Distributed Coordinated Tracking Control for Multiple Uncertain Euler-Lagrange Systems With Time-Varying Communication Delays

SUN, Yanchao; DONG, Dingran; QIN, Hongde; WANG, Ning; LI, Xiaojia

Published in: IEEE Access

Published: 01/01/2019

Document Version: Final Published version, also known as Publisher's PDF, Publisher’s Final version or Version of Record

Publication record in CityU Scholars: Go to record

Published version (DOI): 10.1109/ACCESS.2019.2893261


Citing this paper Please note that where the full-text provided on CityU Scholars is the Post-print version (also known as Accepted Author Manuscript, Peer-reviewed or Author Final version), it may differ from the Final Published version. When citing, ensure that you check and use the publisher's definitive version for pagination and other details.

General rights Copyright for the publications made accessible via the CityU Scholars portal is retained by the author(s) and/or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights. Users may not further distribute the material or use it for any profit-making activity or commercial gain.

Publisher permission Permission for previously published items are in accordance with publisher's copyright policies sourced from the SHERPA RoMEO database. Links to full text versions (either Published or Post-print) are only available if corresponding publishers allow open access.

Take down policy Contact lbscholars@cityu.edu.hk if you believe that this document breaches copyright and provide us with details. We will remove access to the work immediately and investigate your claim.
Distributed Coordinated Tracking Control for Multiple Uncertain Euler–Lagrange Systems With Time-Varying Communication Delays

YANCHAO SUN$^{1*}$, DINGRAN DONG$^2$, HONGDE QIN$^{1*}$, NING WANG$^3$, AND XIAOJIA LI$^1$

$^1$Science and Technology on Underwater Vehicle Laboratory, Harbin Engineering University, Harbin 150001, China
$^2$Department of Biomedical Engineering, City University of Hong Kong, Hong Kong
$^3$School of Marine Electrical Engineering, Dalian Maritime University, Dalian 116026, China

Corresponding author: Hongde Qin (qinhd@hrbeu.edu.cn)

This work was supported by the National Natural Science Foundation of China under Grant U1713205 and Grant 61803119.

ABSTRACT In this paper, under-directed topology, distributed coordinated tracking control schemes are proposed for multiple Euler–Lagrange systems with time-varying communication delays, nonlinear uncertainties, and external disturbances. Concerning with different leader velocities, both constant leader velocity case and time-varying leader velocity case are addressed by designing two different distributed observers. Combining with the proposed distributed leader velocity observers, two coordinated tracking control schemes are developed by the effort of neural network approximation and sliding mode technique, which can compensate the nonlinearities and uncertainties. For the first case, tracking errors are rigorously proved to be globally asymptotically converged by using Lyapunov–Krasovskii method. To further eliminate chattering caused by the discontinuous sign function, the saturation function is used for the second case, and the proposed control algorithm ensures the same convergence of tracking errors via Lyapunov analysis. Finally, the effectiveness of the proposed distributed tracking control schemes is verified by the numerical examples.

INDEX TERMS Multi-agent systems, Euler-Lagrange systems, distributed tracking control, time-varying communication delay, neural network.

I. INTRODUCTION

Multi-agent system is a combination of several agents which cooperate with each other in the environment. These agents can be controlled independently to achieve given tasks cooperatively. Recently, coordinated control of multi-agent systems has received increasing attention from various scientific fields including unmanned aerial vehicles [1]–[4], robotic systems [5]–[9], autonomous underwater vehicles [10], [11], sensor networks [12]–[14], and spacecraft formations [15], [16].

At present, to facilitate the analysis and design procedures, most multi-agent systems are simplified to be composed of members with first-order or second-order linear dynamics. In [17], the consensus control for linear multi-agent systems whose dynamics with first-order and second-order dynamics has been investigated. The consensus was achieved with sufficient conditions under fixed and switching topologies. For second-order linear multi-agent systems, formation tracking control problems were studied in [18] under switching topologies. The proposed control protocol achieved time-varying formation tracking. However, in practice, most actual systems are nonlinear, the models mentioned above limit the applicability of multi-agent systems. Actually, Euler-Lagrange equations can be used to describe the dynamics model of most nonlinear systems, and can be called Euler-Lagrange (EL) systems. The research on coordinated control of EL systems has been developed rapidly in recent years [19]–[23]. Considering whether the desired information could be obtained by all agents, the control structures of the multiple EL systems can be divided into two types, namely, centralized and distributed control cases. Compared with centralized control, distributed control has higher reliability, lower energy consumption, and small communication burden [24]. Therefore, the investigation of distributed control has higher research value. For multiple EL systems, the distributed robust control algorithm was studied in [25] where the agents could communicate with their local neighbours bidirectionally. To compensate for the uncertainties, a disturbance observer and a sliding mode control term were introduced. Under the undirected topology, a distributed control
protocol for multiple EL systems was developed in [26] to realize the robust consensus tracking control where the nonlinear identifiers were proposed to deal with the nonlinear dynamics and disturbances. It should be noted that the communication topologies in aforementioned results are undirected cases which are not suitable to handle packet losses and network faults of the systems. The directed topology could be used to describe these communication conditions well. Based on the directed topology, an adaptive distributed control algorithm was proposed for multiple EL systems with communication constraints in [27], so that the states of the followers could be gradually synchronized with the leader state under the condition that the directed interconnection graph contained a spanning tree. Based on the directed topology, the distributed adaptive tracking control strategy for multiple EL systems with unknown trajectory information was investigated in [28], where virtual control inputs were designed, extra information transmission of local parameter estimates was adopted, and adaptive gain technique was used. The system achieved consensus tracking with the proposed algorithms.

According to the number of leader in the EL systems, the distributed control problems can be classified as leaderless and leader-following control cases. For the leaderless case, three consensus control algorithms for multiple EL systems were developed in [29] and achieved the generalized coordinates and their derivatives consensus. However, for the leaderless cases, there is a limitation for the final states of the agents. Therefore, more and more systems introduce the leader, and are more suitable for applications [30], [31]. The synchronization problems for networked uncertain EL systems under directed graph were considered in [32]. Two control protocols were designed and analyzed by a systematic way. For the systems with a dynamic leader, a distributed tracking controller was designed to achieve coordinated tracking by using Lyapunov methods. The finite-time tracking control problem for multiple EL systems was addressed in [33]. Based on the measurements of relative position and relative velocity, a distributed control algorithm which guaranteed finite-time tracking with control input constraints was proposed first. Based on the relative position measurements only, a new control strategy using state feedback analysis and second-order sliding-mode observer was developed. Yang et al. [34] investigated the distributed coordinated tracking control problem for multiple EL systems under directed topology. In order to estimate the leader states, the sliding mode observers were designed first. Considering the lacking of velocity measurements, the distributed observers were proposed. Based on the above observers, the coordinated tracking control algorithm was proposed to guarantee the followers track the dynamic leader.

