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Robust Hybrid Beamforming in Millimeter Wave Relay Networks With Imperfect CSI

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ABSTRACT Millimeter-wave (mmWave) communication offers rich spectrum resources and acts as a key enabling technology for future wireless communication systems. Hybrid (digital and analog) beamforming and relay techniques are important for mmWave implementations considering the characteristics of mmWave signals, as well as the practical limitations of equipment size, power consumption, and hardware cost. In this paper, a robust hybrid beamforming scheme is presented for mmWave multiple-input multiple-output relay networks adopting amplify-and-forward strategy at the relays. Unlike most existing designs that are based on the perfect channel-state information (CSI), CSI imperfectness is considered in the proposed robust beamforming scheme. An accurate approximation of the average received signal-to-noise ratio is derived and used as the design criterion for the developed iterative beamforming optimization at different nodes. An orthogonal matching pursuit-based algorithm is then utilized to design the hybrid beamforming schemes. Simulation results show that the proposed robust beamforming scheme with affordable computational complexities provides substantial performance gains compared with the existing non-robust designs.

INDEX TERMS Millimeter wave communication, MIMO, relays.

I. INTRODUCTION

The ever-increasing demand for data rates in the mobile system requires higher spectrum bands except for the currently available ones (sub 6 GHz). Therefore, the millimeter-wave (mmWave) technology with the vast unlicensed band (from 30 GHz to 300 GHz) has gained tremendous research interest recently [1], [2]. However, the mmWave signal suffers from the severe path loss and is sensitive to propagation blockages due to its short wavelength. Fortunately, the millimeter-scale wavelength enables the transceiver to equip a large number of antennas in a small volume, which can provide significant beamforming gains via so-called massive multiple-input multiple-output (MIMO) technologies [3]. On the other hand, relay techniques can also be utilized to establish MIMO relay networks to enhance the system coverage [2].

In mmWave systems, traditional fully digital beamforming techniques with one radio frequency (RF) chain per antenna element, which can adjust both the magnitude and the phase of a signal, are not practically feasible due to the limitations of equipment size, power consumption, and hardware cost. Therefore, the hybrid analog/digital beamforming scheme, which contains an analog processor with phase shifters only and a low-dimensional baseband digital processor, has been widely discussed in [4]–[10].

In [4], an orthogonal matching pursuit-based (OMP-based) algorithm is utilized to design the hybrid beamformer in a point-to-point MIMO system, by exploiting the sparsity of mmWave channels. The OMP-based algorithm approximates the beamforming design as a Frobenius norm minimization problem, which is attractive for its ease of use. In [5], the algorithm is modified to jointly design hybrid beamformers in a three-node amplify-and-forward (AF) relay system. In [6] and [7], the algorithm is further extended to relay networks with multiple-relay. In [6], the hybrid beamforming is used at the source and the destination, while analog beamformers/combiners are adopted at the relays. In [7], all the terminals in the relay network utilize hybrid beamformers.
The fully digital beamformers are solved firstly to minimize the mean square error (MSE), then the OMP-based algorithm is utilized to give the hybrid solutions. In [8], the analog and digital processors are separately designed based on codebooks to avoid solving an intractable searching problem in a single-relay multiple-user system. A codebook-free hybrid beamforming scheme is proposed in [9], where the design is treated as a matrix factorization problem and solved by the alternating direction method of multipliers. In [10], the analog processor is solved by utilizing the simplified Gram-Schmidt process that resorts only to the array response vectors, while the baseband digital processor is optimized separately by solving the MSE-based problem.

The aforementioned works are based on the perfect channel state information (CSI) assumption to simplify the derivation. However, the acquired CSI is imperfect in practice due to implementation issues, such as bandwidth limitation, feedback delay, and channel estimation errors. The system performance inevitably degrades if the CSI imperfection is not properly handled [11]. There are only a few works dealing with the imperfect CSI in mmWave relay systems, and most of them adopt MSE-based criterion due to its mathematical tractability. In [12], a MSE-based robust hybrid beamforming scheme is proposed for multiple-user mmWave interference channels with Gaussian-distributed errors. In [13], the robust design is extended to the mmWave two-way AF relay system with the same channel model. However, minimizing the MSE does not guarantee satisfactory spectral efficiencies [14]. The average received signal-to-noise ratio (SNR) is a better alternative, which is highly related to the spectral efficiency. In [15], a robust hybrid beamforming scheme based on the average received SNR is proposed for three-node AF relay systems. To the authors’ best knowledge, the robust design for mmWave multiple-relay networks under imperfect CSI is still open.

In this paper, we propose a robust hybrid beamforming scheme for mmWave relay networks with imperfect CSI. AF relay strategy is adopted, and the CSI error is assumed to be Gaussian-distributed. To make the objective function more related to spectral efficiencies, we adopt the approximated average received SNR as the design criterion. Unlike the work in [15], beamformers at both the source and the destination employ multiple RF chains, and the equivalent relay processor show a block diagonal structure. We propose an iterative algorithm to optimize the beamformers at different nodes alternatively. In each iteration, we utilize a simple Rayleigh quotient-based method to optimize the corresponding beamformer. After that, we develop modified OMP-based algorithms to obtain hybrid beamformers at different nodes.

