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Cost-Effective Path Planning for Submarine Cable Network Extension

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ABSTRACT We describe a cost-effective approach to path planning for submarine cables connecting a given site to an existing cable network on the Earth's surface. The objective is to minimize the overall life-cycle cost of submarine cables by considering multiple design considerations. With the surface terrain represented by a triangulated manifold, we formulate the problem as one of the variational optimization framed in terms of an Eikonal equation and solved by leveraging the fast marching method. The results based on real-world three-dimensional data are presented to illustrate the performance of this approach in the submarine cable network extension.

INDEX TERMS Submarine cables, path planning, network extension, design considerations.

I. INTRODUCTION

Submarine (telecommunications) cables play a crucial role in the transport of information from one location to another around the world. The latest submarine telecoms industry report [1] shows that 99% of international communications are carried over submarine cables. The performance of such a critical infrastructure has great impact on businesses and consumers. For example, Amazon found that every 100 milliseconds of latency (i.e., the time delay in sending data from one location to another, which is directly related to the cable length) cost them 1% of profit, and Google estimated that every 100 milliseconds of latency reduce traffic by 4% [2].

To date, over 1.2 million kilometers of submarine cables have been constructed [3]. It was predicted that a total of \$9.2B (all costs are in US dollars) would be spent on submarine cable projects all over the world between 2016 and 2018, representing a five-fold increase from the previous three years [4]. It is also known that the service capacity of

submarine cable networks is currently a bottleneck for 5G networks [5], and that the data bandwidth demand is envisioned to double every two years for the foreseeable future [1]. These facts bolster the need for developing cost-effective approaches to the extension of submarine cable networks.

One problem of significant practical interest is to lay a new branch from a given site to an existing submarine cable system. This problem arises in bringing submarine cable connectivity to regions such as islands and remote areas, and also in increasing the data transmission capacity by adding new branches to ones that already exist. Examples of the former include the construction, started in 2017, of branches from the existing SEA-US (South-East Asia to the United States) submarine cable to the islands of Palau, Yap, and Chuuk [6]. Examples of the latter include the new cable branch from Qingdao to the existing EAC (East Asia Crossing) network in 2006 [7], funded by the Chinese Government, and a plan dating from 2016 to connect Hanko, the southernmost port of Finland, with C-Lion1, a German-Finnish submarine cable system [8].

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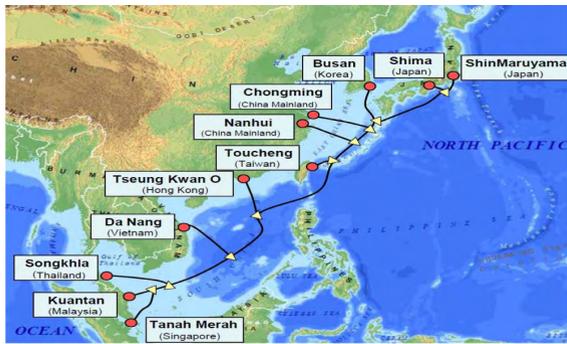


FIGURE 1. The Asia Pacific gateway submarine cable system (<http://www.nec.com>), with nine branching units represented by yellow triangles, connects Japan, Korea, Mainland China, Hong Kong, Taiwan, Thailand, Vietnam, Singapore, and Malaysia.

In general, two engineering choices are possible in such a context of cable network extension. The new cable can be connected from the given site either to a *cable landing station* of the existing network or to a *branching unit* built into an existing cable. A cable landing station is an interface between the submarine cable system and the overland terrestrial transmission system. A branching unit is a device that enables splitting of the cable to serve multiple destinations. It is possible to insert a new branching unit into existing submarine cable systems, but this may require the existing system to be shut down during the insertion operation, resulting in financial losses. In general, as illustrated by the example in Fig. 1, branching units are installed on submarine cables before the cable is laid, which enable cable owners to add new branches to the system.

Cable path planning is a vital procedure to achieve cost effectiveness and resilience because the path of a cable has a direct impact on its costs and its resilience. Note that, in our context of cabling, the term *path planning* is used for finding the optimal path of a cable in a static environment. This is somewhat different from the same term commonly used in many other applications, such as autonomous mobile robots and unmanned aerial vehicles, where it is important to find an optimal path/route in a dynamic environment [9]–[11].

The life-cycle cost of an infrastructure is the total cost over its entire life span and it includes cost induced by the risk and its potential adverse effect on users [12], [13]. It is known that the life-cycle cost incurred in the construction, maintenance and repair of submarine cables is significant. Most existing submarine cable systems cost hundreds of millions of dollars to build [14]. The annual maintenance cost is around 3-5% of the capital cost, and a single cable break can cost hundreds of thousands of dollars to fix [15].

While length certainly adds to the basic construction cost of a submarine cable, its life-cycle cost depends also largely on various natural and human factors that are of concern to the submarine telecoms industry. They typically include volcanic eruptions, earthquakes, water depth, seabed slope, sediment hardness, human activities such as fishing and anchoring. In addition, in path planning for submarine cables,

it is important to avoid environmentally sensitive regions and prohibited areas. For ease of maintenance and repair, it is also necessary to keep a safe distance from existing man-made submarine infrastructures. Section III discusses more on the design considerations taking account of such risk factors for submarine cable network extension.

In this paper, we address the problem of physical path optimization for a submarine cable connecting a given site to an existing cable network on the Earth's surface, with the aim of minimizing the life-cycle cost of the submarine cable that accommodates multiple design considerations. Our main contributions are summarized as follows:

- We discuss design considerations taking account of the various factors that affect the life-cycle cost of submarine cables.
- We define the life-cycle cost function as a weighted sum of cost items attributed to the corresponding design considerations. In determining the weight of each cost item, we follow the general industry practice and prioritize the design considerations qualitatively. Then, we apply the analytical hierarchical process (AHP) introduced in [16] to translate the qualitative prioritization into weight values.
- We formulate the problem as one of variational optimization and show that it can be transformed into an eikonal equation. This enables us to develop an efficient algorithm by leveraging the well-established fast marching method (FMM) [17]–[19] for solving the eikonal equation.
- We perform experiments on real-world three-dimensional data. Comparing our proposed method with one that is based on the Dijkstra's algorithm, the results demonstrate considerable cost savings of up to 17.5%.