For multi-agent systems, another challenge is the communication delay. Each agent needs to communicate with others while even small communication delays will have large impact to the whole closed-loop system [36]. Thus, when developing the distributed control protocols, to consider the communication delays could improve the control performance. For multiple EL systems with constant time delays, the states of the agents could be measured in [37] where the parameters were unknown. With a distributed adaptive control algorithm, the global full-state synchronization was realized. However, in practice, delays cannot be constant. For networked EL systems with time-varying transmission delays, the sampled-data communication method was used in [38] to exchange information. It not only achieved synchronization, but also reduced energy consumption under the proposed strategy. In [39], a distributed adaptive control scheme for multiple uncertain EL systems by using feedforward control technique, which could accommodate time-varying delays well. When designing distributed control laws for the multiple EL systems, model uncertainties and external disturbances should also be considered. Parametric uncertainties which can be linearized have been considered and investigated in some existing works [40]–[42]. However, nonlinear uncertainties and external disturbances could not be dealt with by utilizing the parameter linearization technique [43], [44]. Due to the good approximation capabilities for nonlinear functions, neural network (NN) can be used to compensate for the nonlinear system uncertainties [45]–[47]. To deal with the nonlinear uncertainties, adaptive NN for the multiple EL systems was used in [48]. By combining with the high-gain observer method, an output feedback coordinated tracking control algorithm was proposed. For the multiple uncertain EL systems, nonlinear uncertainties and external disturbances were taken into account in [49], and were approximated by the NN technique. On this basis, an improved distributed adaptive control algorithm was proposed, which made all the tracking errors ultimately bounded. For the distributed tracking control for multiple EL systems, some studies utilized sign function of the errors to cancel disturbance terms [50], [51]. Considering the case that external disturbances were bounded and the leader’s trajectory could not be known to all the followers in the multiple EL systems, sign function methods were used in [51] to design the distributed control strategy to achieved semiglobal synchronization asymptotically. However, the sign function is discontinuous, and will cause the chattering problem. To overcome this problem, saturation function can be used to replace it in the distributed tracking control protocol design. In [52], a comparison of distributed control algorithms using these two functions respectively showed the superiority of the saturation function case, which solved the chattering problem and achieved coordinated tracking control for the multiple EL systems.

In this paper, the distributed coordinated tracking control problem for multiple EL systems is investigated under directed topologies. Time-varying communication delays, nonlinear uncertainties, and external disturbances are considered, simultaneously. According to the state of the leader, two cases are addressed by designing effective control
algorithms, respectively. For the first case where the leader velocity is constant, distributed observers of leader velocity are designed to facilitate the coordinated control algorithm. By using NN and sliding mode control approach, distributed tracking control scheme is established whereby nonlinearities and uncertainties can be dominated well. The Lyapunov-Krasovskii technique is utilized to prove the stability of the tracking errors. For the case where the leader velocity is time-varying, a modified distributed observer is proposed via Lyapunov-Krasovskii method. In order to avoid the chattering arisen from sliding mode, the saturation function is utilized to replace the discontinuous sign function, and could also guarantee the convergence property of the tracking errors. Finally, the effectiveness of both proposed control strategies is verified by the numerical examples. The main contributions of this study are concluded as follow.

1. For multiple EL system with time-varying communication delay, two new control algorithms are proposed separately when the leader velocity is constant and time-varying.
2. The NN and sliding mode control techniques are utilized to estimate and compensate and the nonlinearities and uncertainties.
3. For the chattering problems when using sliding mode control, the continuous saturation function is introduced in the second case.

II. BACKGROUND AND PRELIMINARIES

In this section, the dynamics model of multiple EL systems and its important properties are introduced. Preliminaries of graph theory will also be recalled. Important related lemmas and assumptions are given to facilitate the following research and control law designs.

A. EULER-LAGRANGE DYNAMICS MODEL

Consider a multi-agent system consisting of $n$ followers (denoted by $v_F = \{1, 2, \ldots, n\}$) and one leader (denoted by 0). The dynamics model of the $i$th follower can be described by the following EL equation

$$M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + g_i(q_i) = \omega_i + \tau_i (i \in v_F)\quad (1)$$

where $q_i \in \mathbb{R}^p$ is the generalized position, $M_i(q_i) \in \mathbb{R}^{p \times p}$ is the system inertia matrix, which is symmetrical positive definite, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{p \times p}$ is the Coriolis and centrifugal force matrix, $g_i(q_i) \in \mathbb{R}^p$ is the gravitational force, and $\tau_i \in \mathbb{R}^p$ is the generalized control force, and $\omega_i \in \mathbb{R}^p$ represents the external disturbance. We assume $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, and $g_i(q_i)$ are unknown in this study.

EL system (1) has the following important properties:

- Property 1: $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, and $g_i(q_i)$ are bounded, i.e., for any $q_i \in \mathbb{R}^p$, there exist $0 < M_{m,i} \leq \|M_i(q_i)\| \leq M_{M,i}$, $\|C_i(q_i, \dot{q}_i)\| \leq C_{M,i}$, $\|\dot{q}_i\| \leq M_{M,i}$, and $\|g_i(q_i)\| \leq g_{M,i}$.

- Property 2: The matrix $M_i(q_i)x - 2C_i(q_i, \dot{q}_i)x$ is skew-symmetric, i.e., for any $x \in \mathbb{R}^p$, $x^T [M_i(q_i) - 2C_i(q_i, \dot{q}_i)] x = 0$.

In order to analyze the system further, two assumptions are given as follows:

**Assumption 1**: The generalized position of the leader is bounded, i.e., there exists a constant $q_M$, such that $\sup_t \|q_0(t)\| \leq q_M < \infty$.

**Assumption 2**: The disturbance $\omega_i$ is bounded, i.e., there exists an unknown positive constant $\omega_{M,i}$ such that $\|\omega_i\| \leq \omega_{M,i}$.

B. GRAPH THEORY

In this study, directed graph $G = \{v, e, A\}$ is used to describe the communication topology and information exchange among the agents. Graph $G = \{v, e, A\}$ is composed of a number of nodes and edges, where $v = \{1, 2, \ldots, n\}$ is the node set, and $e \subseteq v \times v$ is the edge set. Node $v_i$ denotes the $i$th agent and every edge of $G = \{v, e, A\}$ has two nodes $(v_i, v_j)$. Edge $(v_i, v_j) \in e$ denotes agent $j$ can obtain information from the $i$th agent, where $v_i$ is called the child node of $v_j$ and $v_j$ is the parent node of $v_i$. In the directed graph, the directed path is defined as the ordered sequence of edge set $(v_1, v_2), (v_2, v_3), \ldots$. If every node has exactly one parent node except for one node, called the root node, and there exists directed paths from the root node to the other nodes, then the directed graph is called the directed tree. The directed tree containing all nodes of the directed graph is called the directed spanning tree. If a directed graph contains a subgraph which is a directed spanning tree, the directed graph is said to have a directed spanning tree. The adjacency matrix of $G$ denotes as $A = [a_{ij}] \in \mathbb{R}^{n \times n}$, when $(v_j, v_i) \in e$, $a_{ij} = 1$, and otherwise $a_{ij} = 0$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n \times n}$ associated with the graph $G$ is defined as $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$.

Consider an augmented graph $\tilde{G}$ which is used to denote the communication topology for the one leader and $n$ followers in this research. The access of followers to the leader is denoted by a diagonal matrix $\text{diag}(a_{10}, \ldots, a_{n0})$, where $a_{0i}$ represents the information transfer weight from the leader to the $i$th follower. Define $H = L + \text{diag}(a_{10}, \ldots, a_{n0})$.

