td>
</tr>
<tr>
<td>Transaction_type</td>
<td>Indicate either in or out of station</td>
</tr>
<tr>
<td>Transaction_timestamp</td>
<td>Date/Time Timestamp of transaction</td>
</tr>
</tbody>
</table>

According to Table 1, a smart card transaction record can be described as a quadruple \( (\text{Card}_i, \text{Station}_i, \text{Transaction}_i, \text{Transaction}_t) \), where \( \text{Card}_i \) denotes the cardholder (i.e., passenger), \( \text{Station}_i \) denotes station, \( \text{Transaction}_i \) denotes the timestamp of transaction, and \( \text{Transaction}_t \) denotes either in or out of station.

The count of average daily transaction records of weekdays is about 2,513,330, and that of weekends is about 2,480,499, which is less than that of weekdays, indicating the daily trip frequency of weekdays is higher than that of weekends.

We conduct preliminary statistical analysis using metro AFC data on October 14. Fig. 2(a) shows the spatial distribution of AFC data transaction records in one day. It presents that the transaction records are most densely distributed at Grand Theatre Station and Luohu Station, closely followed by Huaqiang Road station and Luohu station, and the transaction records of other stations have relatively sparse distribution.

Fig. 2(b) is about temporal distribution of AFC data transaction records, which shows that the spatial distribution of records has a peak value at both 8:00 and 18:00 on no matter weekdays or weekends, and the records in rush hours on weekends are significantly less than those on weekdays. In addition, it is found that the peak value in the morning rush hour is generally less than that in the evening rush, which may result from people’s travel time (such as commuting time) inconsistency in the morning while consistency in the evening. Besides, the records of other periods except for rush hours of weekends are more than those of weekdays. A possible reason could be that people take more non-commuting trips on weekends than on weekdays.

Additionally, the characteristics of temporal distribution of records on weekdays and weekends are quite similar, which suggests that there are similar metro travel patterns with morning and evening peaks on weekdays and weekends in Shenzhen. Moreover, from the perspective of distribution of records in a single day, it is also found that the maximum evening peak value falls on Friday. It indicates that more people tend to take trips in addition to daily commutes on Friday evening because of the last workday.

**D. DATA PROCESSING**

To calculate the running time of vehicles in transit assignment process, geographical information of the stations including latitude and longitude is employed to calculate the distance between two stations. Considering the effect of the radius of the earth, we calculate the distance \( d(i, j) \) between two stations \( i \) and \( j \) as the following (3):

\[
d(i, j) = R \arccos(\cos(\text{Lat}_i) \cdot \cos(\text{Lat}_j) \\ \cdot \cos(\text{Lon}_i - \text{Lon}_j) + \sin(\text{Lat}_i) \cdot \frac{\pi}{180})
\]

where, \( R \) is the radius of the earth, \( (\text{Lat}_i, \text{Lon}_i) \) and \( (\text{Lat}_j, \text{Lon}_j) \) are the latitude and longitude of two stations \( i \) and \( j \), respectively.

The relative data was collected from Google map.\(^1\) Fig. 3(a) is the schematic map\(^2\) of Shenzhen metro in 2013, and Fig. 3(b) truly reflects the spatial distribution of those stations with the involvement of the geographical information collected.

We divide a whole day by 1-hour aggregation segment. In this way, we construct trip data profiles from the raw data of AFC records, and divide the trip data by 1-hour time segments of a whole day. Consequently, we generate OD demand per hour and complete transit assignment to obtain the passenger volume per hour of each metro link according to the transfer cost based logit model introduced in SI.1. In the context of Shenzhen metro network, it is noted that the

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\(^1\) Source: https://maps.google.com

result of transit assignment through the transfer cost based logit model is similar to that by AON assignment method (see details in SI.2). The reasons might be Shenzhen metro network in 2013 is not really complex, and the transfer plans are not various. It is worth noting that when conduct transit assignment in other metro networks, network sizes and complexity should be considered by choosing the reasonable routing algorithm and transit assignment method. Here, to exclude the abnormal crowded scenario, we consider the limited maximal passenger volume $f_{ij}^{L}$ for each metro link as the 95 percentile of its passenger volume values during the whole week (from October 14 to October 20) (see SI Figure S.3 for details).