Notations: Bold lower case and upper case letters denote vectors and matrices, respectively. \( A^{(i,j)} \) denotes the \((i,j)\)-th entry of matrix \( A \); \( A^{(i)} \) denotes the \(i\)-th column of matrix \( A \). \((\cdot)^{\ast}, (\cdot)^{T}, (\cdot)^{H}, \text{tr} (\cdot)\) denote the complex conjugate, matrix transpose, Hermitian transpose, matrix trace, and vectorization operator which stacks the columns of the matrix into a vector, respectively. \( I_{K} \) denotes the \( K \times K \) identity matrix. bd \([A_{1}, \ldots, A_{K}]\) is a block diagonal matrix with matrices \( A_{k}, \ k = 1, \ldots, K \) on its main diagonal. \( a \sim \mathcal{CN}(m, \Sigma) \) means that \( a \) is a circularly symmetric complex Gaussian random vector with mean \( m \) and covariance matrix \( \Sigma \). \( \| \cdot \|_{F} \) denotes the Frobenius norm. \( E_{a}[\cdot] \) stands for the expectation over \( a \). \( \otimes \) denotes the Kronecker product.

**II. SYSTEM DESCRIPTION**

In this section, we introduce the mmWave multiple-relay network and channel model considered in this paper.

**A. SYSTEM MODEL**

A two-hop one-way multiple-relay network is shown in Fig. 1, in which a source with \( N_{s} \) antennas communicates to a destination with \( N_{d} \) antennas through the help of \( K \) relays with \( N_{r} \) antennas on each relay. The source, the destination, and each of the relays are equipped with \( N_{s, rf}, N_{d, rf} \), and \( N_{r, rf} \) RF chains, respectively, which enables the node to process signals in both analog and digital domains. The numbers of RF chains at different nodes are subject to the constraint that the total number of RF chains is less than or equal to the number of antennas at each node.

**FIGURE 1. A two-hop one-way multiple-relay network.**

The accuracy of the approximated average received SNR is verified by numerical simulations. Simulation results also show that the proposed beamforming scheme prevails existing non-robust designs with affordable computational complexities.

The rest of this paper is organized as follows. Section II describes the system and channel models. The proposed robust hybrid beamforming schemes for different nodes are presented in Section III. Simulation results are provided in Section IV. The conclusion is given in Section V.
\( N_{s,rf} \leq N_s, N_{r,rf} \leq N_r, N_{d,rf} \leq N_d \). We assume that there is no direct link between the source and the destination, which is typical for mmWave systems because of the limited coverage.

AF strategy is adopted in this paper, and all the nodes are assumed to work in half-duplex mode. It needs two time slots to perform the data transmission. In the first time slot, the source transmits the \( N \times 1 \) signal \( s \) to the relays, where \( N \) is the number of data streams with \( N \leq N_{s,rf} \). The received signals at the relay nodes are

\[
y_r = \sqrt{\rho_{sr}} H_{sr} F_r s + z_{sr},
\]

where \( E[ss^H] = I; F_s = F_x,F_{s,bb} \) is the \( N_s \times N \) hybrid precoder at the source, with \( F_{s,bb} \) being the \( N_s,rf \times N \) baseband processor and the \( N_s \times N_{s,rf} \) analog processor, respectively; \( H_{sr} = [H_{sr,1}, \ldots, H_{sr,K}]^T \), with \( H_{sr,k} (k = 1, \ldots, K) \) being the \( N_r \times N_{r,rf} \) channel matrix between the source and the \( k \)-th relay (S-Rk link); \( y_r = [y_{r,1}, \ldots, y_{r,K}]^T \), with \( y_{r,k} \) being the received signal vector at the \( k \)-th relay;

\[
z_{sr} = [z_{sr,1}, \ldots, z_{sr,K}]^T ,
\]

where \( z_{sr,k} \) is the additive complex white Gaussian noise (AWGN) vector independent, identically distributed (i.i.d.) entries of zero mean and unit variance at the \( k \)-th relay; \( \rho_{sr} \) is the total transmit power to noise ratio of the source-relay (S-R) link. Notice that, \( F_{s,bb} \) is implemented using analog phase shifters, so every element has equal norm, i.e., \( |F_{i,j}^{(l)}|^2 = 1 \); while \( F_{s,bb} \) has no hardware constraints except the transmit power constraint at the source.