**Assumption 3**: There exist the directed paths from the leader to all followers, i.e., the directed graph $G$ has a directed spanning tree.

**Lemma 1** [32]: When the directed graph $\tilde{G}$ has a directed spanning tree, all eigenvalues of $H$ have positive real parts.

**Lemma 2** [32]: Define a symmetric positive definite matrix $P$. If the graph $\tilde{G}$ has a directed spanning tree, the matrix $R = PH + H^TP$ is symmetric positive definite.

C. GRAPH THEORY CONTROL OBJECTIVE

In the multiple EL systems (1), the leader’s generalized position and velocity are denoted as $q_0$ and $\dot{q}_0$, respectively. Considering time-varying communication delays and nonlinear model uncertainties, we plan to design distributed coordinated tracking control strategies which could ensure that all the followers can track the leader, i.e., the tracking errors and their derivatives could converge to zero asymptotically, i.e.,

$$\lim_{t \to \infty} (q_i - q_0) = 0, \quad \lim_{t \to \infty} (\dot{q}_i - \dot{q}_0) = 0\quad (2)$$

$$\lim_{t \to \infty} (\ddot{q}_i - \ddot{q}_0) = 0, \quad \lim_{t \to \infty} (\dddot{q}_i - \dddot{q}_0) = 0\quad (2)$$
III. DISTRIBUTED COORDINATED TRACKING CONTROL DESIGN

A. CONSTANT LEADER VELOCITY

We assume the leader’s velocity is constant in this subsection. To ensure that each follower could obtain the leader’s state information, especially when there exist time-varying communication delays, we first develop a novel distributed observer for each follower as follows:

\[
\dot{\hat{v}}_i = -\beta \left[ \sum_{j=1}^{n} a_{ij}(\hat{v}_j(t - T) - \dot{v}_j(t - T)) + a_{ii}(\hat{v}_i(t - T) - \dot{v}_i(t - T)) \right] \tag{3}
\]

where \(\beta\) is a positive constant, and \(T\) is the time-varying communication delay with \(T_0\) being its upper bound.

Design the following auxiliary variables

\[
\dot{q}_i = \hat{v}_i - \alpha \sum_{j=0}^{n} a_{ij}(q_i(t - T) - \dot{q}_j(t - T)) \tag{4}
\]

\[
s_i = \hat{q}_i - \hat{q}_r \tag{5}
\]

where \(\alpha\) is a positive constant.

Substituting (5) into (1), we can obtain

\[
M_i(q_i) \dot{s}_i + C_i(q_i, \dot{q}_i) s_i = -M_i(q_i) \ddot{q}_r - C_i(q_i, \dot{q}_i) \ddot{q}_r - g_i(q_i) + \omega_i + \tau_i \tag{6}
\]

Since \(M_i(q_i), C_i(q_i, \dot{q}_i), \) and \(g_i(q_i)\) are all assumed to be unknown, then (6) can be rewritten as

\[
M_i(q_i) \dot{s}_i + C_i(q_i, \dot{q}_i) s_i = f_i + \omega_i + \tau_i \tag{7}
\]

where \(f_i = -M_i(q_i) \ddot{q}_r - C_i(q_i, \dot{q}_i) \ddot{q}_r - g_i(q_i)\) denotes the nonlinear uncertainties. Due to the properties of NN, \(f_i\) can be denoted as

\[
f_i = W_i^T \phi_i (q_i, \dot{q}_i, \ddot{q}_i, \dot{q}_r) + \vartheta_i \tag{8}
\]

where \(W_i\) is the optimal NN weight matrix, \(\phi_i (q_i, \dot{q}_i, \ddot{q}_i, \dot{q}_r)\) is the active function, and \(\vartheta_i\) is the approximation error.

Assumption 4: The approximation error \(\vartheta_i\) is bounded, i.e., there exists bounded constant \(\vartheta_{Mi}\), such that \(\|\vartheta_i\| \leq \vartheta_{Mi}\).

For the \(i\)th follower, the active function can be expressed as \(\phi_i(v) = [\phi_i(v) \cdots \phi_r(v)]^T\). In this study, we choose Gaussian function as the active function

\[
\phi_{ij}(v) = \exp \left( -\frac{\|v - c_{ij}\|^2}{\sigma_{ij}^2} \right), \quad j = 1, \cdots, r \tag{9}
\]

where \(v = [q_i^T, \dot{q}_i^T, \ddot{q}_i^T, \dot{q}_r^T]^T \in \mathbb{R}^{4p}\), \(c_{ij} \in \mathbb{R}^{4p}\) denotes the center of the receptive field, \(\sigma_{ij} > 0\) is the width of the Gaussian function, and \(r\) represents the number of the neurons.

Although \(W_i\) is the optimal NN weight matrix, in practical NN applications, it is often known. Thus its estimates \(\hat{W}_i\) are often used such that the nonlinear term \(f_i\) can be expressed as

\[
\hat{f}_i = \hat{W}_i^T \phi_i (q_i, \dot{q}_i, \ddot{q}_i, \dot{q}_r) \tag{10}
\]

For the \(i\)th follower, the distributed tracking control algorithm is designed as

\[
\tau_i = -K_i s_i - \hat{W}_i^T \phi_i - \hat{\delta}_i \text{sgn}(s_i) \tag{11}
\]

where \(K_i\) is a symmetric positive definite matrix, \(\phi_i(q_i, \dot{q}_i, \ddot{q}_i, \dot{q}_r), \gamma_i\) and \(\delta_i\) are positive constants, and \(\|\cdot\|_1\) represents the sum norm of a vector.

To study further, two lemmas are given as follows.

Lemma 3 [53]: For any differentiable function \(f(t)\), if \(t \to \infty\), there exists a finite limit, and \(\dot{f}(t)\) is consistent, then when \(t \to \infty, \dot{f}(t) \to 0\) holds.

Lemma 4 [54]: Give a positive definite matrix \(M \in \mathbb{R}^{n \times n}\), two constants \(\gamma_1\) and \(\gamma_2, (\gamma_1 < \gamma_2)\), and one vector function \(\omega : [\gamma_1, \gamma_2) \to \mathbb{R}^p\), then the following inequality holds.

\[
\left( \int_{\gamma_1}^{\gamma_2} \omega(s) ds \right)^T M \left( \int_{\gamma_1}^{\gamma_2} \omega(s) ds \right) \leq (\gamma_2 - \gamma_1) \left( \int_{\gamma_1}^{\gamma_2} \omega^T(s) M \omega(s) ds \right) \tag{12}
\]

To analyze the system stability, we choose the following Lyapunov function candidate

\[
V_1 = \frac{1}{2} \sum_{i=1}^{n} s_i^T M_i \vartheta_i + \frac{1}{2 \gamma_1} \text{tr} \left( \hat{W}_i^T \hat{W}_i \right) + \frac{1}{2 \delta_i} \tilde{k}_i^2 \tag{13}
\]

where \(\hat{W}_i W_i - \hat{W}_i \) and \(\tilde{k}_i k_i - \tilde{k}_i\).