The rest of the paper is organized as follows. Section II discusses the related research work. Section III provides the model of the surface terrain, details of the design considerations, and the definition of the life-cycle cost function. Section IV describes the problem formulation and the proposed path planning methodology. Section V presents the numerical results. Finally, conclusions are drawn in Section VI.

II. RELATED WORK

Burnett *et al.* [20] provided guidance on submarine cable route selection in industry. Using data for the region of interest such as charts, satellite gravity bathymetric data, etc., experts manually produce several reasonable routes to connect two points; this consists of a series of route position lists and straight-line diagrams. For a designated route, a preliminary survey is executed along the route to evaluate its availability and rationale. If some constraints on the route cannot be avoided or eliminated, alternative routes will be considered and surveyed. A final path is determined by carefully checking all survey data along each route and comparing all the available routes. Such a manual approach cannot guarantee an optimal path, is very time-consuming, and

relies on, albeit expert, subjective judgments of a complicated scenario.

A numerical solution based on integer linear programming for optimally selecting cable paths from a given set of path alternatives was presented in [21]; this aimed to minimize the cost for both the affected society and cable owners in case of a disaster. Using earthquake hazard information, Tran and Saito [22] presented a method to maximize the robustness of a network by finding suitable geographical routes from various path alternatives under a cost constraint. In [23], Tran and Saito proposed to use dynamic programming to find new links and their routes to a network in a way that minimizes the total end-to-end disconnection probability under a given cost constraint. A node/link replacement strategy was proposed in [24] for a given planar physical network affected by a disaster. In contrast, our work in this paper is focused on physical path optimization for submarine cables on the Earth's surface.

The research work presented in [25]–[27] provided geometric methods to evaluate the vulnerability of a geographic network by identifying the worst-case location of a disaster using probabilistic analysis and certain connectivity measures. The authors of [28], [29] considered a circular disk failure model, assuming that a disaster occurred randomly in a circular disk of a given radius. They evaluated the robustness of connectivity of a network to such a randomly placed disaster based on certain network performance metrics. Considering also a circular disk failure model, Cao *et al.* [30] formulated an optimization problem on a two-dimensional plane for path planning to minimize the total cable cost under certain constraints on resilience. In [31], direction-dependent effects of earthquakes were considered. In this work, an earthquake hazard was assumed to occur in an elliptical area and a path planning method was proposed for cables that lie on a plane. Agrawal *et al.* [32] considered earthquake risk of a backbone optical network based on a stochastic model for occurrence of earthquakes. They also provided a scheme to improve network survivability against earthquake induced failures by relocating a node in the nearby surrounding area. In [33], the use of shielded links (e.g., armored telecommunication cables) was considered to enhance the network resilience in risk-prone areas. Zhang and Modiano [34] gave a method to evaluate the robustness of interdependent networks by obtaining the minimal number of node removals that disconnect the network. The work in [35] provided a method applicable to optical networks for dynamically reconfiguring traffic capacities when a natural hazard strikes, which improves the resilience of the networks and mitigates the impact of the disaster. In the work of [25]–[35], the cables/links are assumed to be laid on a plane and regarded as either straight lines or curved lines with rounded corners. In addition, all these publications considered planar failure models in calculating the cable/link break probability.

The work in [36]–[40] addressed the fundamental problem of cable path optimization between two sites on the

Earth's surface. Aiming for cost minimization and earthquake resilience, Zhao *et al.* [36] provided a raster-based path method to find the least accumulative cost path using the Dijkstra's algorithm. A major shortcoming of the raster-based path method is that a path is restricted to traverse along the edges of the discrete graph when moving between neighboring cells. In [37], an FMM-based approach was presented considering earthquake risks to solve the cable path optimization problem. In [38] and [39], Wang *et al.* proposed optimal path planning with consideration to different protection levels available for cables. The methods used in [38] and [39] were based on a discrete graph and triangulated manifolds, respectively. Wang *et al.* [40] considered the potential overturn risk of a remotely operated vehicle as it buries the cable in an uneven seabed, which is related to the slope of the terrain and the path direction. In this paper, we extend the methodology of [37] to address the more challenging problem of submarine cable network extension taking into account a wide range of design considerations.

III. MODELS

A. SURFACE TERRAIN

As the elevation data about the Earth's surface (including the sea floor) is usually available in a discrete grid, we model the Earth's surface under consideration as a *triangulated piecewise-linear two-dimensional manifold* \mathbb{M} in three-dimensional Euclidean space \mathbb{R}^3 . Each point on \mathbb{M} is described by three-dimensional coordinates (x, y, z) , where $z = \xi(x, y)$ is the elevation of the geographic point (x, y) [41]. Note that, in geographic information science and other related fields, triangulated manifolds are widely adopted to represent topography and surface terrain [42]. Comparing with other available models (e.g., the regular grid model), triangulated manifolds make it easier to accommodate rough surfaces and irregularly spaced elevation data [42].

B. DESIGN CONSIDERATIONS

Here, we describe in more details the design considerations that take account of the various factors for submarine cable network extension. A dollar cost is ascribed to each design consideration to evaluate its budget effects. Our approach requires the setting of a range of parameter values. This is done empirically based on the experience and expertise of path planning cable designers. Examples of specific functions for cost evaluations of the design considerations are provided in Section V.

1) BASIC CONSTRUCTION COST

For any point $X = (x, y, z) \in \mathbb{M}$, the basic construction cost per unit length at location X is defined as $h_1(X)$. The function $h_1(X)$ enables consideration of location-related factors that affect the basic construction cost. Examples of such factors include materials, labor and permits (e.g., right of way). The cable length related cost may also include indirect cost, such as those associated with effect of latency on users' quality of experience.

2) VOLCANIC ERUPTIONS

Volcanic eruptions can damage submarine cables through lava flows and avalanches of hot debris directly [43]. Let n_1 represent the set of all volcanoes in \mathbb{M} , $d(X, i_1)$ represent the distance from location X to the vent of a volcano $i_1 \in n_1$, and $h_2(X, i_1)$ denote the cost associated with the volcano i_1 at location X . In general, $h_2(X, i_1)$ is a decreasing function of the distance $d(X, i_1)$ and an increasing function of the magnitude of potential eruptions. We define the summary cost associated with all volcanoes at location X as $h_2(X) = \sum_{i_1 \in n_1} h_2(X, i_1)$.