The received signals at the \( k \)-th relay are combined through the \( N_{r,rf} \times N_r \) analog beamforming matrix \( F_{r,k} \). In the second time slot, the relays multiply the beamformers’ outputs by the \( N_{r,rf} \times N_{r,rf} \) baseband processing matrix \( F_{r,bb,k} \) and forward the resulting signals to the destination through the \( N_r \times N_{r,rf} \) analog beamforming matrix \( F_{r,k} \), while the source keeps silent. The received signals at the destination are

\[
y_d = \sqrt{\rho_{rd}} H_{rd} F_r y_r + z_{rd}
= \sqrt{\rho_{rd}} \rho_{sr} H_{rd} F_r H_{sr} F_s s + \sqrt{\rho_{rd}} H_{rd} F_r z_{sr} + z_{rd},
\]

where \( F_r = \text{bd} [F_{r,1}, \ldots, F_{r,K}] \) is the equivalent block diagonal relay processing matrix, with \( F_{r,k} = F_{r,t,k} F_{r,bb,k} F_{r,r,k} \) being the processing matrix at the \( k \)-th relay; \( H_{rd} = [H_{rd,1}, \ldots, H_{rd,K}] \), with \( H_{rd,k} \) being the \( N_d \times N_r \) channel matrix between the \( k \)-th relay and the destination (Rk-D link); \( z_{rd} \) is the AWGN vector with i.i.d. entries of zero mean and unit variance; \( \rho_{rd} \) is the total transmit power to noise ratio of the relay-destination (R-D link) link. Similarly, \( F_{r,t,k} \) and \( F_{r,r,k} \) are implemented using analog phase shifters, i.e., \( |F_{r,t,k}^{(l)}|^2 = |F_{r,r,k}^{(l)}|^2 = 1 \); while \( F_{r,bb,k} \) has no hardware constraints except the total transmit power constraint at the relays.

At the destination, the received signals are combined using the \( N_d \times N \) hybrid receiver \( W \). The estimated signal is given by

\[
\hat{s} = W^H y_d = \sqrt{\rho_{rd}} \rho_{sr} W^H H_{rd} F_r H_{sr} F_s s + \sqrt{\rho_{rd}} W^H H_{rd} F_r z_{sr} + W^H z_{rd},
\]

where \( W = W_s, W_{bb} \) with \( W_{bb} \) and \( W_s \) being the \( N_{d,rf} \times N \) baseband processor and the \( N_d \times N_{d,rf} \) analog processor, respectively. \( W_s \) is implemented using analog phase shifters, i.e., \( |W_s^{(l,j)}|^2 = 1 \); while \( W_{bb} \) has no hardware constraints.

### B. CHANNEL MODEL

In this paper, we adopt a narrow-band clustered channel model, which is based on the Saleh-Valenzuela model [17], to characterize the mmWave channel. The channel matrices of the \( S-R \) and \( R-D \) link can be expressed as

\[
H_{l,k} = \frac{1}{\sqrt{N_{l,k} N_{l,ray}}} \sum_{i=1}^{N_{l,ray}} \alpha_{l,k}^{(i)} \hat{a}_r(\theta_{l,k}^{(i)}) a_l(\phi_{l,k}^{(i)})^H ,
\]

where \( l \in \{sr, rd\} \); \( N_{l,k} \) and \( N_{l,ray} \) denote the number of clusters and the number of rays in each cluster of the corresponding link; \( \alpha_{l,k}^{(i)} \) denotes the gain of the \( j \)-th ray in the \( l \)-th cluster of the corresponding link, which is assumed to be a complex Gaussian random variable with zero mean and variance \( \sigma_{\alpha_{l,k}}^2 \); \( a_r(\theta_{l,k}^{(i)}) \) and \( a_l(\phi_{l,k}^{(i)}) \) are the receive and transmit array response vectors of the corresponding link, with \( \theta_{l,k}^{(i)} \) and \( \phi_{l,k}^{(i)} \) being the angle of departure (AoD) and the angle of arrival (AoA), respectively.

In this paper, we adopt the \( M \)-element uniform linear array (ULA), which can be easily extended to the uniform planar array (UPA) [4]. The array response vector is given by

\[
a(\theta) = \frac{1}{\sqrt{M}} \left[ 1, e^{j2\pi d \sin(\theta)}, \ldots, e^{j2(M-1)\pi d \sin(\theta)} \right]^T ,
\]

where \( d \) is the antenna spacing and \( \lambda \) is the mmWave signal wavelength. Without loss of generality, we omit the subscripts and superscripts here for notation simplicity.

In the next, we consider Gaussian-distributed errors of the channel matrices. The actual CSIs processed at the transceiver sides are modeled as

\[
H_{sr,k} = \hat{H}_{sr,k} + \Delta_{sr,k},
\]

\[
H_{rd,k} = \hat{H}_{rd,k} + \Delta_{rd,k},
\]

where \( \hat{H}_{sr,k} \) is the estimated CSI of the \( S-R_k \) link; \( \hat{H}_{rd,k} \) is the estimated CSI of the \( R_k-D \) link; \( \Delta_{sr,k} \) and \( \Delta_{rd,k} \) are the error matrices of the corresponding links, with \( \Delta_{sr,k} \sim \mathcal{CN}(0, \sigma_{\Delta_{sr,k}}^2 I) \) and \( \Delta_{rd,k} \sim \mathcal{CN}(0, \sigma_{\Delta_{rd,k}}^2 I) \), respectively.