Take the derivative of (12), and substitute (11) into it. According to Property 2, we can obtain

\[
\dot{V}_1 = \sum_{i=1}^{n} s_i^T \left( -K_i s_i + \hat{W}_i^T \phi_i + \vartheta_i + \omega_i - \hat{\delta}_i \text{sgn}(s_i) \right) + \frac{1}{\gamma_1} \text{tr} \left( \hat{W}_i^T \hat{\dot{W}}_i \right) + \frac{1}{\delta_i} \left( \tilde{k}_i - \tilde{k}_i \right) \cdot \left( \tilde{k}_i \right) \tag{13}
\]

where \(\hat{W}_i \) is the estimate of \(W_i\). Then

\[
\dot{V}_1 = \sum_{i=1}^{n} s_i^T \left( -K_i s_i + \hat{W}_i^T \phi_i + \vartheta_i + \omega_i - \hat{\delta}_i \text{sgn}(s_i) \right) - \text{tr} \left( \hat{W}_i^T \hat{\phi_i s}_i^T \right) + \frac{1}{\delta_i} \left( \tilde{k}_i - \tilde{k}_i \right) \cdot \left( \tilde{k}_i \right) \tag{14}
\]

Since \(s_i^T \hat{W}_i^T \phi_i\) is a scalar, then

\[
s_i^T \hat{W}_i^T \phi_i = \text{tr} \left( s_i^T \hat{W}_i^T \phi_i \right) = \text{tr} \left( \hat{W}_i^T \phi_i s_i^T \right) \tag{15}
\]

Substituting it into (14), we have

\[
\dot{V}_1 \leq - \sum_{i=1}^{n} s_i^T K_i s_i + \frac{1}{\gamma_1} \left( \vartheta_i + \omega_i \right) - (\vartheta_{Mi} + \omega_{Mi}) \|s_i\|_1 \tag{16}
\]
According to the definition of $\| \cdot \|_1$ and Assumptions 2 and 4, we can obtain
\begin{equation}
\dot{V}_1 \leq - \sum_{i=1}^{n} s_i^T K s_i \tag{17}
\end{equation}

From (17), $V_1(t) \leq V_1(0)$ can be obtained. Then according to (12), $s_i \in L_\infty$ and $\Theta_i \in L_\infty$. Further, according to (5), $\dot{q}_i \in L_\infty$ and $\sum_{j=0}^{\infty} [a_{ij}(q(t-T) - q_j(t-T))] \in L_\infty$. Then from (3) and (4), we can obtain $\dot{q}_i$ and $M_i(q_i)$ are bounded while $C(q_i, \dot{q}_i)$ is also bounded when $\dot{q}_i$ is bounded. Then from (6), one has $\dot{s}_i$ is also bounded. Since $V_1 \geq 0$ and $\dot{V}_1 \leq 0$, there exists $V(\infty) \in [0, V(0))$ such that $\lim_{t \to \infty} V(t) = V(\infty)$. Integrating (17), we can obtain
\begin{equation}
\lambda_{\min}(K) \int_0^{\infty} \dot{s}_i(T) s_i(T) dT \leq V_1(0) - V_1(\infty) \tag{18}
\end{equation}

where $\lambda_{\min}(\cdot)$ denotes the minimum singular value of a matrix. From (18), we can know $s_i \in L_2$. Therefore, $s_i \in L_2 \cap L_\infty$ and $\dot{s}_i \in L_\infty$. According to Lemma 3, when $t \to \infty$, $s_i \to 0$.

Define $\ddot{q} = q - I_n \otimes q_0$ and $\ddot{v} = \ddot{v} - I_n \otimes \dot{q}_0$. Then (3) and (5) can be written in the vector forms as follows:
\begin{align}
\dot{v} &= -\beta(H \otimes I_p) \ddot{v}(t-T) \tag{19} \\
\ddot{x} &= -\alpha(H \otimes I_p) \ddot{q}(t-T) + \ddot{v} \tag{20}
\end{align}

Consider the following system
\begin{equation}
\begin{cases}
\dot{\ddot{q}} = -\alpha(H \otimes I_p) \ddot{q}(t-T) + \ddot{v} \\
\dot{\ddot{v}} = -\beta(H \otimes I_p) \ddot{v}(t-T) 
\end{cases} \tag{21}
\end{equation}

Let $x = [\ddot{q}^T, \ddot{v}^T]^T$, then (21) can be written as
\begin{equation}
\dot{x} = \begin{bmatrix} 0 & I_{3n} \\ 0 & 0 \end{bmatrix} x - \begin{bmatrix} \alpha(H \otimes I_3) & 0 \\ 0 & \beta(H \otimes I_3) \end{bmatrix} x(t-T) = Gx - Fx(t-T) \tag{22}
\end{equation}

where $G = \begin{bmatrix} 0 & I_{3n} \\ 0 & 0 \end{bmatrix}$ and $F = \begin{bmatrix} \alpha(H \otimes I_3) & 0 \\ 0 & \beta(H \otimes I_3) \end{bmatrix}$.

According to Lemma 1, the eigenvalues of $H$ have positive real parts. Therefore, all the eigenvalues of matrix $G-F$ have negative real parts.

**Theorem 1:** Under Assumptions 1-4 for the multiple EL systems (1) with time-varying communication delays, the derivative of delay $T$ satisfies $0 \leq \dot{T}(t) \leq d < 1$, if there exist $P > 0$, $Q \geq 0$, and $R > 0$ to make the following LMI hold,
\begin{equation}
M = \begin{bmatrix} PF + (1-d)Q & F \bar{T}_0 + (1-d)Q \\ F \bar{T}_0 + (1-d)Q & R + (1-d)Q \end{bmatrix} + T_0^2 \begin{bmatrix} (G-F)^T \\ F \end{bmatrix} \begin{bmatrix} (G-F) & F \end{bmatrix} \leq 0 \tag{23}
\end{equation}

then the followers could track the leader asymptotically under distributed tracking control algorithm (11).

**Proof:** Choose the following Lyapunov-Krasovskii function candidate
\begin{equation}
V_2 = x^T P x + \int_{t-T}^{t} x^T(Q x)(\xi)d\xi + T_0 \int_{t-T_0}^{t} (\xi - t + T_0) x^T(\xi) R \tilde{x}(\xi)d\xi \tag{24}
\end{equation}

Taking the derivative of (24), we can obtain
\begin{equation}
\dot{V}_2 = 2x^T P \dot{x} + x^T Q x - (1 - \dot{T}) x^T(t-T) Q x(t-T) + T_0^2 \dot{x}^T(t-T) R \tilde{x}(t-T)
\end{equation}

Defining $\ddot{x} = x - x(t-T)$, we have $\int_{t-T_0}^t \ddot{x}(\xi)d\xi = x(t) - x(t-T_0) = \ddot{x}(t)$.

