Variational Regularized Tree-Structured Wavelet Sparsity for CS-SENSE Parallel Imaging

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ABSTRACT Both compressed sensing magnetic resonance imaging (MRI) and parallel MRI have emerged as effective techniques to accelerate MRI data acquisition in various clinical applications. The hybrid parallel imaging reconstruction methods by combining these two techniques have been developed for providing further acceleration. However, the widely used $L_1$-norm of wavelet coefficients and total variation (TV) regularizer in traditional hybrid imaging methods limited further improvement in image quality. To further enhance imaging quality and reduce acquisition time, we proposed a regularized parallel imaging reconstruction method by incorporating sparsity-promoting wavelet prior and total generalized variation (TGV) regularizer. Specifically, the wavelet sparsity is effectively promoted through the $L_0$ quasi-norm of wavelet coefficients and tree-structured wavelet representation. This sparsity-promoting wavelet prior is capable of representing a better measure of sparseness to guarantee high-quality reconstruction even for high degrees of undersampling. Unlike TV regularizer, which preserves sharp edges but suffers from staircaselike artifacts, TGV regularizer can balance the tradeoff between edges preservation and artifacts suppression. Numerous experiments have been conducted on both simulated and in vivo MRI data sets to compare our proposed method with some state-of-the-art reconstruction methods. Experimental results have demonstrated its superior imaging performance in terms of both quantitative evaluation and visual quality.

INDEX TERMS Compressed sensing (CS), magnetic resonance imaging (MRI), SENSE, total generalized variation (TGV), tree-structured sparsity.

I. INTRODUCTION

Magnetic resonance imaging (MRI) has been extensively studied for its ability to visualize the anatomical and physiological structures as well as functional information. However, MRI commonly suffers from an inherently slower data acquisition process in clinical practice [1]. To accelerate MRI data acquisition, both parallel MRI (pMRI) and compressed sensing MRI (CS-MRI) have attracted considerable research activities for reducing the number of acquired $k$-space measurements [2]. In pMRI, $k$-space measurements are obtained simultaneously using multiple receiver coils with complementary sensitivity profiles such that they can be sampled below the Nyquist sampling rate. The most commonly known pMRI methods include image-domain methods like PILS (Partially Parallel Imaging with Localized Sensitivities) [3], SENSE (SENSitivity Encoding) [4], [5] and $k$-space methods like GRAPPA (GeneRalized Auto-calibrating Partially Parallel Acquisition) [6], [7], SMASH (SiMultaneous Acquisition of Spatial Harmonics) [8]. Under ideal imaging conditions, the maximum achievable acceleration factor can be up to the number of receiver coils. However, this maximum usually cannot be achieved due to practical limitations such as apparent noise and imperfect coil geometry.

CS-MRI enables MRI signal reconstruction based on measurements sampled at significantly below the Nyquist sampling rate if the signal can be represented sparsely in a particular transform domain. SparseMRI, the pioneering work of CS-MRI to accelerate MRI signal acquisition, can be found in [9]. CS-MRI takes advantage of the fact...
that MR images meet two key assumptions underlying the CS reconstruction: (i) MR images are sparse (or compressible) in certain transform domains, and (ii) Fourier encoding is sufficiently incoherent with these sparsifying transforms [10]–[13]. CS-MRI and pMRI accelerate imaging reconstruction through exploiting different redundancies (image sparseness for CS-MRI and coil sensitivity information for pMRI), thus it is necessary to combine them together to further increase imaging speed. The current combination methods can be mainly classified into two types. One is SparseSENSE and its equivalence [14]–[16], considered as a straightforward combination, which directly replaces Fourier encoding in SparseMRI with sensitivity encoding comprising Fourier encoding and coil sensitivities. The other is CS-SENSE [17] and its modified versions [18]–[20] which separate CS-MRI and SENSE reconstruction into two individual steps. For instance, CS-SENSE [17] sequentially carries out SparseMRI [9] for reconstructing each aliased image coil-by-coil in the first step and SENSE reconstruction [4] for generating the final non-aliased image in the second step. That is to say, current CS-MRI methods could be easily incorporated into the original CS-SENSE parallel imaging framework [17] to further improve MRI reconstruction. Similar to [17] and [18], we focus this paper on CS-MRI to reconstruct the aliased images in what follows.

Undersampled MRI benefits from scan time reduction but often at the expense of noise and artifacts amplification. To overcome these potential limitations, many CS-MRI methods were proposed by enforcing sparsity in both wavelet domain and total variation (TV) of MR image. In the wavelet domain, the use of the $L_1$-norm of wavelet coefficients as a sparsity-promoting penalty has been popularized [9], [21]–[23]. $L_1$-norm-based reconstruction methods are often extremely powerful, but the corresponding sampling rate is higher than the theoretical minimum to guarantee exact reconstruction. Recent studies [24]–[30] have shown that the measurements required in MRI can be further reduced by replacing the $L_1$-norm with the $L_p$ quasi-norm, where $0 \leq p < 1$. Especially, $L_0$ quasi-norm requires the least number of $k$-space measurements since it ensures the sparsest representation [31]–[33]. In addition, $L_0$ quasi-norm minimization performs exceptionally well in penalizing small wavelet coefficients and encouraging large ones so that the noise and artifacts can be suppressed in reconstructed images. Therefore, there exists a great potential to incorporate the $L_0$ quasi-norm of wavelet coefficients into CS-MRI to further improve reconstructed image quality. From a statistical point of view, the wavelet coefficients are not only sparse, but also satisfy a tree structure because the coefficients exhibit statistically significant dependencies within each subband and across scales [34]. Many results in the literature have reported that it was more accurate to replace (or combine) the standard wavelet sparsity with tree-structured wavelet sparsity to enhance conventional CS-MRI reconstruction [18], [35], [36]. The most recently developed method [18] by combining $L_1$-norm of wavelet coefficients and tree-structured wavelet sparsity achieved superior results and outperformed other tree-based CS reconstruction methods [37], [38].

Motivated by the advantages of $L_0$ quasi-norm and tree-structured sparsity, we propose to combine them in a hybrid representation architecture for promoting wavelet sparsity in CS-MRI. This sparsity-promoting wavelet prior is capable of providing a better sparse representation in wavelet domain. Accordingly, the accuracy of image reconstruction can be guaranteed even for highly undersampled $k$-space data. As for TV regularizer, it has been extensively used for CS-MRI due to its ability to preserve edges [9], [39]. However, this commonly chosen regularizer may cause staircase-like artifacts due to its nature in favoring piecewise constant reconstructions. To overcome this limitation, several extensions of TV, e.g., total generalized variation (TGV) [40]–[43] and non-local total variation (NLTV) [44], [45], have been proposed to enhance reconstructed image quality. Compared with TV, TGV prefers piecewise polynomial solutions rather than piecewise constant solutions. It thus has the property of suppressing staircase-like artifacts and preserving sharp edges in reconstructed images. MR images essentially tend to have the non-local self-similarity property [46], NLTV is able to guarantee the best reconstruction performance. However, NLTV regularized reconstruction methods suffer from high computational complexity owing to the cost of non-local weights computation for all pixels during image reconstruction. To balance the trade-off between computational cost and accuracy, we propose to incorporate the TGV regularizer into CS-MRI in this work to make image reconstruction more robust in clinical applications.