### III. HYBRID ROBUST BEAMFORMING SCHEME

In this section, we first approximate the average received SNR to derive a tractable optimization problem. Then we propose an iterative algorithm to design the beamforming schemes at different nodes alternatively. The OMP-based algorithms are developed for different nodes to fulfill the equal norm constraints of the analog processors at last.
A. DESIGN PROBLEM FORMULATION

At the destination, the average received SNR $\hat{\rho}$ is given by:

$$\hat{\rho} = E_{H_{rd},H_{sd}} \left[ \frac{\rho_{rd}\rho_{sr}\|W^HH_{rd}F_rH_{sr}F_s\|_F^2}{\rho_{rd}\|W^HH_{rd}F_r\|_F^2 + \|W\|_F^2} \right].$$  (8)

The first order Taylor’s series expansion in [15] can be applied to $\hat{\rho}$. In this paper, we take the first term after the expansion as the design criterion, which is given by

$$\hat{\rho} \simeq \hat{\rho} = \frac{\rho_{rd}\rho_{sr}E_{H_{rd},H_{sd}}[\|W^HH_{rd}F_rH_{sr}F_s\|_F^2]}{\rho_{rd}E_{H_{rd}}[\|W^HH_{rd}F_r\|_F^2 + \|W\|_F^2]}.$$  (9)

In Section IV, we will show that $\hat{\rho}$ is very close to $\hat{\rho}$ by Monte Carlo simulations. Combined with the equal norm constraints of the analog processors and the total transmit power constraints at different nodes, a joint design of the beamforming schemes is developed as the following optimization problem

$$\max_{\{F_{r,t},F_{r,k},W_{r,t},W_{r,k}\}} \hat{\rho}$$

s.t. $F_{r,t} \in F_{r,t}$, $W_{r,t} \in W$, $F_{r,k} \in F_{r,k}$, $F_{r,k} \in F_{r,k}$,

$$\|F_{r,t}F_{r,k}\|_F^2 = 1,$$

$$E_{H_{sub}} \left[ \|F_{r,t}F_{r,k}F_{r,r}\|_F^2 \right] = 1.$$  (10)

where $F_{r,t}, W, F_{r,k}$, and $F_{r,r}, k = 1, \ldots, K$ are the feasible vector sets induced by the equal norm constraints of the analog processors. The estimated array response vectors are usually utilized as the feasible vector sets in existing non-robust beamforming designs when the perfect CSI is available. To deal with the imperfect CSI, we take the quantized response vectors in [4] as the feasible vector sets in this paper. Defining $A$ as the matrix containing all elements of a feasible vector set, the $i$-th column of $A$ is given by

$$A^{(i)} = a(\theta_{q,i}) = \frac{1}{\sqrt{M}} \left[ 1, e^{j2\pi \sin(\theta_{q,i})}, \ldots, e^{j(M-1)2\pi \sin(\theta_{q,i})} \right]^T$$  (11)

where $\theta_{q,i} = \theta_{\min} + (2i-1)(\theta_{\max} - \theta_{\min})$, with $\theta_{\min}, \theta_{\max}$ being the azimuth sector and $B$ being the quantization bit.

The joint optimization in (10) is known to be intractable [4]. In this paper, we use an iterative algorithm to obtain the beamforming matrices at different nodes. Specifically, we first solve the optimal digital processors $F_{r,opt}, F_{s,opt}$, and $W_{opt}$, alternatively. After that, we utilize OMP-based algorithms to solve the analog and baseband processors at different nodes respectively.

B. HYBRID PROCESSOR DESIGN AT THE RELAYS

Firstly, we solve the fully digital relay processor $F_{r,opt}$ with fixed $F_s$ and $W$. The algorithm in [14] can be utilized. To begin with, we introduce two lemmas for further derivations as follows

**Lemma 1** [18]: For random matrix $X \sim CN(\hat{X}, \sigma^2I)$,

$$E_X[XAX^H] = \hat{X}A \sigma^2 tr(A)I.$$  (12)

**Lemma 2** [19]: For matrices $A \in C^{m \times n}$, $B \in C^{n \times m}$, and $C \in C^{n \times n}$, we have the following identities

$$\text{vec}(ACB) = (B^T \otimes A) \text{vec}(C),$$  (14)

$$\text{tr}(ACB) = \text{vec}^H \left( A^H (I_m \otimes C) \text{vec}(B) \right).$$  (15)

By applying Lemma 1, the expectations in the numerator and denominator of (9), and the last power constraint in the optimization problem (10) can be expanded as

$$E_{H_{rd},H_{sd}} \left[ \|W^HH_{rd}F_rH_{sr}F_s\|_F^2 \right]$$

$$= \text{tr} \left( W^HH_{rd}[H_{rd}F_rH_{sr}] \right)$$

$$= \text{tr} \left( W^HH_{rd}[H_{rd}F_rH_{sr}] \right) + \sigma_{e_{rd}}^2 \text{tr}(W^HW) \text{tr}(F_rF_r^H),$$  (16)

where $\Phi_1 = \hat{H}_{sr}F_sF_s^H \hat{H}_{sr} + \sigma_{e_{rd}}^2 \text{tr}(F_sF_s^H)I$.  (19)