III. DYNAMIC EVOLUTION ANALYSIS OF METRO NETWORK CONNECTIVITY

A. PERCOLATION NETWORK CONSTRUCTION

First, with the data we processed and the framework we proposed, we can construct the percolation network of Shenzhen metro. Through the statistical analysis of the largest cluster and the second-largest cluster as $q$ decreases, it is found that the number of available links composing the largest cluster decreases, and the size of the second-largest cluster reaches the maximum at a certain $q$. According to percolation theory, the phase transition of network connectivity occurs when the second-largest cluster reaches the maximum, and the $q$ value at this moment (during the $t$-th time slot) is defined as the critical threshold $q_c(t)$. The critical threshold $q_c(t)$ can be regarded as a quantitative indicator of the network availability limit. A passenger can travel most of the metro coverage parts (giant component of metro network) only if the relative passenger volume he or she can tolerate (tolerance index $q$) is above $q_c(t)$, otherwise, this passenger will fall into small isolated clusters when the tolerance index is below $q_c(t)$. Hence, given a metro network, $q_c(t)$ measures the minimal tolerance index that enables a passenger to take a trip on most metro links with low crowdedness (the links of the largest cluster) within the $t$-th time slot. The larger $q_c(t)$ is, the fewer passengers can travel most of the metro coverage parts, since a passenger can travel over the parts only if his/her tolerance index is above $q_c(t)$. In other words, $q_c(t)$ indicates the friendliness of a metro network for the passengers. The smaller $q_c(t)$ means the friendlier network which is available for more passengers.

Fig. 4 illustrates size (the number of vertices) variation of the largest cluster and the second-largest cluster with $q$ decreasing (orange and green represent the sizes of the largest cluster (G) and second-largest cluster (SG), respectively). Note that as $q$ decreases, the size of the largest cluster decreases, and the second-largest cluster reaches a maximum at the critical threshold ($q_c(t)$), indicating the percolation transition from the connectivity phase to the dis-connectivity phase of the metro network.

We also calculate the critical threshold of dynamic metro networks in different periods of all available days. Due to passenger volume and network connectivity evolving, $q_c(t)$ varies dramatically during the day as demonstrated in Fig. 5. Different patterns and network statuses between weekdays and weekends can also be observed. During weekdays, $q_c(t)$ has 2 local maxima corresponding to rush hours in the morning and evening, and the networks then are quite unfriendly. It is noted that $q_c(t)$ is smaller during morning rush hours than that of evening, showing metro congestion is severer in evening rush hours than morning rush hours, and connectivity judgements by most people are poorer then. Moreover, it is found that the curve is steeper in the morning rush hours while smoother in the evening rush hours, which indicates the evening rush lasts longer than morning rush. During weekends, especially on Sunday, the absence of obvious congestion in the morning rush hours is well reflected by the only one local maximum.

B. DYNAMIC EVOLUTION ANALYSIS OF METRO NETWORK CONNECTIVITY

1) PERCOLATION PROCESS OF METRO NETWORK (THE PERCOLATION PROCESS WITH VARYING PASSENGERS’ TOLERANCE INDEX $q$)

In this section, we simulate the percolation process with varying passengers’ tolerance index $q$ in a fixed time period. The percolation characteristics of the metro network in the evening (from 20:00 to 21:00 on October 14) are discussed under three scenarios ($q = 0.80, q = 0.50, q = 0.20$, representing the states of high, medium, and low tolerance index, respectively). Given the definition of tolerance index $q$, the larger $q$ is, the severer congestion one can tolerate.