In this paper, we formulate CS-MRI in the first step of CS-SENSE as a minimization task by combining sparsity-promoting wavelet prior (comprising $L_0$ quasi-norm of wavelet coefficients and tree-structured wavelet sparsity) with TGV regularizer. This combined regularization constraint can promote artifacts suppression and edges preservation, but makes the reconstruction problem more difficult to solve. To guarantee solution stability and efficiency, an alternating minimization algorithm will be proposed to optimize the reconstruction problem. The superior performance of our proposed method over current state-of-the-art reconstruction methods will be illustrated using both simulated and in vivo MRI datasets.

The rest of this paper is organized as follows. Section II briefly explains the preliminary concepts of SENSE, CS-SENSE and tree-structured wavelet sparsity in CS-SENSE. In Section III, we introduce the proposed MRI reconstruction method with sparsity-promoting wavelet prior and TGV constraints. Section IV is dedicated to several numerical experiments in terms of both quantitative evaluation and visual quality. Finally, we conclude this paper by summarizing our contributions in Section V.
II. BASIC FRAMEWORK

A. SENSITIVITY Encoding (SENSE)

In this section, we give a brief summary of SENSE technique for pMRI. For an arbitrary sampling trajectory, the sampled \( k \)-space data received from all receiver coils can be expressed as the following imaging equation [4], [5]

\[
\mathbf{E} \mathbf{f} = \mathbf{d},
\]

where \( \mathbf{d} \) is the vector formed from the \( k \)-space data recorded by each receiver coil, \( \mathbf{f} \) denotes the unknown vector defining the desired full field of view (FOV) image to be reconstructed, and \( \mathbf{E} \) is the sensitivity encoding operator which consists of the product of Fourier encoding and coil sensitivities modulation over the image, i.e.,

\[
\mathbf{E}_{[c,m],n} = e^{-i2\pi \bar{k}_m \bar{r}_n} S_c \left( \bar{r}_n \right),
\]

where \( \bar{k}_m \) and \( \bar{r}_n \) respectively represent the coordinates of the \( k \)-space data and image domain pixels, and \( S_c \) represents the spatial sensitivity profile of the \( c \)th receiver coil. A successful SENSE reconstruction is heavily dependent on an accurate knowledge of the coil sensitivity profiles. To accurately reconstruct the desired image \( \mathbf{f} \), the coil sensitivity information required for SENSE pMRI can be effectively obtained using any established sensitivity estimation technique [4], [47], [48].

B. COMPRESSED SENSING SENSE (CS-SENSE)

Basic SENSE reconstruction (1) can be considered as a two-step reconstruction scheme. In this case, the sensitivity encoding operator \( \mathbf{E} \) can be decomposed into two sequential linear operators [17]. The first step in SENSE is the reconstruction of aliased images from all receiver coils. For the \( c \)th receiver coil, the aliased image \( \mathbf{f}_{A,c} \) can be obtained by considering the following formula

\[
\mathbf{F}_u \mathbf{f}_{A,c} = \mathbf{d}_{u,c},
\]

for \( c = 1, 2, \cdots, C \). Here, \( C \) is the total number of receiver coils, \( \mathbf{F}_u \) is the undersampled Fourier encoding matrix, and \( \mathbf{d}_{u,c} \) represents the undersampled \( k \)-space data. Based on the known coil sensitivity profiles, the final non-aliased full FOV image \( \mathbf{f} \) in the second step can be obtained from the aliased images \( \mathbf{f}_{A} \), i.e.,

\[
\tilde{\mathbf{S}} \mathbf{f} = \mathbf{f}_{A},
\]

where \( \tilde{\mathbf{S}} \) is a sensitivity modulation matrix given by \( \tilde{\mathbf{S}} = [S_1^H, S_2^H, \cdots, S_C^H]^T \) with \( (\cdot)^T \) denoting the Hermitian-transpose, and \( \mathbf{f}_{A} = [\mathbf{f}_{A,1}^H, \mathbf{f}_{A,2}^H, \cdots, \mathbf{f}_{A,C}^H]^T \) is a column vector corresponding to the aliased images from \( C \) receiver coils. The basic SENSE reconstruction (1) makes full use of the aforementioned two-step formulation. It initially reconstructs the aliased images from all receiver coils and generates the final non-aliased image by solving Eq. (3). Many efforts in the literature [17], [18], [49] have been made to exploit CS to improve the first step or both steps of SENSE. In this paper, similar to [17] and [18], we mainly focus on the use of CS to reconstruct the aliased images in the first step. The original CS-SENSE method proposed by Liang et al. [17] reconstructed the aliased images directly using SparseMRI [9] as follows

\[
\arg\min_{\mathbf{f}_{A,c}} \left\{ \frac{1}{2} \left\| \mathbf{F}_u \mathbf{f}_{A,c} - \mathbf{d}_{u,c} \right\|_2^2 + \alpha \left\| \Psi \mathbf{f}_{A,c} \right\|_1 + \beta \text{TV} \left( \mathbf{f}_{A,c} \right) \right\},
\]

where \( \alpha \) and \( \beta \) are positive regularization parameters, \( \Psi \) denotes the wavelet transform, \( \left\| \cdot \right\|_1 \) represents the standard \( L_1 \)-sparse-enforcing norm, and the TV regularizer is defined as \( \text{TV} \left( \mathbf{f}_{A,c} \right) = \sum_i \sqrt{\sum_{\alpha} \left| \partial_\alpha \mathbf{f}_{A,c} \right|^2} \) with \( \partial_x \) and \( \partial_y \) being the finite difference along \( x \) and \( y \), respectively. The final reconstructed image from the aliased images was obtained using the inherent spatial sensitivity information. The superior performance of CS-SENSE benefits from the sequential implementation of SparseMRI and SENSE instead of a straightforward combination. From a statistical point of view, the wavelet coefficients of MR images are statistically dependent along the branches of the wavelet tree. By capturing the dependencies between locations of the wavelet coefficients, the reconstruction performance of CS-SENSE could possibly be improved significantly.

C. TREE-STRUCTURED WAVELET SPARSITY IN CS-SENSE

The dependencies between wavelet coefficients have been studied intensively in the image processing literature. An example of the wavelet tree structure of a real T2w brain MR image is illustrated in Fig. 1. The coefficients correspond to “root nodes” at the coarsest scale and “leaf nodes” at the finest scale. Each coefficient without “leaf nodes” generally serves as a “parent” and has four “children” coefficients at the finer scale below it. If a parent coefficient is zero, it is likely that its children also tend to be zero [35]. If this prior information is fully exploited in CS reconstruction, it is able to guarantee high image quality from far fewer data samples.