Then, the traces containing $F_s$ in (16)-(18) can be vectorized by applying Lemma 2 as

$$\text{tr} \left( W^HH_{rd}F_rF_r^H \hat{H}_{rd} W \right)$$

$$= \sum_{i=1}^{K} \sum_{j=1}^{K} \text{tr} \left( W^HH_{rd,i}F_{r,i} \Phi_1 F_{r,j}^H \hat{H}_{rd,j}^H \right) W$$

$$= \sum_{i=1}^{K} \sum_{j=1}^{K} \text{vec}^H \left( F_{r,i}^H \left( \hat{H}_{rd,i}^T W^T \hat{H}_{rd,j} \right) \otimes \Phi_{1,ij} \right) \text{vec}(F_{r,j})$$

$$= \text{vec}^H \left( F_{r}^H \right) \Phi_2 \text{vec}(F_{r}).$$  (20)
\[
\text{tr} \left( F_r \Phi_1 F_r^H \right) \\
= \sum_{i=1}^{K} \text{tr} \left( F_r, i \Phi_{1,ii} F_r^{ii} \right) \\
= \text{vb}^H \left( F_r^H \right) \text{bd} \left[ I \otimes \Phi_{1,ii} \right] \text{vb} \left( F_r^H \right), \tag{21}
\]

\[
\text{tr} \left( W_r \tilde{H}_{rd} F_r \tilde{H}_{rd}^H W_r \right) \\
= \sum_{i=1}^{K} \text{vec}^H \left( F_r^H \right) \left[ \left( \tilde{H}_{rd,i} W_r W_r^T \tilde{H}_{rd,i}^T \right) \otimes I \right] \text{vec} \left( F_r^H \right) \\
= \text{vb}^H \left( F_r^H \right) \text{bd} \left[ \left( \tilde{H}_{rd,i} W_r W_r^T \tilde{H}_{rd,i}^T \right) \otimes I \right] \text{vb} \left( F_r^H \right), \tag{22}
\]

where \( \Phi_{1,ii} \) is the \((i,j)\)-th block of \( \Phi_1 \); \( \text{vb}^H \left( F_r^H \right) = \left[ \text{vec}^H \left( F_r^H \right), \ldots, \text{vec}^H \left( F_r^H \right) \right] \) is the vector formed by stacking the vectorized sub-matrices \( F_r, k \) and \( \Phi_{2,ij} = \left( \tilde{H}_{rd,i} W_r W_r^T \tilde{H}_{rd,i}^T \right) \otimes \Phi_{1,ij} \) is the \((i,j)\)-th block of \( \Phi_2 \).

Substituting (16)-(23) into (10), the optimization problem of the relay processor \( F_r \) with fixed \( F_2 \) and \( W \) can be rewritten as

\[
\max \left\{ \frac{\text{vb}^H \left( F_r^H \right) A_1 \text{vb} \left( F_r^H \right)}{\text{vb}^H \left( F_r^H \right) A_2 \text{vb} \left( F_r^H \right)} \right\} \\
\text{s.t.} \begin{cases} 
F_r, k \in F_{r,t,k}, F_r, k \in F_{r,r,k}, \\
\text{vb}^H \left( F_r^H \right) \text{bd} \left[ I \otimes \left( \rho_{sr} \Phi_{1,ii} + I \right) \right] \\
\text{vb} \left( F_r^H \right) = 1 
\end{cases} \tag{24}
\]

where

\[
A_1 = \Phi_2 + \sigma_{e,rd}^2 \text{tr} \left( W_r W_r^T \right) \text{bd} \left[ I \otimes \Phi_{1,ii} \right], \tag{25}
\]

\[
A_2 = \rho_{rd} \text{bd} \left[ \left( \tilde{H}_{rd,i}^T W_r W_r^T \tilde{H}_{rd,i} \right) \otimes I \right] \\
+ \rho_{rd} \sigma_{e,rd}^2 \text{tr} \left( W_r W_r^T \right) I \\
+ \text{tr} \left( W_r W_r^T \right) \text{bd} \left[ I \otimes \left( \rho_{sr} \Phi_{1,ii} + I \right) \right]. \tag{26}
\]

Notice that, the objective (24) is a generalized Rayleigh quotient, with \( A_1 \) being Hermitian and \( A_2 \) being positive definite Hermitian. Therefore, the optimal processor \( \text{vb} \left( F_r^H \right) \) can be solved by

\[
\text{vb} \left( F_r^H \right) = \mu_1 A_2^{-1/2} u_{1,max}, \tag{27}
\]