Then according to Lemma 4, the following inequality can be obtained
\begin{equation}
T_0 \int_{t-T_0}^{t} \ddot{x}^T(t) R \tilde{x}(\xi)d\xi \geq \ddot{x}^T(t) R \ddot{x}(t) \tag{26}
\end{equation}

Since time delay $T$ has the upper bound and $0 \leq \dot{T}(t) \leq d < 1$, then according to (22),(26), and Lemma 2, (25) can be written as
\begin{equation}
\dot{V}_2 \leq 2x^T P([Gx - Fx(t-T)] + x^T Q x - (1 - \dot{T}) x^T(t-T) Q(x(t-T) + T_0^2 \ddot{x}^T R \tilde{x} - \ddot{x}^T R \tilde{x})
\end{equation}

\begin{equation}
\leq 2x^T P([G - F]x + F \ddot{x}) + \dot{\tilde{x}}^T Q \ddot{x}
\end{equation}

\begin{equation}
+ T_0^2 \ddot{x}^T R([G - F]x + F \ddot{x}) - \ddot{x}^T R \ddot{x}
\end{equation}

\begin{equation}
= [x^T \ddot{x}] M [x^T \ddot{x}] \dot{T}^T \tag{27}
\end{equation}

According to (23) and (27), there exists a positive constant $\lambda$, such that $\dot{V}_2 \leq -\lambda \|x\|^2$. According to Lyapunov-Krasovskii theorem, system (21) is asymptotically stable. Moreover, from the above analysis, $s_i \in L_2$ and $s_i \to 0$. Then according to [55, Th. 2.15], system (20) is also asymptotically stable, i.e., $\lim \ddot{q}(t) = 0$ and $\lim \tilde{q}(t) = 0$. This completes the proof of Theorem 1.

**B. TIME-VARYING LEADER VELOCITY**

In this section, the leader’s generalized velocity $\dot{q}_0$ is supposed to be time-varying. In this case, we assume that the leader’s trajectory can be expressed in the following form [56]
\begin{equation}
\dot{v} = S v \tag{28}
\end{equation}
\begin{equation}
\dot{q}_0 = C v \tag{29}
\end{equation}

where $v \in R^m$ is a constant vector, and $S \in R^{m \times m}$ and $C \in R^{m \times m}$ are constant matrices.

Here we give the last assumption for this study.

**Assumption 5:** All the eigenvalues of the matrix $S$ are semi-simple with zero real parts.

**Remark 1:** If Assumption 5 holds, all the components of $q_0$ are sinusoidal functions. Without loss of generality, we suppose that there exist integers $m_0$, $m_1 \geq 0$ which satisfy $m_0 + 2m_1 = m$, and positive real number $S_0^1, \ldots, S_0^{m_1}$ such that
\begin{equation}
S = \text{block diag}(0_{m_0 \times m_0}, \text{diag}(S_0^1, \ldots, S_0^{m_1}) \otimes a) \tag{29}
\end{equation}
where \( a = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \). It should be noted that \( S \in R^{n \times m} \) is skew-symmetric in \((29)\). Define \( \eta = \text{col}(\eta_1^1, \ldots, \eta_m^m) \).

In order to solve the problem that the leader’s time-varying velocity information is only available to some followers, we first proposed the following distributed adaptive observers

\[
\hat{\eta}_i = -\mu_1 \sum_{j=0}^{n} \left[ a_{ij}(\hat{\eta}_i(t) - \eta_j(t)) \right] \\
\hat{\nu}_i = \hat{S}_i \hat{\nu}_i - \mu_2 \sum_{j=0}^{n} \left[ a_{ij}(\hat{\nu}_i(t) - \nu_j(t)) \right] 
\]

where \( i = 1, \ldots, n, \hat{\eta}_i = \text{col}(\hat{\eta}_i^1, \ldots, \hat{\eta}_i^m) \in R^{m}, \nu_0 = \nu, \mu_1, \mu_2 > 0, \) and

\[
\hat{S}_i = \text{block diag}(0_{m_0 \times m_0}, \text{diag}(\eta_1^1, \ldots, \eta_i^m) \otimes a) 
\]

To proceed further, we give the following lemma.

Lemma 5: Consider system \((28)\) under Assumption 3 and 5, if communication delay \( T \) satisfies \( 0 \leq T(t) \leq d < 1 \), and there exist symmetric positive definite matrices \( Q, G, \) and \( F \) such that

\[
M = \begin{bmatrix} Q & -2\mu_1 (H \otimes I_{m_1}) \\ 0 & -(1 - \hat{T})Q \end{bmatrix} < 0, \\
N = \begin{bmatrix} G & -2\mu_2 (H \otimes I_{m_1}) \\ 0 & -(1 - \hat{T})G \end{bmatrix} < 0, \\
E = \begin{bmatrix} F & -2\mu_2 (H \otimes I_{m_1}) \\ 0 & -(1 - \hat{T})F \end{bmatrix} < 0, 
\]

then \( \lim_{t \to \infty} \hat{S}_i - S = 0, \lim_{t \to \infty} \hat{\nu}_i - \nu = 0 \) can be achieved under the distributed observer \((30)-(31)\).

Proof: Let \( \hat{S}_i = \hat{S}_i - S, \hat{\nu}_i = \hat{\nu}_i - \nu, \hat{S} = \text{col}(\hat{S}_1, \ldots, \hat{S}_n), \hat{\nu} = \text{col}(\hat{\nu}_1, \ldots, \hat{\nu}_n), \).

Equation \((31)\) can be written in the vector form as

\[
\dot{\hat{S}} = -\mu_1 (H \otimes I_{m_1}) \hat{S}(t - T) \\
\dot{\hat{\nu}} = \hat{S}_d \hat{\nu} + (I_n \otimes S) \hat{\nu} - \mu_2 (H \otimes I_{m_1}) \hat{\nu}(t - T) 
\]

First, we will prove \( \lim_{t \to \infty} \hat{S} = 0 \). Let \( \hat{\eta}_i = \hat{\eta}_i - \eta, \hat{\eta} = \text{col}(\hat{\eta}_1, \ldots, \hat{\eta}_n) \). Equation \((30)\) can be expressed in vector form as

\[
\dot{\hat{\eta}} = -\mu_1 (H \otimes I_{m_1}) \hat{\eta}(t - T) 
\]

From \((32)\), it is worth noting that only \( \hat{\eta}_i^1, \ldots, \hat{\eta}_i^m \) are nonzero elements in \( \hat{S}_i \). Thus the stability problem of system \((34)\) could be transformed into the stability problem of system \((36)\). Continue the following Lyapunov-Krasovskii function candidate

\[
V_1 = \hat{\eta}^T \hat{\eta} + \int_{t - T}^{t} \hat{\eta}^T (\xi) Q \hat{\eta}(\xi) d\xi 
\]

Taking the derivative of \((37)\), one can obtain

\[
\dot{V}_1 = 2\hat{\eta}^T \hat{\eta} + \hat{\eta}^T Q \hat{\eta} - (1 - \hat{T}) \hat{\eta}^T (t - T) Q \hat{\eta}(t - T) \\
= -2\mu_1 \hat{\eta}^T (H \otimes I_{m_1}) \hat{\eta}(t - T) + \hat{\eta}^T Q \hat{\eta} \\
- (1 - \hat{T}) \hat{\eta}^T (t - T) Q \hat{\eta}(t - T) \\
= [ \hat{\eta}^T - \hat{\eta}^T (t - T) ] M [ \hat{\eta}^T - \hat{\eta}^T (t - T) ]^T 
\]

where \( M = \begin{bmatrix} Q - 2\mu_1 (H \otimes I_{m_1}) \\ 0 & -(1 - \hat{T})Q \end{bmatrix} \). If \( M \leq 0 \) holds, \( V_1 < 0 \). Then system \((36)\) is asymptotically stable. Therefore, \( \lim_{t \to \infty} \hat{\eta} = 0 \), and further, \( \lim_{t \to \infty} \hat{S} = 0 \).