In the literature [34], [35], many efforts have been devoted to improve CS reconstruction by directly replacing the standard wavelet sparsity with tree-structured wavelet sparsity. However, the wavelet coefficients of MR images may not
exactly follow the theoretical assumption of tree-structured sparsity. To improve the reconstructed image quality, there is a great potential to combine both standard and tree-structured wavelet sparsity to further promote sparsity in a unified framework. Chen and Huang [49] proposed to promote sparsity in wavelet domain by combining the $L_1$-norm of wavelet coefficients and tree-structured wavelet sparsity to enhance CS reconstruction as follows

$$\arg\min_{f_{A,c}} \left\{ \frac{1}{2} \left\| F_d f_{A,c} - d_{A,c} \right\|^2 + \alpha \left( \| \Psi f_{A,c} \|_1 + \sum_{g \in G} \| (\Psi f_{A,c})_g \|_2 \right) + \beta TV (f_{A,c}) \right\},$$

(5)

where $G$ represents the set of all parent-child groups for the wavelet tree structure, and $g$ is one of such groups. The sparsity-promoting wavelet prior introduced in (5) has been extended to CS-SENSE pMRI (termed WaTMR) [18]. Experimental results have demonstrated the superior performance of WaTMR compared with existing state-of-the-art reconstruction methods. However, the undesirable staircase-like artifacts commonly produced by TV regularizer could constrain the further improvement in image quality [41], [42]. To overcome the model-dependent deficiency, it is necessary to incorporate a valid extension of TV into the CS reconstruction step (5) of WaTMR. As a measure of standard wavelet sparsity, $L_1$-norm of wavelet coefficients (i.e., $\| \Psi f_{A,c} \|_1$) is exploited in (5) to make CS reconstruction robust. Recent work has shown that non-convex optimization with $L_p$ quasi-norm minimization ($0 \leq p < 1$) can further reduce the number of data measurements for undersampled MRI reconstruction [28], [29], [31], [32]. Therefore, the $L_p$ quasi-norm of wavelet coefficients can potentially be used in (5) to further improve MRI reconstruction.

III. PROPOSED METHODOLOGY

A. PROPOSED SENSE PMRI RECONSTRUCTION

It is generally thought that the most natural measure of the sparsity of wavelet coefficients is its $L_0$ quasi-norm [31], [32]. Although it is more intractable to handle the $L_0$ quasi-norm minimization problem, we still focus on $L_0$ quasi-norm instead of $L_1$-norm to promote the sparsity of wavelet coefficients. In particular, the $L_0$ quasi-norm is capable of penalizing small wavelet coefficients and encouraging large wavelet coefficients so that high-quality image reconstruction can be guaranteed. As for TV regularizer in (5), it is known for its capability of preserving edges but it sometimes causes undesirable staircase-like artifacts. TGV regularizer [50], a higher-order extension of TV, has been proposed to overcome this limitation. In particular, TGV generalizes TV and prefers piecewise polynomial solutions to describe more precisely the intensity variation in MR images. Therefore, reconstructed images with TGV regularizer can benefit from both staircase-like artifacts suppression and edges preservation. We only consider the second-order total generalized variation (i.e., $TGV^2_2$) regularizer in this work since it is adequate for undersampled MRI reconstruction [40]–[42]. To further improve image quality, it would be fairly straightforward to extend our proposed method to higher-order total generalized variation regularizers. For a scalar field $f$, the definition of $TGV^2_2$ is given by

$$TGV^2_2 (u) = \arg\min_{w} \left\{ \kappa_1 \| \nabla u - w \|_1 + \kappa_0 \| \mathcal{E} (w) \|_1 \right\},$$

(6)

where $\kappa_1$ and $\kappa_0$ are positive tuning parameters, and $\mathcal{E} (w) = \frac{1}{2} (\nabla w + \nabla w^T)$ denotes the symmetrized gradient of a complex-valued vector field $w$. To make our reconstruction method more robust on practical MR images, reconstruction of aliased images is formulated as an unconstrained minimization problem regularized by $L_0$ quasi-norm of wavelet coefficients, tree-structured sparsity constraint and $TGV^2_2$ regularizer, i.e.,

$$\arg\min_{f_{A,c}} \left\{ \frac{1}{2} \left\| F_d f_{A,c} - d_{A,c} \right\|^2 + \alpha \left( \| \Psi f_{A,c} \|_0 + \sum_{g \in G} \| (\Psi f_{A,c})_g \|_2 \right) + \beta TGV^2_2 (f_{A,c}) \right\},$$

(7)

for $c = 1, 2, \ldots, C$. Here, $\| \Psi f_{A,c} \|_0$ represents the $L_0$ quasi-norm of $\Psi f_{A,c}$ which counts the number of nonzero elements in $\Psi f_{A,c}$. The obtained aliased images from all receiver coils are then unfolded to reconstruct the final image using coil sensitivities.

Our proposed method (7) contains three different regularization terms, thus their influences on image reconstruction should be carefully investigated. Take the T2w brain MR image (Fig. 1, left) as an example, we implement the CS reconstruction using different combinations of regularization terms. As shown in Fig. 2, we produce the reconstruction results in the case of Cartesian sampling pattern at different reduction factors of $R = 3$ and 4. It can be observed that our image reconstruction is dependent on all three regularization terms. The quality of reconstructed image is much more sensitive to $TGV^2_2$ regularizer. Both $L_0$ quasi-norm of wavelet coefficients and tree-structured wavelet sparsity are also beneficial for improving image quality. The superior performance of our proposed method will be further demonstrated in Section IV in terms of both quantitative evaluation and visual quality.

B. OPTIMIZATION ALGORITHM

Due to the non-smooth, non-convex and non-separable terms, it is difficult to directly minimize (7) using commonly-used numerical algorithms. To achieve solution stability, an alternating minimization algorithm (AMA) is proposed in this section. AMA has attached increasing attention in many fields such as information theory, machine learning and image processing owing to its iterative nature and simplicity [51], [52]. The key idea behind our numerical algorithm...
is that the $L_0$ quasi-norm, tree-structured sparsity constraint and TGV$_k^2$ regularizer can be decoupled. For the sake of better reading, the variables $F_{i,c}, f_{i,c}$ and $d_{i,c}$ are respectively replaced by $F$, $f$ and $d$ throughout this paper. We introduce two auxiliary variables $v = \Psi f$ and $z = \Psi \Psi f$ in (7), which can be rewritten as an equivalent unconstrained minimization problem, i.e.,

$$
\arg \min_{v, x, f} \left\{ \frac{1}{2} \| F f - d \|_2^2 + \frac{\mu}{2} \| \Psi f - v \|_2^2 + \frac{\lambda}{2} \| G \Psi f - z \|_2^2 \\
+ \alpha \left( \| v \|_0 + \sum_{i=1}^{s} \| z_{gi} \|_2 \right) + \beta \text{TGV}_k^2 (f) \right\}. 
$$

(8)

where $\mu$ and $\lambda$ are positive penalty parameters which control the weights of penalty terms, $G$ is a binary matrix to duplicate the overlapped entries, $g_i$ is the $i^{th}$ group and $s$ is the total number of parent-child groups. The variables $v$, $z$ and $f$ are coupled together in minimization of (8), thus it is computationally intractable to solve them simultaneously. We propose to decompose the complex minimization problem (8) into several subproblems and then solve them separately. We now alternatively solve it with respect to $v$, $z$ and $f$ until the obtained solution converges to the optimal one.