**Algorithm 1** OMP-Based Algorithm to Design the Hybrid Relay Processors

**Require:** \( F_{r, opt} \), \( A_{r,t,k} \) and \( A_{r,r,k} \)

1. for \( k = 1 : K \) do
2. \( F_{r,t,k} = F_{r,r,k} = \left[ \right]; \)
3. \( F_{r, res} = F_{r, opt}; \)
4. for \( i = 1 : N_{r,s} \) do
5. \( \Gamma_{r,r} = F_{r, res} A_{r,r,k}; \)
6. \( m = \text{arg max}_{i=1, \ldots, N_{r,s}} \left[ \left( \Gamma_{r,r} \right)^{i,j} \right] ; \)
7. \( F_{r,r,k} = \left[ \Gamma_{r,r,k} \left| A_{r,r,k} \right| \right]; \)
8. \( \Gamma_{r,t} = A_{r,t,k}^H(\Gamma_{r,r,k} F_{r,r,k})^{-1}; \)
9. \( n = \text{arg max}_{i=1, \ldots, N_{r,s}} \left( \Gamma_{r,t,k} \left| F_{r,t,k} \right| \right); \)
10. \( F_{r,t,k} = \left[ F_{r,t,k} \left| A_n \right| \right]; \)
11. \( F_{r, bb,k} = \left( F_{r,t,k} F_{r,t,k}^H \right)^{-1} F_{r,t,k} F_{r, opt,k} F_{r,r,k}; \)
12. \( F_{r, res} = \left[ F_{r, opt,k} F_{r, bb,k} F_{r, bb,k} F_{r, res,k} \right]; \)
13. end for
14. end for
15. \( F_{r, bb} = \left[ \sqrt{u_{1,max}} A_2^{-H/2} \right] \text{bd} \left[ I \otimes \left( \rho_{sr} \Phi_{1,ii} + I \right) \right] A_2^{-1/2} u_{1,max}. \)

By re-organizing the elements of \( \text{vb} \left( F_{r, opt}^H \right) \) as a block diagonal matrix, we obtain the optimal digital relay processor \( F_{r, opt} \).

The hybrid relay processors at different relays, i.e., \( F_{r,t,k}, F_{r, bb,k}, \) and \( F_{r,r,k} \), can be obtained based on the sparse approximation. The well known Frobenius norm minimization problem [4], [20] is adopted to solve the cascaded processors as

\[
\min \left\{ \left\| F_{r, opt,k} - F_{r,t,k} F_{r, bb,k} F_{r,r,k} \right\| F \right\} \\
\text{s.t.} \begin{cases} 
F_{r,t,k} \in F_{r,t,k}, \quad F_{r,t,k} \in F_{r,t,k}, \\
\text{vb}^H \left( F_r^H \right) \text{bd} \left[ I \otimes \left( \rho_{sr} \Phi_{1,ii} + I \right) \right] A_2^{-1/2} u_{1,max}. \tag{28}
\end{cases} \tag{29}
\]

where \( F_{r, opt,k} \) is the \( k \)-th diagonal block of \( F_{r, opt} \). We define \( A_{r,t,k} \) and \( A_{r,r,k} \) as the matrices containing all elements of \( F_{r,t,k} \) and \( F_{r,r,k} \), respectively.

The pseudo code of the OMP-based algorithm to solve the above optimization problem is given in Algorithm 1. The hybrid processor at the relays is solved one by one. Step 15 guarantees that the total transmit power constraint at the relays is satisfied.


**C. HYBRID PROCESSOR DESIGNS AT THE SOURCE AND DESTINATION**

In this subsection, we solve the fully digital receiver $W_{opt}$ with fixed $F_r$ and $F_t$. Following the same procedure in Section III-B, we reformulate the trace terms in (16)-(18) by applying Lemma 2 as

$$\text{tr} \left( W^H \hat{H}_{rd} F_r^H F_t^H W \right) = \text{vec}^H \left( W \right) \left[ I \otimes \hat{H}_{rd} F_r^H \right] \text{vec} \left( W \right).$$  \hspace{1cm} (30)

Then the optimization problem of $W$ can be rewritten as

$$\begin{align*}
\max_{\left(W_{s},W_{bb}\right)} & \quad \text{vec}^H \left( W \right) \mathbf{A}_3 \text{vec} \left( W \right) \\
\text{s.t.} & \quad W_{s}^{(i)} \in \mathcal{W},
\end{align*}$$

where

$$\begin{align*}
\mathbf{A}_3 &= I \otimes \hat{H}_{rd} F_r^H F_t^H \hat{H}_{rd}^H + \sigma_{e,rd}^2 \text{tr} \left( F_r \Phi_1 F_t^H \right) I, \\
\mathbf{A}_4 &= \rho_n I \otimes \hat{H}_{rd} F_r^H F_t^H \hat{H}_{rd}^H + \rho_n \sigma_{e,rd}^2 \left( F_r \Phi_1 F_t^H \right) I + I.
\end{align*}$$