As shown in Fig. 6, when $q = 0.80$, the network breaks into a large cluster and a small cluster. Connectivity is almost perfect for passengers with tolerance index $q$ equalling 0.80. When $q = 0.20$, a few small clusters are formed, indicating the links of low crowding only occupy a small part of the...
whole network, and some links are unavailable for bridging these parts and finally developing into poor network connectivity. When \( q = 0.50 \), five different sizes of clusters are formed. It is worth noting that the largest cluster suddenly breaks, and the size of the second-largest cluster (Fig. 6(b) in blue) reaches a maximum. We conclude that 0.50 is the critical threshold in the period from 20:00 to 21:00 on October 14 (see Fig. 6(d)). Besides, clusters internal connectivity is good, but the lack of connection among clusters brings congestion and affects the entire metro network connectivity.

This phenomenon indicates that metro network connectivity is pretty good for passengers with tolerance index \( q \) equalling to 0.8 or above. They can travel through most parts of the metro network (red lines). For passengers with tolerance index above 0.5, there is still a relatively large cluster for them to travel through. However, for passengers with tolerance index equalling to or slightly below 0.5, regions they can travel through become significantly small. At the same time, in order to achieve the condition of travel that is not too crowded, it is necessary to identify bottlenecks and improve them. Otherwise, there will be so few links available with the tolerance level of passengers decreasing, such as the percolation network shown in Fig. 6(c). Passengers with 20% tolerance index can travel through most parts of the metro network only through increasing their tolerance index.

2) TEMPORAL EVOLUTION OF METRO NETWORK CONNECTIVITY (DYNAMIC EVOLUTION OF METRO NETWORK CONNECTIVITY OVER TIME WITH FIXED \( q \) VALUE)

In this section, dynamic evolution of metro network connectivity is revealed as time changes with the fixed \( q \) value. Different states of the metro network during the period from 6:00 to 21:00 on October 14 for passengers with the same tolerance index \( q \) (\( q = 0.50 \)) are analysed. The distribution of percolation metro networks is shown in Fig. 7.

According to Fig. 7, the networks in the periods 11:00 ~ 12:00 and 13:00 ~ 14:00 form giant connected clusters, and network connectivity is pretty good for these passengers in these periods, which are lunch breaks around noon. Most links in the metro network can maintain available for this group of passengers. The networks in the periods 6:00 ~ 7:00 and 8:00 ~ 9:00 can form relatively large clusters, and metro network connectivity is not too bad. Major parts in the metro network can maintain available for them. 18:00 ~ 19:00 is the evening rush hour, and the metro network connectivity for them then is obviously worsened. Few links in the metro network can maintain available, and the whole network is split into several independent small regions (clusters). This means it comprises lots of congested links (relative passenger volume is greater than 50%). In addition, the network is quite crowded compared with that in the morning rush hour (8:00 ~ 9:00). There is the most obvious second largest cluster in the period 20:00 ~ 21:00, the evening time. The connectivity in the period 20:00 ~ 21:00 from their judgment is poorer than that at noon. Generally speaking, Fig. 7 reflects the temporal evolution during a day of Shenzhen metro network connectivity for a group of passengers with the same tolerance index.

IV. RESULTS AND DISCUSSION OF BOTTLENECK IDENTIFICATION

A. BOTTLENECK IDENTIFICATION OF METRO NETWORK

There are two ways to improve metro network connectivity: the one is to improve passengers’ individual tolerance index of congestion by increasing \( q \) so that enable the giant coverage parts of metro network to be available for them; the other is to improve friendliness of metro network by decreasing \( q^{(t)} \). The former way can hardly address underlying problems of congestion in metro systems, while the latter one can resolve the issue of metro network fundamentally. Bottleneck identification provides a low cost and efficient way to improve friendliness of metro network (e.g., improving a single metro link). Therefore, bottleneck identification of metro network is crucial to optimize the global network friendliness of metro system.