3) EARTHQUAKES

Earthquakes can result in significant displacements of the seabed and destabilization of the seabed sediment by liquefaction, surface faulting and landslides [20], [44], [45], which can potentially damage submarine cables. Earthquake risk at a given location, for example, is measured by predicted peak ground velocity (PGV), or magnitude of historical earthquakes over the last 60 years. PGV is known to be strongly correlated with the index *repair rate*, which represents the expected number of repairs (or cable breaks) per unit length resulting from earthquake damage [46]. Let n_2 denote the set of all earthquakes that have occurred in \mathbb{M} . We use $h_3(X, i_2)$ to denote the cost caused by an earthquake $i_2 \in n_2$ at location X and $h_3(X, i_2)$ has a strong positive correlation with repair rate at location X . The summary cost associated with all earthquakes at location X is $h_3(X) = \sum_{i_2 \in n_2} h_3(X, i_2)$.

4) WATER DEPTH

Generally, the risk level and the budget of a cable have a close relationship with the water depth where the cable is laid. Coast and inshore areas down to approximately 300 m water depth are not only most frequently exposed to natural hazards caused by steep slopes and weather, but also incur over 72% of annual cable faults caused by fishing and anchoring [47]. Cables laid in shallow sea areas are, consequently, generally double armored and buried to a depth of 1.5 m to provide the necessary protection, resulting in additional costs. As water depth increases, the threats to cables from human activities decreases. At depths in excess of 1000 m, route designers usually prefer light weight cable because the threats to cables are relatively rare [20], [44], [45]. Let $h_4(X)$ denote the cost caused by water depth $D(X)$ (in units of kilometers, where $D(X) \leq 0$ means on the land) at location X . In common practice, route designers usually determine a specific water depth \tilde{D} ($\tilde{D} > 0$), referred to as “End of Burial” limit, which depends on environmental features (e.g., how long the continental shelf extends) and the project. That is, for example, when $0 \leq D(X) < \tilde{D}$, cables are usually double armored and buried, and $h_4(X)$ decreases significantly with the growth of $D(X)$. In areas where $D(X) \geq \tilde{D}$, $h_4(X)$ decreases relatively slowly as $D(X)$ increases.

5) SEABED SLOPE

Seabed areas of higher slope are more prone to generate hazards such as debris flow [44]. Commonly, in relatively steep areas, it is necessary for cable routes to be orthogonal to the slope and to have minimal course alterations. Such course changes reduce the lateral contact surface area of the cable with the seabed. With a lesser tension on the cable, it is more likely that sediment failures could be avoided. And, of course, the cable ship prefers straight laying, optimizing the chances of having the cable touchdown point in the desired location. In addition, a remotely operated vehicle (ROV) is usually used to assist burial of cables, and a steeper slope increases the likelihood of overturning the ROV [48]. In areas with very steep slope, it may not be possible to use an ROV or even to deploy it at all. When cable laying but not burying is possible, protection of the cable is augmented, resulting in added laying costs [49]. Let $h_5(X)$ be the cost associated with the slope $p(X)$ at location X . To compute this, experts usually determine two specific slope degrees \tilde{p}_1 and \tilde{p}_2 ($\tilde{p}_1 < \tilde{p}_2$) as “threshold” (e.g., $\tilde{p}_2 = 20^\circ$ in [48], [50]) in common practice. The slope $p(X) \leq \tilde{p}_1$ is considered sufficiently low to have a negligible effect on the laying cost. Areas where $\tilde{p}_1 < p(X) \leq \tilde{p}_2$ are problematic, and $h_5(X)$ increases as $p(X)$ increases to accommodate possible augmented protection. For steep areas where the slope $p(X) > \tilde{p}_2$, $h_5(X)$ increases rapidly with $p(X)$, indicating higher levels of protection [20], [44] and possible overturn of the ROV.

6) SEDIMENT HARDNESS

In areas where the cable needs to be buried, soft and loose sediment types are commonly preferred, and rocky seabed terrains need to be avoided because of the difficulty of burying [20], [44], [45]. Where rocky regions are unavoidable, other forms of protection for cables are available including double armoring of the cable, articulated pipes and cable anchors. The cost caused by sediment hardness at location X is defined as $h_6(X)$ where, the harder the seabed, the less its suitability for burying cables, and the higher $h_6(X)$.

7) ENVIRONMENTALLY SENSITIVE REGIONS AND PROHIBITED AREAS

Cable routes need to avoid environmentally sensitive regions as much as possible [20], [44], [45]. For example, some vulnerable species living in the seabed ecosystem, such as corals and seagrass, are adversely affected by operations of cable deployment and repair. In addition, coral reefs provide habitats and nursery grounds for other marine organisms. Any cable laying activity in these areas is likely to trigger requirements for stringent environmental impact assessments, which implies additional costs. Known prohibited areas such as military areas and disputed territories also need to be avoided. It is extremely hard and time consuming to request permissions for cable operations in these prohibited areas from relevant authorities [20], [45]. For safety considerations, sea areas potentially suffering or in the process of a war,

terrorism or piracy attacks are totally avoided. Let $h_7(X)$ denote the cost at location X caused by this issue. If location X is in an environmentally sensitive region or prohibited area, a penalty cost is assigned to $h_7(X)$.

8) FISHING AND ANCHORING

Fishing and anchoring are two principal causes for cable faults. According to the statistical data [47], there are about 150 to 200 cable faults annually worldwide, and around 72% of these faults are attributed to fishing and anchoring. Over 80% of fishing and anchoring faults occur in shallow water (depth of 300 m or less). As mentioned earlier, as water depth increases, the threat to cables from fishing and anchoring decreases [20], [44], [45].

The fishing equipment used in bottom trawling activities and the dredge rakes along the trench to harvest products such as clams and scallops can dig dozens of centimeters into the seabed [20], [44], [45]. This behavior can directly and severely damage unburied or partially buried cables. Let $h_8(X)$ represent the cost caused by the fishing activities at location X , and $h_8(X)$ has a strong positive correlation with water depth and the frequency of fishing activities.

Anchoring damages cables in two ways: heavy anchors dig into soft seabed floors crushing cables directly, and laid anchors can be dragged over a long distance by a vessel before being hoisted [20], [44], [45]. In the latter case, cables laid in the path of the dragged anchor can be severely damaged. Let $h_9(X)$ be the cost related to anchoring at location X . As in the case of fishing, the cost $h_9(X)$ has a strong positive correlation with water depth and the frequency of anchoring activities.