The generalized Rayleigh quotient-based method can be applied to solve the optimal digital receiver $W_{opt}$, which is given by

$$\text{vec} \left( W_{opt} \right) = \mathbf{A}_4^{-1/2} \mathbf{u}_{2,max},$$  \hspace{1cm} (35)

where $\mathbf{u}_{2,max}$ is the eigenvector of $\mathbf{A}_4^{-H/2} \mathbf{A}_3 \mathbf{A}_4^{-1/2}$ corresponding to the largest eigenvalue. After re-organizing vec($W_{opt}$) into $W_{opt}$, the cascaded hybrid processors $W_s$ and $W_{bb}$ can be obtained by solving the following Frobenius norm minimization problem

$$\begin{align*}
\min_{\left(W_r,W_{bb}\right)} & \quad ||W_{opt} - W_r W_{bb}||_F \\
\text{s.t.} & \quad W_r^{(i)} \in \mathcal{W}.
\end{align*}$$

Notice that there is only one analog component in the hybrid receiver. Defining $A_r$ as the matrix containing all elements of the feasible vector set $\mathcal{W}_r$, the OMP-based method in Algorithm 1 is modified to solve the above problem, which is summarized in Algorithm 2.

The hybrid structure of the precoder at the source is similar to the receiver at the destination. So, we can obtain the optimal digital processor $F_{s,opt}$ and the cascaded hybrid processors $F_{s,t}$ and $F_{s,bb}$ with fixed $F_r$ and $W$ in a same manner. The optimization problem of $F_{s,opt}$, and the Frobenius norm minimization problem of $F_{s,t}$ and $F_{s,bb}$ can be established with slight modification of (32) and (36). The generalized Rayleigh quotient-based method and Algorithm 2 can be applied directly to solve the above optimization problems. We omit details here for brevity.

**Algorithm 2 Modified OMP-Based Algorithm to Design the Hybrid Receiver**

**Require:** $W_{opt}$, and $A_r$;

1: $W_r = [ ]$

2: $W_{bb} = W_{opt}$

3: for $i = 1 : N_s, r, f$ do

4: \hspace{1cm} $F_r = A_r^I W_{res}$

5: \hspace{1cm} $n = \arg \max_{n=1, \ldots, N_s} \left( \mathbf{F}_r \mathbf{F}_r^H \right)_{(i,i)}$

6: \hspace{1cm} $W_r = \left[ W_r | A_r^n \right]$.

7: \hspace{1cm} $W_{bb} = \left( W_r^H W_r \right)^{-1} W_r^H W_{opt}$

8: \hspace{1cm} $W_{res} = \frac{W_{opt} - W_r W_{bb}}{||r||}$

9: end for

10: return $W_r$ and $W_{bb}$.

**D. ALTERNATING ALGORITHM FOR PRECODER DESIGNS**

We utilize an alternating algorithm to design the processors at different nodes. The pseudo code is given in Algorithm 3. At each step, we optimize the processing matrix at one node by keeping the processing matrices at other nodes fixed. The iteration will end when a stopping criterion triggers, e.g., the change of the average received SNR is below a threshold.

Notice that, each update in Algorithm 3 will increase the computational complexity is comparable with other benchmark designs.

**Algorithm 3 Alternating Maximization Algorithm for Hybrid Transceiver Designs**

**Initialization:** Construct $F_{s,t}$, $W_s$ with random phases and set $F_{s,bb}$, $W_{bb}$ as identity matrices;

1: repeat

2: \hspace{1cm} Optimize $F_r$ with fixed $F_s$ and $W$;

3: \hspace{1cm} Solve $F_{s,t},k$, $F_{s,bb,k}$, and $F_{s,r,k}$ using Algorithm 1, update $F_r = \text{bd}(F_{s,t}, F_{s,bb}) F_{r,1}$, $\ldots, F_{r,k} F_{s,bb,k}$

4: \hspace{1cm} Optimize $W_r$ with fixed $F_r$ and $F_t$;

5: \hspace{1cm} Solve $W_r$ and $W_{bb}$ using Algorithm 2, and update $W = W_r W_{bb}$

6: \hspace{1cm} Optimize $F_s$ with fixed $F_r$ and $W$;

7: \hspace{1cm} Solve $F_{s,t}$ and $F_{s,bb}$ using Algorithm 2, and update $F_s = F_{s,t} F_{s,bb}$

8: until a stopping criterion triggers

**IV. SIMULATION RESULTS**

In this section, we evaluate the performance of the proposed hybrid beamforming scheme. The mmWave channel is modeled to be a $N_{cl} = 8$ cluster environment with $N_{ray} = 10$ rays per cluster for every individual link. All clusters are assumed...
to be equal power, i.e., $\sigma_{d,k}^2 = 1, \forall i, k$. The azimuth AoAs and AoDs of all nodes follow Laplacian distribution with uniformly distributed means over $[0, 2\pi)$ and angular spread 10 degrees. The quantization bits at all the nodes are set to be equal, i.e., $B = 5$. The number of data streams is assumed to be equal to the number of RF chains at the source for simplicity of exposition, i.e., $N = N_{s,rf}$. The ULA is adopted with antenna elements separated by a half wavelength distance, i.e., $d = \lambda/2$. Transmitted SNRs for all the links are set to be equal, i.e., $\rho_{sr} = \rho_{rd} = \rho$. All simulation results are averaged over 10000 random channel realizations. The stopping criterion of Algorithm 3 is chosen that the changing rate of SNR in each iteration is below $1 \times 10^{-3}$.