In this section, we apply the proposed framework to identify instantaneously those metro links bridging different clusters of low crowding (with respect to the bottleneck). In the case of percolation phase transition occurring in the metro...
network, the key links that connect different clusters can be found at the critical threshold $q_c^{(t)}$. Removal of these metro links will cause disintegration of the metro network into several isolated clusters (Fig. 8 (a) and Fig. 8(b)). Therefore, in the metro network, bottleneck plays a critical role in bridging different available clusters, and it is related to the network friendliness which ensures the metro network connectivity and availability for more passengers. To compare the impact of bottleneck links and other links (e.g., a randomly selected link, the most congested link and the link with the highest betweenness) on the network friendliness ($q_f^{(t)}$), we decrease the ratio ($r_{ij}^{(t)}$) of bottleneck links by a factor of $1 - \alpha$ ($\alpha > 0$) and measure the friendliness ($q_f^{(t)c^\alpha}$) of the modified metro network (Fig. 8(c)).

FIGURE 8. Bottlenecks of the metro network ($t = 17:00 \sim 18:00, 0.50 < q < 0.51$). (a) The percolation metro network just above critical threshold, where two links (in black circles) are removed at threshold. (b) Same metro network after removal of the two links. (c) The improvement of $q_f^{(t)}$ by decreasing separately the ratio ($r_{ij}^{(t)} = \frac{q_f^{(t)}}{q_f^{(0)}}(1 - \alpha)$) of bottlenecks marked in Fig. 8(a) and a randomly selected link, the most congested link and the link with the highest betweenness.

During the evening rush hour of Shenzhen metro ($t = 17:00 \sim 18:00, October 14$), the largest cluster in the percolation network just above the critical threshold $q_c^{(t)}$ ($q = 0.50$) is decomposed into two clusters (Fig. 8 (b)) when removing two links (in black circles of Fig. 8 (a)). For the percolation metro networks under different $q$ values in the evening rush hour, we find the connected clusters one by one, and identify the removed metro link when the second-largest connected cluster reaches a maximum, and that is the bottleneck. Meanwhile, $q$ value reaches the critical threshold $q_c^{(t)}$ at the moment. The bottlenecks identified in Fig. 8(a) are bottleneck A “Minzi → Shenzhen North Railway Station” and bottleneck B “Window of the World → Baishizhou”. Then, to compare the impact of bottlenecks with that of a randomly selected link (nonterminal link) (“Zhuzilin → Chegongmiao”), the link with the highest betweenness (“Xiangmihu → Shopping Park”) and the most congested link (with the highest relative passenger volume: “Lianhua North → Shangmeilin”) of the global metro system, we decrease the relative passenger volume ($r_{ij}$) of those links by a factor of $1 - \alpha(0 < \alpha < 1)$ and measure the new friendliness $q_f^{(t)c^\alpha}$ of the improved metro network. It is found in Fig. 8(C) that the new friendliness ($q_f^{(t)c^\alpha}$) of the metro network obviously decreases only through decreasing the relative passenger volume of the bottlenecks we identified. And the improvement of friendliness of the whole network is negligible when decreasing the relative passenger volume of a randomly selected link, the most congested link and the link with the highest betweenness, although the most congested link directly impacts the operational conditions of networks, and the link with the highest betweenness usually plays a critical role in network structure [33]–[35]. It suggests that the bottlenecks identified in our percolation network are different from the ones in conventional sense, because in addition to taking individual cognition towards congestion into account, we are looking for the bottleneck of metro system from the connectivity perspective considering both the network structure and flow.

B. EVOLVING BOTTLENECKS IN DIFFERENT PERIODS

Metro network is a dynamic system, and operation status of metro system and network connectivity are affected by multiple factors, such as the fluctuations in passenger volume. Therefore, the bottlenecks found here are hypothesized to evolve with time and are usually different in different hours (Fig. 9). As shown in Fig. 9, from the bottlenecks identified during different periods on weekday (October 14) and weekend (October 19) can be noted that the bottlenecks differentiate at the levels of time of the day and day of the week, which is due to travel patterns and travel demands varying along with time variation. Thereby, the operation strategies should be adjusted dynamically as time changes.