9) EXISTING MAN-MADE SUBMARINE INFRASTRUCTURES

For ease of maintenance and repair, it is necessary to reserve a sufficient separation, called ‘‘safety distance’’, between the new cable route and existing man-made infrastructures such as submarine cables, pipelines, and offshore energy infrastructures. Near an offshore energy infrastructure or a planned exploitation region, inevitable drilling operations and seabed explorations impose a potential threat to submarine cables, even if the cables are well buried [20], [44], [45]. For example, the International Cable Protection Committee highly recommends reserving at least 1 km separation of a new planned cable route from existing offshore wind farms located in shallow water [51]. In high risk areas, a new cable route needs to keep away from existing cables to prevent multiple cables from being damaged simultaneously by either a natural or human-caused disaster. With n_3 representing the set of man-made infrastructures in \mathbb{M} , the associated cost at location X is $h_{10}(X) = \sum_{i_3 \in n_3} h_{10}(X, i_3)$, where $h_{10}(X, i_3)$ represents the cost attributed to an existing man-made infrastructure $i_3 \in n_3$ and is a decreasing function of the shortest distance $d(X, i_3)$ from location X to the infrastructure i_3 .

10) CONSIDERATION OF OTHER NATURAL HAZARDS

In addition to the design considerations discussed above, other natural hazards (e.g., cyclones, hurricanes and

typhoons) may lead to a damage of the submarine cable systems [20], [44]. For example, hurricanes may destroy terminal stations (e.g., beach manholes and cable landing stations) of cable infrastructures located near the shore and occasionally they can trigger submarine landslides off the continental shelf and damage existing cables near the shore [52]. However, the location selection of terminal stations of cable infrastructures (which is also associated with the locations of the cables near the shore) is beyond the scope of this paper.

C. LIFE-CYCLE COST

Let N denote the indexed set of the design considerations discussed above. We define $h(X)$ as the life-cycle cost per unit length of the cable passing through location X , and it is given by

$$h(X) = \sum_{m \in N} w_m h_m(X), \quad (1)$$

where w_m represents the weight value specified for the cost item attributed to the corresponding design consideration. We discuss in Section V how one may determine the weight values based on general industry practice.

Let $\mathbb{H}(\gamma)$ denote the total life-cycle cost of a cable γ (Lipschitz continuous [53]). To calculate $\mathbb{H}(\gamma)$, according to the *natural parameterization* [54], the cable γ is parameterized as a function of arc length s . That is, each point $X \in \gamma$ can be written as $X = X(s)$. We assume that, for any point $X \in \gamma$, the life-cycle cost per (arbitrarily) small length ds is the product of the cost $h(X)$ and the length ds , i.e., $h(X)ds$. Then $\mathbb{H}(\gamma)$ can be written as

$$\mathbb{H}(\gamma) = \int_0^{l(\gamma)} h(X(s))ds, \quad (2)$$

where $l(\gamma)$ represents the total length of the cable γ .

IV. PROBLEM FORMULATION AND PATH PLANNING METHODOLOGY

Let node A be a fixed point in \mathbb{M} . Let $\gamma_1, \gamma_2, \dots, \gamma_k$ be the existing cables in a cable system network in \mathbb{M} . Our objective is to lay a cable γ from A to an unknown end location B with the minimal life-cycle cost $\min_{\gamma} \mathbb{H}(\gamma)$. The end node B belongs to one of the existing cables γ_t , $t = 1, 2, \dots, k$. The optimization problem is as follows:

$$\begin{aligned} \min_{\gamma} \mathbb{H}(\gamma) &= \min_{\gamma} \int_0^{l(\gamma)} h(X(s))ds \\ \text{such that } &\gamma(A) = A, \gamma(B) = B, \text{ node } B \\ &\text{belongs to one of the existing cables } \gamma_t, \\ &t = 1, 2, \dots, k, \text{ given } \gamma_1, \gamma_2, \dots, \gamma_k. \end{aligned} \quad (3)$$

Note that in this context (3) can be converted into a non-linear partial differential equation, also known as the eikonal equation, which arises in problems of wave propagation and describes a large number of physical phenomena [17]–[19]. In general, there is no analytical solution for the eikonal equation and, indeed, our available data is just the discretized

topographical data. Consequently, we resort to a numerical method to compute an approximate solution. In this paper, we adopt FMM, a continuous version of the Dijkstra's algorithm. Unlike the "discrete" Dijkstra's algorithm that forces the path to walk exclusively along the edges of triangles in the triangulated manifold, this "continuous" version of the Dijkstra's algorithm finds better solutions by allowing the path to traverse through the interiors of triangles, and is more resilient to the particular choice of triangulation.

To use FMM, we discretize the continuous area into a triangulated manifold \mathbb{M} as previously discussed, with mesh points of \mathbb{M} referred to as nodes. The FMM-based approach is based on two phases. In the first phase, we start from the source node and consider other nodes in order according to their distance (cost) from the source node [17], [18]. This is similar to the Dijkstra's algorithm but it is done using a wavefront representing the set of points with equal cost. The wave is generated at the source node A and, when the wavefront touches a node, the node is tagged and its cost (its distance from the source node) is recorded. The speed of the wavefront moving outward from a node is the reciprocal of the cost on the node. The process continues until the wave reaches the destination node B . In the second phase, we start from the destination node B tracking back to the source node A using the information gleaned in the first phase. However, unlike Dijkstra, the path back does not exactly visit the nodes on the path back, but crosses a line between the nodes and their closest neighbors that have been tagged based on the steepest decent principle [17], [18]. This phase is continued until the backward path reaches the source node A . This path backwards is the planned path.

The fundamental principle of FMM is described next. For a site $S \in \mathbb{M}$, we have a cost function $\phi(S)$, representing the minimum life-cycle cost of the cable from node A to node S . That is to say, if we let c be a fixed nonnegative cost, the set $\{S \in \mathbb{M} | \phi(S) = c\}$ is a front. Kimmel and Sethian [19] demonstrated that $\phi(S)$ is the physical solution of the eikonal equation

$$\|\nabla\phi(S)\| = h(S), \quad \phi(A) = 0, \quad (4)$$

where ∇ is the gradient operator and $\|\nabla\phi(S)\|$ indicates the length of the gradient vector $\nabla\phi(S)$. The function $h(S)$ is the life-cycle cost as described by (1). The gradient of $\phi(S)$ can be calculated once $\phi(S)$ is derived for every point S in \mathbb{M} . Based on the gradient of $\phi(S)$, we can obtain the optimal path along the greatest gradient. In other words, the optimal path will be constructed by tracking backwards from S to A along the greatest gradient.