1) $v$-SUBPROBLEM
The first $v$-subproblem is equivalent to solving the following optimization problem

$$
v^{i+1} = \arg \min_{v} \left\{ \frac{\mu}{2} \| \Psi f - v \|_2^2 + \alpha \| v \|_0 \right\}. 
$$

(9)

Since the unknown variable $v$ is componentwise separable, the solution $v^{i+1}$ can be obtained using a hard-thresholding operator $T$ formulated in [53], i.e.,

$$
v^{i+1} = T_{\alpha, \mu} \left( \Psi f \right),
$$

(10)

where $T_{\alpha, \mu}$ is defined componentwisely as

$$
T_{\alpha, \mu} \left( \Psi f \right) = \begin{cases} 0, & \text{if } |\Psi f| < \sqrt{\frac{2\alpha}{\mu}}, \\
\Psi f, & \text{otherwise.} \end{cases}
$$

(11)

2) $z$-SUBPROBLEM
Given the fixed $f$, the minimization of (8) with respect to $z$ can be expressed as the following simple formulation

$$
z_{gi}^{i+1} = \arg \min_{z_{gi}} \left\{ \frac{\lambda}{2} \| (G \Psi f)_{gi} - z_{gi} \|_2^2 + \alpha \| z_{gi} \|_2 \right\},
$$

(12)

for $i = 1, 2, \ldots, s$. In particular, the $L_2$-regularized least squares problem (12) can be effectively solved via a shrinkage operator [54], i.e.,

$$
z_{gi}^{i+1} = \text{shrinkage} \left( \left( G \Psi f \right)_{gi} \left| \alpha \right| \right) \\
= \max \left( \left( G \Psi f \right)_{gi} - \frac{\alpha}{\lambda} \right) \circ \left( G \Psi f \right)_{gi}^2,
$$

(13)

where $\circ$ denotes the pointwise multiplication operator.

3) $f$-SUBPROBLEM
Given the fixed $v^{i+1}$ and $z^{i+1}$, solution $f^{i+1}$ of the $f$-subproblem in (8) is equivalent to solving the following minimization problem

$$
f^{i+1} = \arg \min_{f} \left\{ \mathcal{H} (f) + \beta \text{TGV}_k^2 (f) \right\},
$$

(14)

where

$$
\mathcal{H} (f) = \frac{1}{2} \| F f - d \|_2^2 + \frac{\mu}{2} \| \Psi f - v^{i+1} \|_2^2 + \frac{\lambda}{2} \| G \Psi f - z^{i+1} \|_2^2,
$$

which is a smooth convex function with a positive Lipschitz constant $L_{\mathcal{H}}$, i.e., $\| \nabla \mathcal{H} (x) - \nabla \mathcal{H} (y) \|_2 \leq L_{\mathcal{H}} \| x - y \|_2$ for every $x, y \in \mathbb{R}^N$. Note that TGV$_k^2 (f)$ is a continuous convex but non-smooth function. To guarantee solution stability and efficiency, the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) [54] is introduced here to solve the $f$-subproblem (14). The pseudo-code for the optimization process is given in Algorithm 1.

The efficiency of the FISTA is significantly dependent on the ability to quickly solve the second step $\tilde{f}^{k+1} = \text{prox}_{\mathcal{H}} \left( \beta \text{TGV}_k^2 (f) \right) (\tilde{y})$ in Algorithm 1, i.e.,

$$
\tilde{f}^{k+1} = \arg \min_{f} \left\{ \frac{L_{\mathcal{H}}}{2} \| f - \tilde{y} \|_2^2 + \beta \text{TGV}_k^2 (f) \right\},
$$

(15)

where the Lipschitz constant $L_{\mathcal{H}}$ is conventionally set to 1. In this paper, the proximal function (15) is efficiently solved using the Chambolle and Pock’s primal-dual method [55]. We refer the interested reader to Appendix for more details on this primal-dual method.
Algorithm 1 FISTA for Solving f-Subproblem (14)

1: Input: E, G, Ψ, d, v^{i+1}, z^{i+1}, μ, λ, β and L₇k.
2: Initialize: f⁰ → 1, y⁰ → f and p⁰ → f.
3: for k = 0 to k_max do
4:     y = y⁰ - β ∇L₇k(y⁰).
5:     f^{k+1} → proxL₇k((βTGV²(f))(y)).
6:     k^{k+1} = \frac{1}{2} \left( 1 + \sqrt{1 + 4(k_k)^2} \right)
7:     y^{k+1} = f^{k+1} + \left( \frac{k_k}{k_k+1} \right)(f^{k+1} - f^{k}).
8: end for
9: f^{k+1} = f^{k_max}.

FIGURE 3. From left to right: the illustrations of Cartesian sampling patterns for SENSE with reduction factor of Rₛ = 2, CS with reduction factor of R_C = 3, and combined version CS-SENSE with reduction factor of R = 6.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The proposed method was evaluated on three datasets: simulated, in vivo human brain and cardiac datasets. For all in vivo studies, informed consent was obtained from the volunteer in accordance with the institutional review board policy. All reconstruction methods were implemented using Matlab 2014a (The MathWorks, Natick, Inc., MA) on a machine with 3.10 GHz Intel Core i5-2500 CPU and 4 GB RAM. The root sum-of-squares (SoS) reconstruction from fully sampled data was used as the gold standard for comparison. To quantitatively evaluate the reconstruction results, we simultaneously used two popular metrics, i.e., peak signal-to-noise ratio (PSNR) and structural similarity (SSIM) index [56].