Then we give the stability analysis for system \((35)\). The boundedness of \( \hat{\nu} \) should be proved first. Choose the following Lyapunov-Krasovskii function

\[
V_2 = \nu^T \hat{\nu} + \int_{t - T}^{t} \nu^T (\xi) G \nu(\xi) d\xi 
\]

Taking the derivative of \((39)\) gives

\[
\dot{V}_2 = 2\nu^T \hat{\nu} + \nu^T G \nu - (1 - \hat{T}) \nu^T (t - T) G \nu(t - T) \\
= 2\nu^T [ \hat{S}_d \nu + (I_n \otimes S) \nu - \mu_2 (H \otimes I_{m_1}) \nu(t - T) ] + \nu^T G \nu \\
- (1 - \hat{T}) \nu^T (t - T) G \nu(t - T) \\
= [ \nu^T - \nu^T (t - T) ] N [ \nu^T - \nu^T (t - T) ]^T 
\]

If \( N \leq 0 \) holds, \( V_2 < 0 \). Therefore, we can know \( \nu \) is bounded.

Choose the following Lyapunov-Krasovskii function candidate

\[
V_3 = \nu^T \hat{\nu} + \int_{t - T}^{t} \nu^T (\xi) F \nu(\xi) d\xi 
\]

According to \((35)\) and taking the derivative of \((41)\), we can obtain

\[
\dot{V}_3 = 2\nu^T \nu + \nu^T F \nu - (1 - \hat{T}) \nu^T (t - T) F \nu(t - T) \\
= 2\nu^T [ \hat{S}_d \nu + (I_n \otimes S) \nu - \mu_2 (H \otimes I_{m_1}) \nu(t - T) ] + \nu^T F \nu - (1 - \hat{T}) \nu^T (t - T) F \nu(t - T) \\
= 2\nu^T \hat{S}_d \nu + 2\nu^T (I_n \otimes S) \nu - 2\mu_2 \nu^T (H \otimes I_{m_1}) \nu(t - T) + \nu^T F \nu - (1 - \hat{T}) \nu^T (t - T) F \nu(t - T) 
\]

Let \( d = 2\nu^T \hat{S}_d \nu \) and \( W = V_3 = \int_{t - \infty}^{t} d(\xi) d\xi \), where \( W \) is a continuous scalar function. According to Assumption 5 and \((40)\), \( \nu \) and \( \nu \) is bounded. Therefore, \( \nu \) is also bounded. From \((38)\), \( \lim_{t \to \infty} \hat{S} = 0 \) can be obtained. Therefore, \( \int_{t - \infty}^{t} d(\xi) d\xi \) exists and is also bounded. Further, \( \nu \) is lower bounded. According to \((42)\), we have

\[
\dot{W} = 2\nu^T (I_n \otimes S) \nu - 2\mu_2 \nu^T (H \otimes I_{m_1}) \nu(t - T) \\
+ \nu^T F \nu - (1 - \hat{T}) \nu^T (t - T) F \nu(t - T) \\
= -2\mu_2 \nu^T (H \otimes I_{m_1}) \nu(t - T) + \nu^T F \nu \\
- (1 - \hat{T}) \nu^T (t - T) F \nu(t - T) \\
= [ \nu^T - \nu^T (t - T) ] E [ [ \nu^T - \nu^T (t - T) ]^T 
\]

where \( E = \begin{bmatrix} F & -2\mu_2 (H \otimes I_{m_1}) \\ 0 & -(1 - \hat{T})F \end{bmatrix} \).

If \( E \leq 0 \), then \( \dot{W} \leq 0 \), system \((35)\) is asymptotically stable. According to \((41)\), \( \lim_{t \to \infty} \nu = 0 \) can be proved.
Remark 2: Let \( e_i = \mu_2 \sum_{j=0}^{n} [a_{ij}(\dot{v}_j(t) - T) - \dot{v}_j(t) - T)] \).

Then
\[
\lim_{t \to \infty} (C\dot{v}_i(t) - q_0(t)) = \lim_{t \to \infty} (C\dot{v}_i(t) - C\dot{v}(t)) = \lim_{t \to \infty} C\dot{v}_i(t) = 0
\]

\[
\lim_{t \to \infty} (C\dot{v}_i(t) - \dot{q}_0(t)) = \lim_{t \to \infty} (C\dot{v}_i(t) - C\dot{v}(t)) = \lim_{t \to \infty} C(\dot{S}_i(t)\dot{v}_i(t) - \dot{e}_i(t) - S\dot{v}_i(t)) - \lim_{t \to \infty} C(\dot{S}_i(t)\dot{v}_i(t) - \dot{e}_i(t) + S\dot{v}_i(t)) = 0
\]

According to Remark 2, we can conclude that the distributed observers (30)-(31) for the followers achieve good observation for the leader’s position \( q_0 \) and velocity \( \dot{q}_0 \). Then we define the following auxiliary variables before the coordinated tracking control strategy is given

\[
\dot{q}_{ri} = C\dot{S}_i\dot{v}_i - \gamma(q_i - C\dot{v}_i) \quad \text{and} \quad s_i = \dot{q}_i - \dot{q}_{ri}
\]

where \( \gamma \) is a positive constant.

Substituting (46) into (1), we have

\[
M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i = -M_i(q_i)\dot{q}_{ri} - C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) + \omega_i + \tau_i \quad \text{(47)}
\]

Here, we use the same method as control algorithm (11) to approximate model uncertainties. However, algorithm (11) has utilized the discontinuous sign function, which can lead to undesirable chattering. In order to solve the chattering problem, the following modified coordinated tracking control protocol is designed

\[
\tau_i = -k_s \dot{q}_i + \dot{W}_i^T \phi_i - k_i \text{sat}(s_i) \quad \dot{W}_i = \gamma_i \phi_i s_i^T \quad \text{(48)}
\]

where

\[
k_i \geq \|\omega_i + \theta\| \quad \text{and} \quad \text{sat}(s_i) = \begin{cases} \text{sgn} (\frac{s_i}{\delta}) , & \|\frac{s_i}{\delta}\| \geq 1 \\ \frac{s_i}{\delta} , & \text{otherwise} \end{cases} \quad \text{(50)}
\]

and \( \delta \) is a positive constant.

**Theorem 2:** Under Assumptions 1-5 for the multiple EL systems (1) with time-varying communication delays satisfying \( 0 \leq \hat{T}(t) \leq d < 1 \), if there exist symmetric positive definite matrices \( Q, G, \) and \( F \) such that (33) holds, then the coordinated tracking control algorithm (48) with the distributed observer (30)-(31) can make the followers track the leader asymptotically.