Our goal is to connect node A into the given cable network system with the least life-cycle cost. The methods and ideas we have described above, using the surface triangulation \mathbb{M} and FMM [17]–[19], translate into the following path planning algorithm for planning a path from node A to an existing network:

- 1) Tag all nodes of the given triangulated gridded surface as *far* for initialization.

- 2) Tag the start node A as *dead* and the neighbors (one grid point away) of A as *open*.
- 3) Calculate the values of the *open* nodes by solving the eikonal equation (4) and check whether one of these new *open* nodes belongs to the existing network. If yes, this special node B is the destination and the algorithm moves to **Step 5**. If not, tag the *open* node with minimum value ϕ as *dead* and its neighbors tagged *far* as *open*. If there are multiple nodes with minimum value ϕ , choose one randomly to become *dead* and convert its neighbors with *far* tags into *open*.
- 4) Return to **Step 3**.
- 5) Derive the optimal path by solving $\frac{dX(s)}{ds} = -\nabla\phi$ given $X(0) = B$, i.e., track backwards from B to A along the steepest gradient.

In **Step 3**, an update schema to calculate the value of ϕ of the *open* nodes is required. We adopt the monotone update procedure in [17], [18] to approximate the gradient on a triangulated mesh. In **Step 5**, a finite-difference approximation, for example, the first-order Euler method or a second-order Heun's integration method, can be used to track backwards along the path. Note that the computational complexity of FMM is $O(V \log V)$, where V is the number of discretized grid nodes of \mathbb{M} [17].

V. APPLICATION

In this section, we apply our method to a practical scenario. A region \mathbb{D} from the southwest point (7.5000°N, 122.8000°E) to the northeast point (11.5000°N, 125.8000°E) is located in the Philippines as shown in Fig. 2(a). Bohol Island is in the center of the region \mathbb{D} . West of Bohol Island are Cebu Strait and Cebu Island. To the northeast of Bohol Island is Leyte Island and to the south of Bohol Island, across the Bohol Sea, is Mindanao Island. An existing cable network system represented by the black lines in Fig. 2(a), called the *Domestic Fiber Optic Network* (DFON) and run by PLDT Inc., provides communication services for the major islands of the region \mathbb{D} . The black circles in Fig. 2(a) represent the landing stations of the system. Bohol Island, with more than one million population, is not connected to the DFON cable network.

Our aim is to design a cable path from the start node (9.7287°N, 124.4516°E) on Bohol Island represented by a black square in Fig. 2(a) to the existing DFON cable network. We assume that two branching units have already been installed on a cable whose length is in excess of 200 km, and at most one branching unit has already been installed on a cable less than 200 km. In industry practice, the distance between two branching units should be more than three times water depth. This is because, when a branching unit is broken, the repair vessel needs to lift it and the operation disturbs other branching units if they are set too close. The maximal water depth in the region \mathbb{D} is less than 4 km, and we satisfy the industry practice by keeping the distance between any two branching units in excess of 12 km. The locations of the installed branching units are chosen randomly, though