A. EXPERIMENTAL DATA ACQUISITIONS

1) SHEPP-LOGAN PHANTOM IMAGING EXPERIMENT

In our simulation experiment, a 256 × 256 numerical Shepp-Logan phantom was used as the original image and Biot-Savart law [57] was exploited to generate the eight-channel coil sensitivity profiles. The piecewise-constant phantom can be sparsely represented in finite-difference domain. The objective of this experiment is to evaluate the performance of our proposed method when the sparseness constraint is strictly satisfied. In particular, the simulated k-space data were obtained by taking the Fourier transform of the original image weighted by the coil sensitivities. The Cartesian sampling patterns, used to generate undersampled k-space data, were visually illustrated in Fig. 3. The overall reduction factor was set as R = R_C × R_S for our proposed method, where R_C and R_S were the individual reduction factor for CS and SENSE reconstructions, respectively. The undersampled k-space measurements in this experiment were manually generated using Cartesian sampling pattern at different reduction factors of R = 2.0 × 2, 2.5 × 2, 3.0 × 2, 3.5 × 2 and 4.0 × 2. The similar Cartesian sampling patterns were also adopted in other numerical experiments in this work. To further investigate the influence of noise on image reconstruction, the real and imaginary components of the k-space data were corrupted with different levels of Gaussian noise ranged from 1% to 9% with 2% in step.

2) IN VIVO BRAIN IMAGING EXPERIMENT

This experiment is to evaluate the reconstruction performance of our proposed method when applied to in vivo brain images that are usually not as sparse as phantom but compressible in finite-difference and wavelet-transform domains. The brain dataset was obtained from a 3T commercial scanner (Achieva, Philips Medical Systems, Best, the Netherlands) with an eight-channel SENSE head coil using a 2D T1-weighted spin echo protocol (TR/TE = 700/11 ms, FOV = 24 × 24 cm, matrix = 256 × 256 and 50 slices 1 mm each). Similar to the simulation experiment, the SoS reconstruction was used as the reference image for both quantitative and qualitative comparisons. The center of the 32 × 32 k-space data was obtained to estimate the channel-sensitivity profiles. The undersampled k-space data were generated using Cartesian sampling pattern at different reduction factors of R = 1.5 × 2, 2.0 × 2, 2.5 × 2, 3.0 × 2 and 3.5 × 2. The reconstructed images and their corresponding local magnification views were illustrated for qualitative comparison.

3) IN VIVO CARDIAC IMAGING EXPERIMENT

The purpose of this experiment is to evaluate the reconstruction performance of our proposed method when applied to one frame of in vivo dynamic cardiac data. The dataset was acquired from a clinical 3T MRI scanner (Achieva, Philips Medical Systems, Best, the Netherlands) with an eight-channel receive coil. The scan parameters were TR/TE = 3.8/1.9 ms, flip angle = 45°, matrix = 256 × 256, slice thickness = 8 mm. Similar to the brain imaging experiment, the central 32 × 32 k-space data were fully sampled to estimate the sensitivity map of each receiver coil. The Cartesian sampling pattern at different reduction factors of R = 2.0 × 2, 2.5 × 2, 3.0 × 2, 3.5 × 2 and 4.0 × 2 was exploited to generate the undersampled k-space measurements. The SoS reconstruction from the fully sampled data was exploited as the reference image for visual comparison.

B. COMPARISONS WITH OTHER SENSE PMRI RECONSTRUCTION METHODS

In order to evaluate the MRI reconstruction performance, our proposed method will be compared with several state-of-the-art pMRI techniques as follows:
1) IRGN-TGV [42]: This auto-calibrated pMRI method, proposed based on the second-order TGV regularizer, simultaneously performed image reconstruction and sensitivity estimation. The final reconstructed image was robustly achieved using an iteratively regularized Gauss-Newton (IRGN) method.

2) CS-JSENSE [58]: This CS-based pMRI method was essentially an extension of joint estimation of coil sensitivities and output image (JSENSE) [59] by incorporating the sparsity of multi-channel coil images.

3) CS-SENSE [17]: This CS-based pMRI method sequentially carried out SparseMRI [9] for reconstructing the aliased images and then SENSE [4] for producing the final reconstructed image.

4) WaTMRI [18]: WaTMRI can be considered as a natural generalization of CS-SENSE [17] by adopting the advantage of tree-structured wavelet sparsity in the first step of CS-SENSE.

The first two parallel imaging methods (i.e., IRGN-TGV and CS-JSENSE) aim at simultaneously performing image reconstruction and sensitivity estimation from undersampled k-space measurements. The enhanced image quality can be achieved with improved self-calibration of coil sensitivity profiles. In contrast, it is necessary to pre-estimate the coil sensitivities for both CS-SENSE and WaTMRI. The superior performance of these two pMRI methods benefits from the combination of CS and SENSE reconstructions. If the coil sensitivity profiles can be obtained with sufficient accuracy, these two hybrid parallel imaging methods are capable of producing high-quality image reconstruction. In our experiments, the CS reconstruction problem for both CS-SENSE and WaTMRI was efficiently solved using the fast composite splitting algorithm (FCSA) [22], [60].

C. EXPERIMENTAL PARAMETERS SETTINGS

It is well known that the choice of parameters plays an important role in reconstructing MR images. There are six important parameters involved in our proposed method, i.e., \( \kappa_0, \kappa_1, \alpha, \beta, \mu \) and \( \lambda \). For TGV\(_2\) regularizer, both \( \kappa_0 \) and \( \kappa_1 \) have a crucial role in balancing the trade-off between the first and second derivatives. According to our experience in undersampled MRI reconstruction, selecting \( \kappa_0 \) as \( 2\kappa_1 \) was suitable for most experiments without further tuning. For compatibility with the popular TV regularizer, \( \kappa_1 = 1 \) and \( \kappa_0 = 2 \) were used throughout all our experiments. The other parameters involved in our experiments were tuned manually. The regularization parameters \( \alpha \) and \( \beta \) control the trade-offs between the data-fidelity and regularization terms. We performed exhaustive experiments in this paper to manually determine the satisfactory choices of \( \alpha \) and \( \beta \) for different MRI datasets (i.e., phantom, brain and cardiac). For the sake of simplicity, both \( \alpha \) and \( \beta \) range over the set \( \text{linspace}(1e-7, 1e-3, 50) \) generated by Matlab.