Meanwhile, it is found that the links as bottlenecks appear repeatedly in different periods (see Table 2). It is noted that the metro link with the transfer station serving as its vertex appears as a bottleneck more frequently, since this kind of metro link often plays an important role in bridging different
connected clusters. For example, “Shenzhen North Railway Station” is the transfer station of Shenzhen Metro Line 4 (one of the busiest lines in Shenzhen metro during rush hours) and Line 5. It also includes high-speed railway, road passenger transport, taxi, public transport and other modes of transport transfer, together to form a comprehensive large transportation hub,\(^3\) which plays a critical role in generating travel demands and connecting transit clusters. As shown in Table 2, “Shenzhen North Railway Station ↔ Minzhi” and “Shenzhen North Railway Station → Changlingpi” serve as bottlenecks in several time periods.

**TABLE 2. List of bottlenecks identified in different time periods.**

<table>
<thead>
<tr>
<th>Time Periods</th>
<th>Bottlenecks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekdays(Oct.14)</td>
<td>Weekdays(Oct.19)</td>
</tr>
<tr>
<td>7:00–8:00</td>
<td>“Shenzhen North Railway Station→Minzhi”</td>
</tr>
<tr>
<td></td>
<td>“Shenzhen North Railway Station→Changlingpi” &amp;</td>
</tr>
<tr>
<td></td>
<td>“Daxin-Taoyuan”</td>
</tr>
<tr>
<td>8:00–9:00</td>
<td>“Minle→Shangmeilin”</td>
</tr>
<tr>
<td></td>
<td>“Minle→Shangmeilin”</td>
</tr>
<tr>
<td>11:00–12:00</td>
<td>“Baishizhou→Window of the World”</td>
</tr>
<tr>
<td></td>
<td>“Changlingpi→Shenzhen North Railway Station” &amp;</td>
</tr>
<tr>
<td></td>
<td>“Baishizhou→Window of the World”</td>
</tr>
<tr>
<td>13:00–14:00</td>
<td>“Changlingpi→Shenzhen North Railway Station” &amp;</td>
</tr>
<tr>
<td></td>
<td>“Baishizhou→Window of the World”</td>
</tr>
<tr>
<td>16:00–17:00</td>
<td>“Baishizhou→Window of the World”</td>
</tr>
<tr>
<td>17:00–18:00</td>
<td>“Minzhi→Shenzhen North Railway Station”</td>
</tr>
<tr>
<td>20:00–21:00</td>
<td>“Minzhi→Shenzhen North Railway Station”</td>
</tr>
<tr>
<td></td>
<td>“Window of the World →Baishizhou”</td>
</tr>
<tr>
<td></td>
<td>“Window of the World”</td>
</tr>
<tr>
<td></td>
<td>“Window of the World →Baishizhou”</td>
</tr>
<tr>
<td></td>
<td>“Window of the World”</td>
</tr>
</tbody>
</table>

Besides, “Baishizhou → Window of the World” as a bottleneck exhibits the highest occurrence frequency in a day no matter weekday or weekend. As the vertex, “Window of the World” is a transfer station of Line 1 and Line 2. “Baishizhou”, located on Shennan Avenue, has convenient accessible transportation. Nowadays, Baishizhou is full of rent houses, and about 150,000 people live there, covering an area of only 0.6 square kilometers,\(^4\) which contributes to a large number of travel demands. These facts corroborate our identification of bottleneck “Baishizhou → Window of the World” with the highest occurrence frequency from the network structure and traffic flow standpoint. In addition, through comparing the bottlenecks identified in the same period of October 14 and October 19, we can note that the bottlenecks identified in the periods of 8:00 ~ 9:00 and 16:00 ~ 17:00 of these two days are the same, which demonstrates that the bottlenecks in a certain period will appear both on weekday and weekend, therefore, they should be attached importance to in the certain period.