Fig. 2 shows the spectral efficiencies versus transmitted SNRs with $N_s = N_r = N_d = 8$, $K = 4$, $N_{s,rf} = N_{r,rf} = N_{d,rf} = 4$, $\sigma_{e,sr}^2 = \sigma_{e,rd}^2 = 0.1$. The spectral efficiencies of optimal fully digital designs with perfect CSI, hybrid designs with perfect CSI, and non-robust MMSE-based designs [7] with imperfect CSI are also included for comparison. In the figure, the optimal designs with perfect CSI provide the best spectral efficiencies. As the number of RF chains is sufficient (one RF chain for every two antennas), the hybrid designs with perfect CSI provide spectral efficiencies that are essentially equal to the optimality. However, the perfect CSI assumptions in above two designs are not practical. By considering errors in the acquired CSI, the proposed robust method achieves much higher spectral efficiencies compared with non-robust MMSE-based designs. The figure also shows that the performance gap between the proposed method and the optimal designs gets smaller with the decreasing of the power of errors, e.g., the gap is negligible when $\sigma_e^2 = 0.1$.

Fig. 3 plots the spectral efficiencies with different number of RF chains. Antenna configurations with $N_s = N_d = 32$, $N_r = 8$, $K = 4$, and symmetric errors with $\sigma_{e,sr}^2 = \sigma_{e,rd}^2 = 0.3$ are considered. The numbers of RF chains at different nodes are set to be equal, i.e., $N_{s,rf} = N_{r,rf} = N_{d,rf} = N_{rf}$. As shown in the figure, the optimal fully digital designs with perfect CSI provide the best spectral efficiencies. For hybrid designs, the performances approach the optimality with the increasing of RF chains. Besides, the proposed robust designs with $N_{rf} = 2$ achieve the performance of hybrid designs with perfect CSI when $N_{rf} = 2$. Still, the proposed robust design outperforms the non-robust MMSE-based design.

Fig. 4 shows the spectral efficiencies with asymmetric errors. The configurations are $N_s = N_r = N_d = 4$, $K = 2$, $N_{s,rf} = N_{r,rf} = N_{d,rf} = 2$. The symmetric link corresponds...
to the identical power of errors, i.e., $\sigma_{e, sr}^2 = \sigma_{e, rd}^2 = 0.1$; the strong SR link corresponds to $\sigma_{e, sr}^2 = 0.1, \sigma_{e, rd}^2 = 1$; and the strong RD link corresponds to $\sigma_{e, sr}^2 = 1, \sigma_{e, rd}^2 = 0.1$. In the figure, the proposed design with symmetric link and low errors ($\sigma_{e, sr}^2 = \sigma_{e, rd}^2 = 0.1$) approach the hybrid designs with perfect CSI as expected. The performance of the strong SR link outperforms the strong RD link for both robust and non-robust designs. The proposed robust designs still provide higher spectral efficiencies compared with non-robust designs.

In Fig. 5, the accuracy of the Taylor’s series approximation is evaluated. The average received SNR in (8) and the approximation in (9) are compared. Three antenna configurations are considered, where Scenario I corresponds to $N_s = N_r = N_d = 8, K = 4$, Scenario II corresponds to $N_s = N_r = N_d = 32, N_r = 8, K = 4$, and Scenario III corresponds to $N_s = N_r = N_d = 4, K = 2$.

Next, we evaluate the computational complexities of different designs. In Table 1, we first list the average iteration numbers for all the simulations in Fig. 5. As we can see, it takes only a few rounds for the proposed method to converge to a solution in all the simulations. The iteration number increases with the numbers of antennas and RF chains. The computational complexity per iteration can be found in Table 2. We also include the complexities of the hybrid design with perfect CSI and the non-robust MMSE-based design for comparison. As all the designs we considered in simulations employ the OMP-based algorithm, we only list complexities of matrix operations required per iteration in the table, including the matrix multiplication, inversion, and eigenvalue decomposition (EVD). It is seen that the computational complexities of the proposed method and the hybrid design with perfect CSI are in the same order, and both of them are larger than the non-robust MMSE-based design due to the Kronecker product required in the Rayleigh quotient-based method. However, these two designs provide significant gains compared with the MMSE-based design as shown in previous simulations. Combined with the average iteration numbers in Table 1 and the practical issue to acquire perfect CSI, the proposed method can provide good spectral efficiencies with affordable complexities compared with existing methods, which makes it more attractive in practice.

V. CONCLUSIONS

A robust hybrid beamforming scheme for mmWave AF MIMO relay networks has been presented. The imperfect CSI with Gaussian-distributed errors is dealt with to derive an approximated average received SNR as the design criterion. An iterative algorithm is proposed to optimize the beamformers at different nodes alternatively. A modified OMP-based algorithm is developed to design the hybrid processor at the corresponding node. Numerical results show that the approximated SNR is very close to the average results of Monte-Carlo simulations, and the proposed scheme provides substantial gains over existing schemes with affordable computational complexities.

REFERENCES