The undersampled k-space measurements were generated using Cartesian sampling pattern at a reduction factor of \( R = 3.0 \times 2 \). As shown in Fig. 4, our reconstruction performance was more sensitive to \( \beta \) than \( \alpha \). It means that TGV\(_2\) regularizer proposed in our method (7) can significantly affect the reconstructed image quality. This observation is in agreement with our previous analysis shown in Fig. 2. According to the quantitative analysis results, we manually selected the good parameters \( \alpha = 6.132 \times 10^{-5} \) and \( \beta = 2.051 \times 10^{-5} \) for phantom and cardiac datasets, as well as \( \alpha = 6.132 \times 10^{-5} \) and \( \beta = 6.132 \times 10^{-5} \) for brain dataset. The limitation of our proposed alternating minimization algorithm is that it is highly dependent on the parameters \( \mu \) and \( \lambda \). These two parameters respectively penalize the differences between \( \mathbf{v} \) and \( \Psi f \), as well as \( z \) and \( G \Psi f \) in (8). We empirically determined the good values of \( \mu \) and \( \lambda \) according to the parameter \( \alpha \). In particular, these two parameters were set to be \( \mu = \lambda = 0.01\alpha \) for all MRI datasets. The reconstruction results under the current parameters settings were consistently promising in all our experiments. The way for the stopping criteria in our proposed method was that the relative change of \( f \) was sufficiently small, i.e., \( \| f^{t+1} - f \|_2^2 / \| f \|_2^2 \leq \epsilon \) and \( \epsilon \) was set to be \( 5 \times 10^{-4} \). The reconstruction results of other comparative methods were generated with the manually optimized parameters. Further study on automatically selecting

![Figure 4. From top to bottom: the influences of different regularization parameters (\( \alpha, \beta \)) on image reconstruction for phantom (top), brain (middle) and cardiac (bottom) datasets. The undersampled k-space measurements were generated using Cartesian sampling pattern at a reduction factor of \( R = 3.0 \times 2 \). The good values of \( \alpha, \beta \) were determined using the objective quality metrics PSNR (left) and SSIM (right), respectively.](image-url)
TABLE 1. PSNR/SSIM comparisons of various construction methods on phantom MRI dataset for Cartesian sampling pattern at different reduction factors of $R = 2.0 \times 2$, $2.5 \times 2$, $3.0 \times 2$, $3.5 \times 2$ and $4.0 \times 2$.

<table>
<thead>
<tr>
<th>$R (R_C \times R_S)$</th>
<th>IRGN-TGV</th>
<th>CS-JSENSE</th>
<th>CS-SENSE</th>
<th>WaTMRI</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (2.0 × 2)</td>
<td>34.45/0.9428</td>
<td>47.01/0.9913</td>
<td>43.83/0.9777</td>
<td>44.63/0.9835</td>
<td>49.64/0.9943</td>
</tr>
<tr>
<td>5 (2.5 × 2)</td>
<td>32.41/0.9330</td>
<td>44.63/0.9835</td>
<td>41.06/0.9558</td>
<td>42.04/0.9675</td>
<td>46.38/0.9893</td>
</tr>
<tr>
<td>6 (3.0 × 2)</td>
<td>30.86/0.9819</td>
<td>42.64/0.9780</td>
<td>40.04/0.9519</td>
<td>40.90/0.9561</td>
<td>44.10/0.9831</td>
</tr>
<tr>
<td>7 (3.5 × 2)</td>
<td>29.89/0.8620</td>
<td>39.01/0.9611</td>
<td>36.67/0.9319</td>
<td>37.65/0.9452</td>
<td>43.94/0.9628</td>
</tr>
<tr>
<td>8 (4.0 × 2)</td>
<td>27.34/0.8058</td>
<td>38.32/0.9449</td>
<td>35.13/0.9209</td>
<td>36.04/0.9305</td>
<td>40.81/0.9449</td>
</tr>
</tbody>
</table>

D. PHANTOM IMAGING EXPERIMENT RESULTS

For the simulated Shepp-Logan phantom dataset, undersampled $k$-space data were generated using Cartesian sampling pattern at different reduction factors of $R = 2.0 \times 2$, $2.5 \times 2$, $3.0 \times 2$, $3.5 \times 2$ and $4.0 \times 2$. Fig. 5 presents the images reconstructed using five different methods, i.e., IRGN-TGV, CS-JSENSE, CS-SENSE, WaTMRI and our proposed method. It can be observed that at a reduction factor of less than $R = 3.0 \times 2$, both CS-JSENSE and our proposed method are able to generate satisfactory reconstructed images visually similar to the reference image. However, as reduction factor increases, CS-JSENSE tends to produce blurred edges which lead to image quality degradation. As shown by the arrows in Fig. 5, the reconstructed images generated by IRGN-TGV, CS-SENSE and WaTMRI suffer from the undesirable aliasing artifacts. These artifacts become more noticeable at a higher reduction factor. In contrast, our proposed method is capable of suppressing undesirables artifacts and guaranteeing high image quality. Its superior performance on parallel MRI reconstruction is further illustrated in Figs. 6 and 7. From the 1D profiles in Fig. 6, we can conclude that the intensity values of our proposed method are more structurally similar to the reference image. Both CS-SENSE and WaTMRI lead to significant biases at the image boundaries. For better visualization, we presented the error images resulting from the absolute differences between the reference and reconstructed images shown in Fig. 7. To enhance high image quality, the error images should contain as little structural information as possible. As can be observed, the geometrical structures are noticeable in error images generated by IRGN-TGV, CS-JSENSE, CS-SENSE and WaTMRI, respectively. In contrast, our proposed method produces more random and smaller absolute differences. Quantitatively, both PSNR and SSIM values of our proposed method are higher than those of other competing methods, as shown in Table 1.
FIGURE 7. Error images corresponding to the phantom reconstruction results in Fig. 5. From left to right: error images generated by IRGN-TGV, CS-JSENSE, CS-SENSE, WaTMRI and our proposed method, respectively.

FIGURE 8. Images reconstructed from a set of simulated Shepp-Logan phantom datasets corrupted with different levels of Gaussian noise at different reduction factors of \( R = 2.0 \times 2 \) (left) and \( R = 3.0 \times 2 \) (right). Both PSNR (top) and SSIM (bottom) were used to evaluate the quality of reconstructed images.

To investigate the sensitivity of different reconstruction methods to Gaussian noise, we generated undersampled \( k \)-space data whose real and imaginary components were corrupted with different levels of Gaussian noise ranged from 1% to 9% with 2% in step. In Fig. 8, both PSNR and SSIM were used to evaluate the reconstruction results of different methods. As can be observed, both IRGN-TGV and CS-JSENSE, which simultaneously perform image reconstruction and sensitivity estimation, are particularly sensitive to noise pollution and result in degraded image quality. CS-SENSE, WaTMRI and our proposed method can lead to more robust reconstructions in the presence of Gaussian noise. Our method significantly outperforms all the other competing methods in terms of both PSNR and SSIM. It can be seen that because our proposed method is capable of suppressing noise and artifacts in undersampled MRI reconstruction.