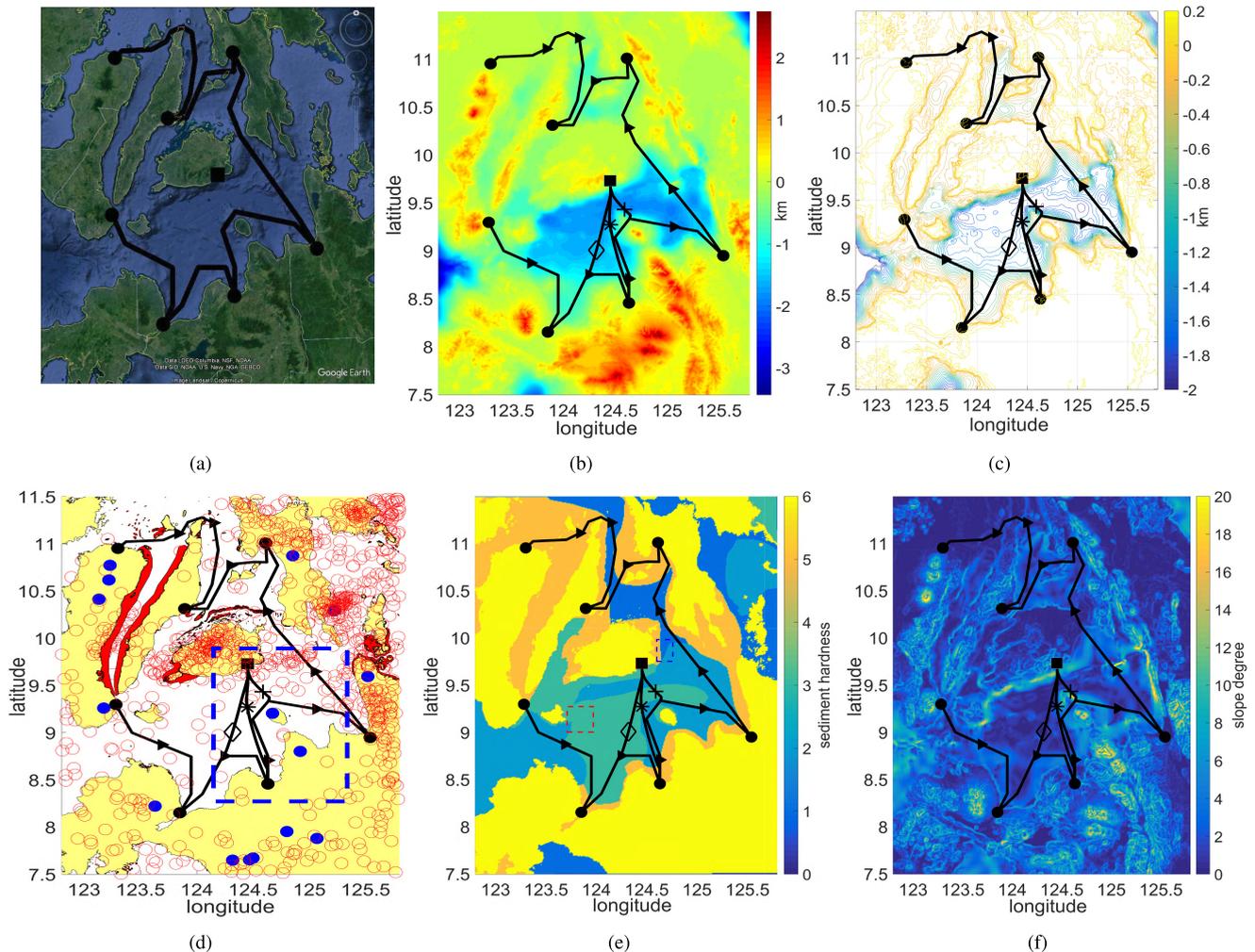


FIGURE 2. Application example. (a) The region \mathbb{D} in Google Earth. The black circles indicate the landing stations. The black lines illustrate the DFON cable network. The black square represents the start node. (b) Elevation map of the region \mathbb{D} . The black triangles represent the branching units on the cables. (c) Elevation contour map of the region \mathbb{D} . (d) Coastline of the region \mathbb{D} . The blue nodes indicate the locations of volcanoes and the red circles illustrate the sites where earthquakes historically occurred. The larger the size of the red circles, the higher the magnitude of the historical earthquake. The red regions represent the coral and seagrass distribution. (e) Sediment map of the region \mathbb{D} . The lower the value of a specific sediment, the looser the seabed sediment. The red rectangle represents fishing areas and the black rectangle represents anchoring areas. (f) Slope map of the region \mathbb{D} .

we exclude the cases where the distance between branching units or between a branching unit and a landing station is less than 12 km.

We consider three scenarios:

Scenario 1: The destination can be an arbitrary location on an existing cable with the addition of a new branching unit.

Scenario 2: The destination can be an installed branching unit or a landing station.

Scenario 3: The destination can only be a landing station.

Scenario 1 suits a submarine cable system that allows inserting a branching unit on an existing cable. Though less practical, this case enables us to obtain a lower bound on the cost of cable laying, a matter of significant practical importance. Scenario 2 works for a practical case in which unused branching units exist in the cable system and the main trunk allows some of the transmission capacity to be diverted from these unused branching units. Scenario 3 is more appropriate for

an end-to-end system and it does not affect the provision of capacity to the current cable system.

In this example, we consider available data on basic construction cost, historical earthquakes, locations of volcanoes, seabed slopes, sediment hardness, water depth, and the distribution of seagrass and coral. Since local data on fishing and anchoring is absent, two regions are postulated as shown in Fig. 2(e). The red rectangle from the southwest corner (9.0000°N, 123.7100°E) to the northeast corner (9.2700°N, 123.9700°E) represents fishing areas, and the black rectangle from the northwest corner (9.7500°N, 124.6000°E) to the southeast corner (9.9800°N, 124.7600°E) represents anchoring areas.

A. DATA SOURCE AND DESCRIPTION

The locations of the DFON cable network and its landing locations are obtained from Google Fusion Tables (<https://support.google.com/fusiontables/>), and the satellite map of

the region \mathbb{D} is from Google Earth. Data sources of the design considerations are:

- Rectangularly gridded data of elevation and water depth is downloaded from the General Bathymetric Chart of the Oceans (GEBCO, <http://www.gebco.net/>) with 30 arc-second increments in longitude and latitude, which is the publicly available data with the highest resolution in \mathbb{D} . Fig. 2(b) is the elevation map of \mathbb{D} and Fig. 2(c) is the elevation contour map.
- Historical earthquake data, from 1957 to 2017 for earthquakes with magnitudes in excess of 3.5 in \mathbb{D} , is available from United States Geological Survey (USGS, <https://earthquake.usgs.gov/>). Such earthquake data is shown by the red circles in Fig. 2(d).
- Volcano locations are downloaded from National Oceanic and Atmospheric Administration (NOAA, <https://www.ngdc.noaa.gov/>). The blue nodes in Fig. 2(d) represent the volcanoes.
- The seabed sediment situation in \mathbb{D} is obtained from a map called “Surface sediments and topography of the North Pacific”, created by J. Frazer *et al.* (<http://nla.gov.au/nla.obj-234313983>).
- The seagrass and coral regions are extracted from two databases, Global Distribution of Seagrasses and Global Distribution of Coral Reefs (<http://data.unep-wcmc.org/datasets/>), respectively. In Fig. 2(d), the red regions represent the coral and seagrass distribution.
- The slope on each gridded node of \mathbb{D} is calculated from the downloaded elevation data and shown in Fig. 2(f).

B. COST FUNCTIONS

The specific functions for cost evaluations of the design considerations in our experiments are provided as follows. In sea areas where the basic construction cost is only related to the cable length, $h_1(X)$ can be set as a constant, e.g., $h_1(X) = 28,000$ [14].

Since the risk caused by volcanic eruptions decreases with growth of the distance, cable route designers usually designate a “No Go Zone” (NGZ), based on local features (e.g., slope) and the estimation of the probability and intensity of a potential volcanic eruption. That is, cables deployed in the NGZ are expected to suffer permanent destruction. This issue is handled by setting a distance $\tilde{d} = 3$ km as in [43] and assuming that $h_2(X, i_1)$ decreases exponentially with the distance if $d(X, i_1) > \tilde{d}$. Then, $h_2(X, i_1)$ is given by

$$h_2(X, i_1) = \begin{cases} a_1, & \text{if } d(X, i_1) \leq \tilde{d}, \\ a_1 e^{3-2d(X,i_1)}, & \text{otherwise,} \end{cases} \quad (5)$$

where a_1 is a large penalty cost for avoiding the NGZ.

For an earthquake $i_2 \in n_2$, we apply an attenuation equation from [55] to obtain the PGV at location X as

$$\log_{10}(\text{PGV}(X)) = 2.04 + 0.422 \times (M_w - 6) - 0.0373 \times (M_w - 6)^2 - \log_{10}(d(X, i_2)), \quad (6)$$

where M_w is the earthquake magnitude of i_2 . Note that the PGV values obtained from (6) are median intensities in a

statistical sense. Then, the PGV is converted to the (expected) repair rate by an empirical equation given in [46], which is

$$\ln(g(X, i_2)) = 1.30 \times \ln(\text{PGV}(X)) - 7.21, \quad (7)$$

and $h_3(X, i_2)$ is given by

$$h_3(X, i_2) = b \times g(X, i_2), \quad (8)$$

where b is the cost of a cable repair.