**Proof:** Choose the Lyapunov function candidate as follow.

\[
V = \frac{1}{2} \sum_{i=1}^{n} s_i^T M_i s_i + \frac{1}{2\gamma_i} \text{tr}(\dot{W}_i^T \dot{W}_i) \quad \text{(51)}
\]

Taking the derivative of (51) and substituting (48) into it, we have

\[
\dot{V} = \sum_{i=1}^{n} s_i^T (-k_s \dot{q}_i + \dot{W}_i^T \phi_i + \theta_i + \omega_i - k_i \text{sat}(s_i)) + \frac{1}{\gamma_i} \text{tr}(\dot{W}_i^T \dot{W}_i) \quad \text{(52)}
\]

where Property 2 is utilized. Since \( \dot{W}_i = -\dot{\dot{W}}_i = -\gamma_i \phi_i s_i^T \), we have

\[
\dot{V} = \sum_{i=1}^{n} s_i^T (-k_s \dot{q}_i + \dot{W}_i^T \phi_i + \theta_i + \omega_i - k_i \text{sat}(s_i)) - \text{tr}(\dot{W}_i^T \phi_i s_i^T) \quad \text{(53)}
\]

Since \( s_i^T \dot{W}_i^T \phi_i \) is a scalar, one has

\[
\dot{V} = \sum_{i=1}^{n} s_i^T (-k_s \dot{q}_i + \dot{W}_i^T \phi_i) = \text{tr}(\dot{W}_i^T \phi_i s_i^T) \quad \text{(54)}
\]

Then, we can obtain

\[
\dot{V} = \sum_{i=1}^{n} s_i^T (-k_s \dot{q}_i + \dot{W}_i^T \phi_i + \theta_i + \omega_i - k_i \text{sat}(s_i)) \quad \text{(55)}
\]

According to (50),

**Case 1:** If \( \|\frac{s_i}{\delta}\| \geq 1 \), then sat \((s_i) = \text{sgn}(\frac{s_i}{\delta}) \) and

\[
\dot{V} \leq \sum_{i=1}^{n} s_i^T (-k_s \dot{q}_i + \dot{W}_i^T \phi_i + \theta_i + \omega_i - k_i \|\frac{s_i}{\delta}\|) \quad \text{(56)}
\]

Further, we can obtain

\[
\dot{V} \leq \sum_{i=1}^{n} \left(-s_i^T k_s \dot{q}_i + \|s_i\| \|\theta_i + \omega_i\| - k_i \|\frac{s_i}{\delta}\| \right) \quad \text{(57)}
\]

Substituting (49) into it, we have

\[
\dot{V} \leq - \sum_{i=1}^{n} s_i^T k_s \dot{q}_i \leq 0 \quad \text{(58)}
\]

**Case 2:** If \( \|\frac{s_i}{\delta}\| < 1 \), sat \((s_i) = \frac{s_i}{\delta} \)

\[
\dot{V} = \sum_{i=1}^{n} s_i^T (-k_s \dot{q}_i + \dot{W}_i^T \phi_i + \theta_i + \omega_i - k_i \frac{s_i}{\delta}) \leq \sum_{i=1}^{n} \left(-s_i^T k_s \dot{q}_i + \|s_i\| \left|\frac{k_i s_i}{\delta} - \|\theta_i + \omega_i\|\right| \right) \quad \text{(59)}
\]

If \( \delta \leq \frac{k_i \|s_i\|}{\|\theta_i + \omega_i\|} \), we can obtain

\[
\dot{V} \leq - \sum_{i=1}^{n} s_i^T k_s \dot{q}_i \leq 0 \quad \text{(60)}
\]
From (58) and (60), we can find $V(t) \leq V(0)$. Therefore, $s_i \in L_\infty$, $\tilde{q}_i \in L_\infty$, and $\tilde{k}_i \in L_\infty$. Since $V(t) \geq 0$, $\dot{V}(t) \leq 0$, there exists $V_\infty \in [0, V(0)]$ such that $t \rightarrow \infty$ $V(t) = V_\infty$. Following the similar procedure as (17)-(18), it can be proved that $s_i \rightarrow 0$ when $t \rightarrow \infty$.

According to (45) and (46), we have

$$
\dot{q}_i - C_i \dot{\tilde{v}} + \gamma(q_i - C_i \tilde{v}) = \dot{q}_i - C_i (\dot{\tilde{v}} + e_i) + \gamma(q_i - C_i \tilde{v}) = s_i + Ce_i \tag{61}
$$

Equation (61) can be seen as a first-order differential equation, whose state is $q_i - C_i \tilde{v}$ and input is $s_i + Ce_i$. According to Lemma 5 and Remark 2, we have for all $t \geq 0$, $s_i + Ce_i \rightarrow 0$ as $t \rightarrow \infty$. Therefore, $\dot{q}_i - C_i \tilde{v}$ and $q_i - C_i \tilde{v}$ both converge to the original points when $t \rightarrow \infty$. According to (44), we can obtain that $\lim_{t \rightarrow \infty} (q_i(t) - q_0(t)) = 0$, $\lim_{t \rightarrow \infty} (\dot{q}_i(t) - \dot{q}_0(t)) = 0$. Thus the proof of Theorem 2 is completed.

**Remark 3:** For the multiple EL systems with time-varying communication delays, the coordinated tracking control law (48) with the distributed observers (30)-(31) can ensure the followers track the time-varying leader even with the restrictions mentioned in Theorem 2.

**Remark 4:** In Theorem 1, the upper bound of the communication delay $T$ can be calculated by the LMI (23). However, there is no constraint on $T$ in Theorem 2 when the velocity of the leader is time-varying. Therefore, $T$ calculated in Theorem 1 can be also used in Theorem 2.

### IV. SIMULATION STUDIES

Consider a group of two-link manipulators with one leader and four followers whose communication topology is shown in Fig. 1, simulations are performed to show the effectiveness of the proposed algorithms.

The dynamics models of the followers are as follow.

$$
M_i (q_i) \ddot{q}_i + C_i (q_i, \dot{q}_i) \dot{q}_i + g_i (q_i) = \omega_i + \tau_i, \quad i = 1, \ldots, 4
$$

where

$$
q_i = [q_{i1}, q_{i2}]^T,
$$

$$
M_i (q_i) = \begin{bmatrix}
\Theta_{i1} + \Theta_{i2} + 2\Theta_{i3} \cos q_{i2} & \Theta_{i2} + \Theta_{i3} \cos q_{i2} \\
\Theta_{i2} + \Theta_{i3} \cos q_{i2} & \Theta_{i2}
\end{bmatrix},
$$

$$
C_i (q_i, \dot{q}_i) = \begin{bmatrix}
-\Theta_{i3} (\sin q_{i2}) \dot{q}_i & -\Theta_{i3} (\sin q_{i2}) (\dot{q}_{i1} + \dot{q}_{i2}) \\
\Theta_{i3} (\sin q_{i2}) \dot{q}_{i1} & 0
\end{bmatrix},
$$

$$
G_i (q_i) = \begin{bmatrix}
\Theta_{i4} \cos q_{i1} + \Theta_{i5} \cos (q_{i1} + q_{i2}) \\
\Theta_{i2} \cos (q_{i1} + q_{i2})
\end{bmatrix}.
$$

Then the initial conditions of the manipulators are given as follow.