E. IN VIVO BRAIN IMAGING EXPERIMENT RESULTS

This experiment is implemented to evaluate our proposed method when the to-be-reconstructed brain MR images contain many fine structures with different contrast. The reconstructed images and their associated magnified views are respectively displayed in Figs. 9 and 10 at different reduction factors of \( R = 1.5 \times 2, 2.0 \times 2, 2.5 \times 2 \) and \( 3.0 \times 2 \). As shown in Fig. 9 by arrows, our proposed method is able to suppress undesirable aliasing artifacts which are present in IRGN-TGV, CS-JSENSE and WaTMRI. As for TV regularizer in CS-SENSE and WaTMRI, it only performs well in reconstructing piecewise constant images and often causes some undesirable artifacts. As reduction factor increases, CS-JSENSE tends to oversmooth the fine details shown by the arrows in local magnification views. In contrast, our proposed method is capable of preserving the fine details while improving the image quality. It means that our proposed method significantly outperforms all the other competing methods in terms of both PSNR and SSIM. It can be seen that because our proposed method is capable of suppressing noise and artifacts in undersampled MRI reconstruction.
method can provide a good balance between artifacts suppression and details preservation. In Fig. 11, the advantage of our method is further confirmed by the surface plots of reconstructed images at a reduction factor of $R = 3.0 \times 2$. It is worth noting that some fine details are lost in all other reconstruction methods. Our proposed method is able to effectively reconstruct the brain images with many fine structures, and produces the most similar surface plot to that of the reference image. This is consistent with the results of Table 2, which depicts the quantitative results with different reconstruction methods and reduction factors. It can be seen that both PSNR and SSIM values of our proposed method are higher than those of IRGN-TGV, CS-JSENSE, CS-SENSE and WaTMRI. These results demonstrate that the proposed method has better reconstruction performance than other competing methods.

F. IN VIVO CARDIAC IMAGING EXPERIMENT RESULTS

For the cardiac dataset, we compared our proposed method with CS-JSENSE, CS-SENSE and WaTMRI for Cartesian sampling pattern at different reduction factors. Fig. 12 visually presents the reconstructed cardiac MR images. The corresponding local magnifications and error images are respectively displayed in Figs. 13 and 14 to reveal more details. We can see that all competing methods in Fig. 12 can preserve the main geometric structures of cardiac MR images. However, CS-JSENSE tends to blur fine image details as the reduction factor becomes larger. This observation is more noticeable in the local magnification views shown in Fig. 13. In comparison, CS-SENSE, WaTMRI and our proposed method are able to preserve much more details. It means that these three methods can produce more robust image reconstructions at high reduction factors. From the error images in Fig. 14, it is seen that the geometrical structures are noticeable in error images generated by CS-JSENSE, CS-SENSE and WaTMRI, respectively. The errors of our
TABLE 2. PSNR/SSIM comparisons of various construction methods on brain MRI dataset for Cartesian sampling pattern at different reduction factors of $R = 1.5 \times 2, 2.0 \times 2, 2.5 \times 2, 3.0 \times 2$ and $3.5 \times 2$.

<table>
<thead>
<tr>
<th>$R (R_C \times R_S)$</th>
<th>IRGN-TGV</th>
<th>CS-JSENSE</th>
<th>CS-SENSE</th>
<th>WaTMRI</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 (1.5 × 2)</td>
<td>32.908/0.9002</td>
<td>38.312/0.9485</td>
<td>39.530/0.9601</td>
<td>40.414/0.9664</td>
<td>44.374/0.9786</td>
</tr>
<tr>
<td>4 (2.0 × 2)</td>
<td>29.772/0.8553</td>
<td>35.204/0.9229</td>
<td>36.887/0.9336</td>
<td>37.733/0.9434</td>
<td>41.645/0.9635</td>
</tr>
<tr>
<td>5 (2.5 × 2)</td>
<td>28.287/0.8406</td>
<td>31.624/0.8795</td>
<td>32.643/0.8893</td>
<td>33.629/0.8851</td>
<td>36.560/0.9176</td>
</tr>
<tr>
<td>6 (3.0 × 2)</td>
<td>27.275/0.7910</td>
<td>30.826/0.8614</td>
<td>31.983/0.8515</td>
<td>32.477/0.8693</td>
<td>35.431/0.9047</td>
</tr>
<tr>
<td>7 (3.5 × 2)</td>
<td>25.364/0.7536</td>
<td>28.965/0.8053</td>
<td>29.136/0.8120</td>
<td>29.638/0.8234</td>
<td>32.288/0.8424</td>
</tr>
</tbody>
</table>

TABLE 3. PSNR/SSIM comparisons of various construction methods on cardiac MRI dataset for Cartesian sampling pattern at different reduction factors of $R = 2.0 \times 2, 2.5 \times 2, 3.0 \times 2, 3.5 \times 2$ and $4.0 \times 2$.

<table>
<thead>
<tr>
<th>$R (R_C \times R_S)$</th>
<th>CS-JSENSE</th>
<th>CS-SENSE</th>
<th>WaTMRI</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (2.0 × 2)</td>
<td>34.061/0.9067</td>
<td>40.029/0.9656</td>
<td>40.895/0.9714</td>
<td>54.247/0.9982</td>
</tr>
<tr>
<td>5 (2.5 × 2)</td>
<td>33.797/0.8896</td>
<td>36.121/0.9333</td>
<td>36.817/0.9426</td>
<td>44.533/0.9858</td>
</tr>
<tr>
<td>6 (3.0 × 2)</td>
<td>31.300/0.8395</td>
<td>34.350/0.9114</td>
<td>35.156/0.9227</td>
<td>42.278/0.9734</td>
</tr>
<tr>
<td>7 (3.5 × 2)</td>
<td>29.761/0.7990</td>
<td>31.679/0.8580</td>
<td>32.213/0.8732</td>
<td>38.650/0.9505</td>
</tr>
<tr>
<td>8 (4.0 × 2)</td>
<td>29.102/0.7339</td>
<td>30.394/0.8243</td>
<td>30.910/0.8493</td>
<td>36.616/0.9298</td>
</tr>
</tbody>
</table>

method are more random and smaller. This is because our proposed method is capable of preserving more fine details and suppressing undesirable artifacts. The superior performance of our method benefits from the sparsity-promoting wavelet prior and $TGV^2_\kappa$ regularizer. The sparsity-promoting wavelet prior represents a better measure of sparseness to guarantee high-quality reconstruction even at a high reduction factor. $TGV^2_\kappa$ regularizer is able to achieve a good balance between artifacts suppression and tissue features preservation.

Table 3 depicts both PSNR and SSIM values of reconstructed images as a function of reduction factors. It is clear that our proposed method produces the highest PSNR and SSIM values among the competing reconstruction methods at the same reduction factor, which is in agreement with the visual observations in Figs. 12-14.

G. ALGORITHM CONVERGENCE AND COMPUTATIONAL TIME

We analyzed the algorithm convergence and computational time on phantom, brain and cardiac datasets at the same reduction factor of $R = 3.0 \times 2$. It is difficult to establish the theoretical convergence of our proposed method due to the non-smooth and non-convex objective function (7). For the sake of simplicity, we only provided empirical evidence to illustrate our convergence behavior. As shown in Fig. 15, both PSNR and SSIM values are visually displayed as functions of CPU computational time for phantom, brain and cardiac datasets. It can be observed that as the iteration number increases, both PSNR and SSIM values monotonically increase until they reach their maximum values. These observations illustrate that the convergence properties of CS-SENSE, WaTMRI and our proposed method can be guaranteed for undersampled MRI reconstruction. Both CS-SENSE and WaTMRI yield the lowest computational cost owing to the strong convergence properties of FCSA [22], [60]. However, they generate lower image quality compared with our proposed method. The $L_1$-norm minimization and TV regularizer in these two methods have limited their further improvements in image quality.