Around 85% of cables faults occur in waters at a depth of 1 km or less [47] and threats to cables decrease rapidly with growth of the depth in these regions [44]. It is appropriate then to set $\tilde{D} = 1$ km. In addition, submarine cables are always laid in the sea except for areas close to the landing stations. This leads to the following form of the function $h_4(X)$ for evaluation of the cost caused by water depth:

$$h_4(X) = \begin{cases} a_2, & \text{if } D(X) \leq 0, \\ a_2 e^{-4D(X)}, & \text{if } 0 < D(X) \leq \tilde{D}, \\ a_2 e^{-3-D(X)}, & \text{otherwise,} \end{cases} \quad (9)$$

where a_2 is a very large penalty cost assigned to $h_4(X)$ to avoid the exposure of the cable to the land.

As for seabed slope, in industry practice, under 10° is considered acceptable but greater than 10° is problematic, and significantly increased cable cost is incurred for slopes in excess of 20° [48], [50]. Accordingly, we set $\tilde{p}_1 = 10^\circ$ and $\tilde{p}_2 = 20^\circ$. For slopes between $10^\circ - 20^\circ$, we assume that the cost associated with a slope is linear with the slope. Therefore, $h_5(X)$ is given by

$$h_5(X) = \begin{cases} a_3 e^{p(X)-20}, & \text{if } p(X) > \tilde{p}_2, \\ a_3(p(X) - 10)/10, & \text{if } \tilde{p}_1 < p(X) \leq \tilde{p}_2, \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

where a_3 is a penalty cost for avoiding areas with steep slopes.

Next, based on the data of seabed sediment situation, we divide the seabed sediment of \mathbb{D} into five hardness levels as shown in Fig. 2(e). For areas without sediment information, the sediment hardness levels are assigned as the highest level. A project report by the Scottish government [56] indicates that in rocky regions the cost per kilometer of cable protection adds another 83% to the basic construction cost per kilometer. In some cases, the cost of cable protection in rocky regions can be more than double that cost for the soft seabed [57]. Accordingly, we use the following function

$$h_6(X) = \begin{cases} C(X) \times h_1(X), & \text{if } D(X) \leq \tilde{D}, \\ 0, & \text{otherwise,} \end{cases} \quad (11)$$

for cost evaluation, where $C(X)$ is associated with the sediment hardness level at X to represent the unsuitability for burying cables. The harder the seabed, the larger the value of $C(X)$. We set $C(X) \in \{0.27, 0.41, 0.55, 0.69, 0.83\}$ [56], [57] with an assumption that $h_6(X)$ increases linearly with the hardness level.

TABLE 1. Comparison matrix for Alternative 1.

design consideration	design consideration								
	water depth	earthquakes	volcanic eruptions	seabed slope	sediment hardness	basic construction cost	coral and seagrass	fishing	anchoring
water depth	1	1/9	1	1/4	1	1/3	1	1	1
earthquakes	9	1	9	2	9	3	9	9	9
volcanic eruptions	1	1/9	1	1/4	1	1/3	1	1	1
seabed slope	4	1/2	4	1	4	1	4	4	4
sediment hardness	1	1/9	1	1/4	1	1/3	1	1	1
basic construction cost	3	1/3	3	1	3	1	3	3	3
coral and seagrass	1	1/9	1	1/4	1	1/3	1	1	1
fishing	1	1/9	1	1/4	1	1/3	1	1	1
anchoring	1	1/9	1	1/4	1	1/3	1	1	1
consistency ratio	0.08 < 0.1								

For the effect of seagrass and coral, we use a simple cost function

$$h_7(X) = \begin{cases} a_4, & \text{if location } X \text{ is located in an} \\ & \text{seagrass or coral habitat,} \\ 0, & \text{otherwise,} \end{cases} \quad (12)$$

where a_4 is a large penalty cost to avoid these environmentally sensitive regions.

Based on the recent data in [47], average cable faults caused by fishing activities per year per 1000 kilometers are about 0.069, 0.011 and 0.004 for water depth less than 300 m, in the 300-1000 m range and greater than 1000 m, respectively. Accordingly, we use the following cost function for fishing activities

$$h_8(X) = \begin{cases} 0.069 \times b \times L \times 10^{-3}, & \text{if } 0 \leq D(X) \leq 0.3, \\ 0.011 \times b \times L \times 10^{-3}, & \text{if } 0.3 < D(X) \leq 1.0, \\ 0.004 \times b \times L \times 10^{-3}, & \text{if } D(X) > 1.0, \end{cases} \quad (13)$$

where L is the designed lifespan of submarine cables, generally around 25 years [44].

Similarly, average cables faults caused by anchoring activities per year per 1000 kilometers are 0.023 and 0.002 for water depth less than 300 m and greater than 300 m [47], respectively. The function $h_9(X)$ is given in a similar manner to (13) by

$$h_9(X) = \begin{cases} 0.023 \times b \times L \times 10^{-3}, & \text{if } 0 \leq D(X) \leq 0.3, \\ 0.002 \times b \times L \times 10^{-3}, & \text{if } D(X) > 0.3. \end{cases} \quad (14)$$

It is worth mentioning that (13) and (14) are obtained from a global analysis of submarine cable system faults [47], but they also provide guidance for local areas where the data of fishing and anchoring is absent.

In general, it is highly recommended that the spacing between an existing man-made submarine infrastructure and the new route should be greater than three times the water

depth to avoid damaging existing cables during installation of the new cable [58]. For an existing infrastructure $i_3 \in n_3$, we use the cost function

$$h_{10}(X, i_3) = \begin{cases} a_5, & \text{if } d(X, i_3) \leq 3D(X), \\ a_5 e^{3D(X)-d(X, i_3)}, & \text{otherwise,} \end{cases} \quad (15)$$

where a_5 is a large penalty cost to avoid the vicinity of the existing infrastructure.

In our experiments, we set a_1, a_2, a_3, a_4, a_5 , and b as \$3M. That is, the penalty cost is that of a potential cable repair [59], [60].

C. WEIGHT VALUES

In determining the weight values w_m for the various design considerations, we follow the general industry practice where one consults experts representing a key stakeholder about qualitative prioritization of the design considerations. Specifically, based on the Saaty's ranking method [16], we use an $n \times n$ comparison matrix to rank the priorities of the design considerations, where n is the number of the chosen design considerations. An element $n_{i,j}$ of the matrix represents the importance of the i th design consideration relative to the j th design consideration. That is, if $n_{i,j} > 1$, the i th design consideration is more important than the j th design consideration, whereas if $n_{i,j} < 1$, the i th design consideration is less important than the j th design consideration. A higher value of $n_{i,j}$ indicates a higher priority of the i th design consideration relative to the j th design consideration, where the maximal value of $n_{i,j}$ is 9 and the minimal value is 1/9.