Having detected the bottlenecks in different time periods and demonstrated their physical meanings with factual evidence, an improved understanding of the response to the bottlenecks should be addressed. Although passengers can increase their tolerance of congestion in order to ensure their network connectivity and availability judgment, it is not an ideal way to root out the core problem of network. Satisfyingly, as the proposed method enables us to identify instantaneously bottleneck at critical threshold \(q_c^{(1)}\), it can provide opportunities to improve the network performance with minor cost (e.g., improving a single link). In operational practice, it is recommended to develop strategies for improving the capacity of bottlenecks effectively, such as strengthening organization of passengers at platforms, developing personalized route planning dynamically for passengers and even improving intermodal transportation access where the bottleneck link located in, etc., which may ensure a friendly metro transportation environment satisfying more passengers.

**V. CONCLUSION**

In this paper, we developed a data-driven framework based on percolation theory to reveal the dynamic evolution of metro network connectivity and identify recurrent bottlenecks. Different from the previous related works, the proposed framework engaged passengers’ heterogeneous cognition towards congestion. We developed a measure, named *network friendliness*, that can well describe metro network connectivity considering both network structure and individual tolerance of congestion of metro system. Besides, we proposed an index, named *tolerance index*, to describe individual tolerance of congestion. By comparing individual tolerance index and the friendliness of metro network, metro network connectivity with regard to different passengers was depicted quantitatively. As we presented connectivity evolution in metro network by characterizing it as a percolation process, the breakdown of global transit was presented as identified bottlenecks were congested from the perspective of individual cognition.

A real-world case study of Shenzhen Metro was carried out to validate the proposed method. It was found that the proposed method can capture the dynamic evolution of Shenzhen metro network connectivity and enable effective identification of transit bottlenecks. In most cases, the bottlenecks identified with the method of this paper were different from the ones in a conventional sense, as the bottlenecks identified with the proposed method not only incorporated individual cognition but also considered both the network structure and flow from the connectivity perspective. Supported by the analysis of typical examples, we demonstrated the physical meanings of bottlenecks identified, which confirmed the rationality of the proposed method in the real metro system. Besides, the network connectivity and friendliness were significantly increased through a small improvement of bottlenecks pinpointed.

\(^3\)https://en.wikipedia.org/wiki/Shenzhen_North_station

\(^4\)Shenzhen BBS.https://szbbs.sznews.com/thread-3499504-1-1.html
In general, the proposed framework can serve to improve the whole metro network connectivity and construct a user-friendly metro network in a highly efficient, low cost way, so that can pave the way for new applications and a near real-time surveillance of patterns and dynamics of metro systems, in particular for the future realization of the “smart city.” Additionally, individual cognition towards congestion presented in this pilot study may encourage more data-driven research on metro network dynamics from the perspective of travel psychology and behavior of individuals.

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REFERENCES


YANG ZHAO received the bachelor’s degree in statistics from the Shandong University of Science and Technology, in 2011, and the Ph.D. degree from the City University of Hong Kong, in 2015. She is currently the Scientific Officer with the Centre for Systems Informatics Engineering, City University of Hong Kong. Her research interests are in machine learning and statistics, especially their application to real problems.

KWOK-LEUNG TSUI was a Professor with the School of Industrial and Systems Engineering, Georgia Institute of Technology. He is currently the Chair Professor of industrial engineering with the School of Data Science, City University of Hong Kong, and the Founder and Director of the Center for Systems Informatics Engineering. His current research interests include data mining, surveillance in healthcare and public health, prognostics and systems health management, calibration and validation of computer models, process control and monitoring, robust design, and Taguchi methods. He is a fellow of the American Statistical Association, the American Society for Quality, and the International Society of Engineering Asset Management, a U.S. representative to the ISO Technical Committee on Statistical Methods. He was a recipient of the National Science Foundation Young Investigator Award. He was the Chair of the INFORMS Section on Quality, Statistics, and Reliability and the Founding Chair of the INFORMS Section on Data Mining.