In contrast, our proposed method is capable of keeping a good balance between computational cost and reconstruction performance. The computational time of IRGN-TGV and CS-JSENSE can be found in Table 4. These two methods suffer from the highest computational cost because it is more challenging to simultaneously reconstruct both MR image and coil sensitivities.

H. DISCUSSION

It is well known that accurate estimation of coil sensitivity maps plays a significant role in SENSE reconstruction [4] in the second step of our proposed method. The sensitivity maps
in our \textit{in vivo} imaging experiments were pre-estimated using the central $k$-space phase-encoding lines and refined through a polynomial regression method. Recent studies [61]–[63] have shown that SENSE reconstruction highly depends on the estimation of sensitivity maps. To further enhance reconstruction performance, it is necessary to incorporate advanced coil sensitivity estimation methods into SENSE reconstruction in our parallel imaging framework. For example, an optimization cost function, defined as the difference between the raw sensitivity map and the desired map, was iteratively performed to estimate the coil sensitivities [61]. An inverse field-based approach based on computational electromagnetics and multi-level optimization was proposed to optimize the sensitivity maps [62]. To eliminate potential aliasing artifacts and noise amplification in SENSE reconstruction, a global magnitude-phase fitting method was further developed to generate accurate coil sensitivity maps [63]. These coil sensitivity estimation techniques could be integrated with our CS-SENSE framework to guarantee high-quality MRI reconstruction.

Over the past several years, deep learning has been rapidly emerging as one of the most successful techniques in medical imaging field [64]. It has the capacity to outperform widely-used dictionary learning-based MRI reconstruction. As one typical deep learning category, convolutional neural network (CNN) [65] and its modified variational version [66] have significantly enhanced MR image quality. To eliminate artifacts from artifact corrupted MR images, two deep residual learning networks [67], which separately trained phase and magnitude data, were developed to handle complex-valued MR data. Generative adversarial network (GAN)-based deep architecture [68] has also been introduced for promoting CS-MRI. Meanwhile, a refinement learning method was designed in [69] to further accelerate GAN-based MRI reconstruction. It is worth noting that deep learning-based CS-MRI methods could be incorporated into the first step of our parallel imaging framework to further improve CS-SENSE.

It should be pointed out that our parallel imaging framework is only available for single (static) MR image reconstruction. Dynamic medical imaging, which plays a more important role in clinical applications, has recently attracted a great deal of research attention. However, the inherently slow imaging speed commonly makes it difficult for dynamic imaging studies (e.g., dynamic contrast-enhanced MRI (DCE-MRI), dynamic susceptibility contrast MRI (DSC-MRI), and dynamic cardiac MRI, etc.) to capture high spatio-temporal resolutions. To overcome the limitation, a temporally constrained reconstruction technique [70] has been presented to perform DCE-MRI from undersampled...
Before we use the Chambolle and Pock’s primal-dual algorithm [55], the TGV\(_k^2\)-regularized denoising subproblem (15) should be rewritten as follows

\[
\min_f \left\{ \frac{L_H}{2} \| f - \bar{y} \|_2^2 + \beta \text{TGV}^2_k (f) \right\}
\]

\[
= \max_{f, w} \left\{ \frac{L_H}{2\beta} \| f - \bar{y} \|_2^2 + \kappa_1 \| \nabla f - w \|_1 + \kappa_0 \| E(w) \|_1 \right\}
\]

where \( p \) and \( q \) denote the dual variables. The sets associated with these variables are given by

\[
P = \{ p = (p_1, p_2) \mid \| p \|_\infty \leq \kappa_1 \},
\]

and

\[
Q = \{ q = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} \mid \| q \|_\infty \leq \kappa_0 \}.
\]

Algorithm 2 Primal-Dual Algorithm for TGV\(_k^2\)-Regularized Denoising (15)

1: **Input:** \( \bar{y}, \kappa_1, \kappa_0, \delta, \tau, \beta \) and \( L_H \).
2: **Initialize:** \( \tilde{f}^0, \tilde{p}^0 \leftarrow 0, \tilde{w}^0 \leftarrow 0, p^0 \leftarrow 0 \) and \( q^0 \leftarrow 0 \).
3: **while** some stopping criterion is not satisfied do
4: \( \tilde{p}^{j+1} = \text{proj}_P (p^j + \delta (\nabla \tilde{f} - \bar{w}^j)) \),
5: \( \tilde{q}^{j+1} = \text{proj}_Q (q^j + \beta E(\tilde{w}^j)) \),
6: \( \tilde{f}^{j+1} = \frac{L_H \bar{y} + \beta \tilde{p}^{j+1} + \tau \text{div} \tilde{q}^{j+1}}{\beta + \tau} \),
7: \( \tilde{w}^{j+1} = \tilde{w}^j + \tau (\tilde{p}^{j+1} + \text{div} \tilde{q}^{j+1}) \),
8: \( \tilde{f}^{j+1} = 2\tilde{f}^{j+1} - \tilde{f}^j \),
9: \( \tilde{w}^{j+1} = 2\tilde{w}^{j+1} - \tilde{w}^j \),
10: **end while**

From [55], the primal-dual algorithm for (15) is summarized in Algorithm 2.

Note that the Euclidean projectors \( \text{proj}_P (\tilde{p}) \) and \( \text{proj}_Q (\tilde{q}) \) onto the convex sets \( P \) and \( Q \) in Algorithm 2 are respectively defined as \( \text{proj}_P (\tilde{p}) = \frac{p}{\max(1,|p|/\kappa_1)} \) and \( \text{proj}_Q (\tilde{q}) = \frac{q}{\max(1,|q|/\kappa_0)} \). The respective definitions for divergence operators \( \text{div} \tilde{p} \) and \( \text{div} \tilde{q} \) read as \( \text{div} \tilde{p} = \partial_{x_1} \tilde{p}_1 + \partial_{x_2} \tilde{p}_2 \) and \( \text{div} \tilde{q} = \partial_{x_1} \tilde{q}_{11} + \partial_{x_2} \tilde{q}_{12} + \partial_{x_1} \tilde{q}_{21} + \partial_{x_2} \tilde{q}_{22} \).

In order to guarantee convergence of the primal-dual algorithm, the parameters \( \delta \) and \( \tau \) in Algorithm 2 are selected as \( \delta = \tau = 1/\sqrt{2} \) throughout this paper.

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The authors would like to thank Chen and Huang [18], Knoll et al. [42], and Xie et al. [58] for kindly providing their codes.

**REFERENCES**


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