$$
q_{i1} (0) = \pi / 5, \quad q_{i2} (0) = -\pi / 3, \quad \dot{q}_{i1} (0) = 0, \quad \dot{q}_{i2} (0) = 2\pi / 5,
$$

$$
q_{i3} (0) = -\pi / 6, \quad q_{i4} (0) = 3\pi / 5, \quad q_{i5} (0) = \pi / 6,
$$

$$
q_{i6} (0) = 4\pi / 5, \quad \dot{q}_{i6} (0) = 0, \quad \dot{q}_{i7} (0) = 0.
$$

For the $i$th follower, the activation function of the NN model is chosen as $\varphi_i (z) = [\varphi_{i1} (z), \ldots, \varphi_{i6} (z)]$, $\varphi_{ij} (z) = \exp \left( -\frac{1}{\sigma_{ij}} ||z-c_{ij}||^2 \right)$, $j = 1, \ldots, 6$, where $c_{ij}$ is the center of the receptive field and is evenly distributed in $[-5, 5]^4 \times [-0.5, 0.5]^4$, $\sigma_{ij}$ denotes the width of the Gaussian function, which is chosen as $\sigma_{ij} = 2$. The initial weights matrix $\tilde{W}(0)$ is chosen as $\tilde{W}(0) = 0_{6 \times 2}$. The time-varying delay is chosen as $T = [0.3 + 0.05 \sin(2\pi t)] s$.

### A. SIMULATION RESULTS FOR DISTRIBUTED CONTROL ALGORITHM (11)

For the first case, the leader has a constant velocity. The initial position and velocity of the leader are $q_0 (0) = [0, 0]^T$ and $\dot{q}_0 (0) = [-0.4, 0.5]^T$. The parameters of the observer (3) and controller (11) are selected as $\alpha = 10$, $\beta = 1$, $\delta_i = 1, \gamma_i = 0.1$, and $K_i = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$. The initial values of $\tilde{v}_i$ and $\tilde{k}_i$ are set to be zero. The simulation results are shown in Figs. 2-7.

Figs. 2 and 3 show the position tracking errors between each follower and the leader, which are defined as

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Manipulator 1</th>
<th>Manipulator 2</th>
<th>Manipulator 3</th>
<th>Manipulator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{i1} (m)$</td>
<td>0.98</td>
<td>1</td>
<td>0.96</td>
<td>1</td>
</tr>
<tr>
<td>$l_{i2} (m)$</td>
<td>1</td>
<td>0.95</td>
<td>1</td>
<td>1.02</td>
</tr>
<tr>
<td>$m_i (kg)$</td>
<td>1.02</td>
<td>0.96</td>
<td>1.01</td>
<td>1.04</td>
</tr>
<tr>
<td>$m_i (kg)$</td>
<td>1.12</td>
<td>1.15</td>
<td>1.07</td>
<td>1.09</td>
</tr>
<tr>
<td>$J_i (kg \cdot m^2)$</td>
<td>0.23</td>
<td>0.21</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td>$J_i (kg \cdot m^2)$</td>
<td>0.41</td>
<td>0.40</td>
<td>0.42</td>
<td>0.41</td>
</tr>
</tbody>
</table>
\[ q_{ei_k} = q_{i_k} - q_{0k}, \ i = 1, \ldots, 4, \ k = 1, 2. \] It can be seen that the followers can converge to the leader within 10s while the tracking errors can decrease to less than 0.2. Figs. 4 and 5 show the velocity tracking errors of the followers, which are defined as \[ v_{ei_k} = \dot{q}_{i_k} - \dot{q}_{0k}, \ i = 1, \ldots, 4, \ k = 1, 2. \] It can be found that the velocity errors can also converge to zero. The control inputs of the followers are shown in Figs. 6 and 7. They can decrease to less than 15Nm when each follower tracks the leader asymptotically.

**B. SIMULATION RESULTS FOR DISTRIBUTED CONTROL ALGORITHM (48)**

For the second case, the leader has a time-varying velocity. The trajectory of the leader is chosen as

\[ q_{01}(t) = \pi / 6 \sin (\omega t + \pi / 2) + \pi / 2, \]
\[ q_{02}(t) = 2\pi / 3 \sin (\omega t + \pi / 2), \]

where \( \omega = 0.1\pi \) and \( q_0(0) = [0, 0]^T. \)
The parameters of the observer (30)-(31) and controller (48) are selected as \( \mu_1 = 1, \mu_2 = 1, K_i = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \gamma_i = 100, k_i = 80, \delta_i = 0.2 \).

The initial value of \( \hat{\nu}_i, \hat{W}_i, \) and \( \hat{\eta}_i \) are set to be zero. The simulation results are shown in Figs. 8-13.

Figs. 8 and 9 show the position tracking errors between each follower and the leader. The tracking errors can decrease to less than 0.01 in 10s. Figs. 10 and 11 show that the velocity tracking errors of the followers can decrease to less than 0.5. From Figs. 12 and 13, we can find that the control inputs of the followers can decrease to 20Nm when each follower tracks the leader asymptotically.
V. CONCLUSION

This study investigates distributed coordinated tracking control strategies for multiple EL systems with time-varying communication delays, nonlinear uncertainties, and external disturbances under the directed topology. Two cases are investigated according to the state of the leader. For the first case where the velocity of the leader is constant, the observers are designed for the followers firstly. Then, based on NN, the coordinated control algorithm is proposed to achieve coordinated tracking, which is proved by using Lyapunov–Krasovskii method. For the second case where the velocity of the leader is time-varying, firstly, modified distributed observers are proposed and the stability is proved by Lyapunov–Krasovskii method. Then, the control algorithm is proposed while in order to reduce the chattering, the saturation function is utilized to replace the discontinuous sign function in the controller design. The convergence property is proved by the Lyapunov method. Finally, the simulation results verify the effectiveness of the proposed algorithms.

REFERENCES


YANCHAO SUN received the B.S. degree in flight vehicle design and engineering and the M.S. and Ph.D. degrees in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2010, 2012, and 2016, respectively.

He is currently an Associate Professor with the Science and Technology on Underwater Vehicle Laboratory, Harbin Engineering University. His research interests include distributed cooperative control of multiple Euler–Lagrange systems, multi-agent system control, and multiple AUV systems control.

DINGRAN DONG is currently pursuing the Ph.D. degree with the Department of Biomedical Engineering, City University of Hong Kong.

Her research interests include the distributed control of multiple Euler–Lagrange systems and consensus control of multi-agent systems.

HONGDE QIN received the Ph.D. degree from Harbin Engineering University, Harbin, China, in 2003.

He is currently a Professor and the Director of the Science and Technology on Underwater Vehicle Laboratory, Harbin Engineering University. His current research interests include underwater vehicle system control and bionic robot system control.

NING WANG received the B.Eng. degree in marine engineering and the Ph.D. degree in control theory and engineering from Dalian Maritime University (DMU), Dalian, China, in 2004 and 2009, respectively.

He is currently a Full Professor with the School of Marine Electrical Engineering, DMU. His research interests include fuzzy neural networks, nonlinear control, and unmanned vehicles and autonomous control.

XIAOJIA LI is currently pursuing the M.S. degree in naval architecture and ocean engineering with Harbin Engineering University.

His research interests include multiple AUV systems control and multiple Euler–Lagrange systems control.

* * *