Table 1 provides the comparison matrix for the case of Alternative 1 where we choose nine design considerations for qualitative prioritization. The priorities in this case are based on the experience of cable route engineering obtained from several industry partners. For example, earthquakes are considered as a top priority among the design considerations because \mathbb{D} is an earthquake-prone area as shown in Fig. 2(d). The priority of seabed slope is lower than

TABLE 2. Weight values for design considerations.

	water depth	earthquakes	volcanic eruptions	seabed slope	sediment hardness	basic construction cost	coral and seagrass	fishing	anchoring	existing cable
Alternative 1	0.046	0.405	0.046	0.180	0.046	0.142	0.046	0.046	0.046	-
Alternative 2	0.039	0.348	0.039	0.156	0.039	0.123	0.039	0.039	0.039	0.100

earthquakes because the importance of seabed slope has been already included in its near infinity cost for high slope regions, which guarantees that cables will avoid them. The priority of volcanic eruptions is low because most volcanoes in \mathbb{D} are located inland as shown in Fig. 2(d) and they have little impact on submarine cable path planning. We then apply AHP [16] to translate the qualitative prioritization into weight values for the corresponding design considerations. The results are shown in Table 2.

In practice, in congested areas with submarine cables such as Singapore Straits and coastlines of Algeria, Egypt and Taiwan, route designers avoid deploying new cables in the proximity of existing cables to prevent multiple cables from being damaged simultaneously by either a natural or human-caused disaster [61]. We consider this issue in the case of Alternative 2 where it is assumed that the start node on Bohol Island is in the vicinity of an existing cable represented by the magenta dash line marked by a hexagram as shown in Fig. 3. Accordingly, a design consideration associated with the existing cable is further added. We set the weight value of this additional design consideration to 0.100, and modify the weight values of the other design considerations obtained for Alternative 1 proportionately. The results for Alternative 2 are shown in Table 2.

D. PATH PLANNING RESULTS AND EXPLANATIONS

We compare the results obtained by our proposed FMM-based method and the one based on the Dijkstra’s algorithm [62] using the gridded graph of the same triangulated manifold representing the region \mathbb{D} .

In Fig. 2, the curves marked by plus, diamond and asterisk correspond to the results for Alternative 1 obtained by FMM for Scenarios 1-3, respectively. From Fig. 2(d), we can observe that the three routes avoid dense earthquake areas as much as possible.

The path planning results for Alternative 2 are presented in Fig. 3. Note that, for easier observation, the blue rectangular region in Fig. 2(d) is magnified in Fig. 3. The curves marked by plus, diamond, and asterisk again correspond to the results obtained by FMM for Scenarios 1-3 in this case, respectively. Note that the blue dot curve marked by pentagon in Fig. 3 is the result for Alternative 1 for Scenario 2, which does not consider the effect of the existing cable. If a new cable is laid following the blue dot curve, the probability of simultaneously breaking the new cable and the existing cable is greatly increased when a strong earthquake (or other hazards) occurs near the two cables. In contrast, with the effect of

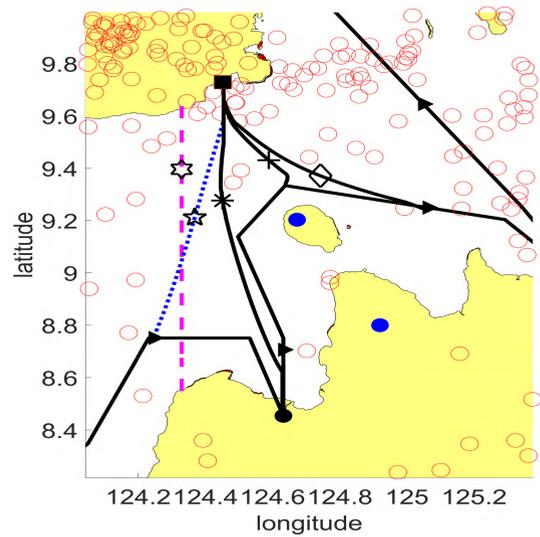


FIGURE 3. Results for Alternative 2.

the existing cable considered in Alternative 2, the solid curve marked by diamond is the result obtained for Scenario 2, which is much farther away from the existing cable. The risk that the two cables are damaged simultaneously by a strong earthquake (or other hazards) reduces and the resilience of the whole network is improved.

We compare the Dijkstra’s algorithm and FMM with respect to the total length, cost and running time in Table 3 for Alternative 1 and in Table 4 for Alternative 2, respectively. The results are obtained using a Lenovo Think-Center M900 Tower desktop (64GB RAM, 3.4 GHz Intel(R) Core(TM) i7-6700 CPU) for running the codes in Matlab R2016b. The core computational codes of FMM and the Dijkstra’s algorithm are written in C++. The relative difference in cost for a cable γ between FMM and the Dijkstra’s algorithm is defined as

$$\frac{\mathbb{H}(\gamma)_{\text{Dijkstra}} - \mathbb{H}(\gamma)_{\text{FMM}}}{\mathbb{H}(\gamma)_{\text{FMM}}} \tag{16}$$

For both the Dijkstra’s algorithm and FMM, the results in Table 3 and Table 4 demonstrate the cost saving in connecting a cable from a given site to a network through a branching unit (Scenario 2) versus connecting it directly to a landing station (Scenario 3). This suggests that reasonable location design for branching units may reduce the life-cycle cost of the new cable effectively. From Table 3 and Table 4, for the same scenario of each alternative, we can observe that the

TABLE 3. Comparison between the Dijkstra's algorithm and FMM for Alternative 1.

	Scenario 1		Scenario 2		Scenario 3	
	Dijkstra	FMM	Dijkstra	FMM	Dijkstra	FMM
length (km)	66.40	50.58	117.36	110.76	160.37	144.02
cost (10 ⁶ dollars)	14.78	12.59	20.88	19.98	27.46	25.54
running time (s)	10.26	17.06	10.28	17.17	10.71	22.74
relative difference in cost	17.4%		4.5%		7.5%	

TABLE 4. Comparison between the Dijkstra's algorithm and FMM for Alternative 2.

	Scenario 1		Scenario 2		Scenario 3	
	Dijkstra	FMM	Dijkstra	FMM	Dijkstra	FMM
length (km)	66.40	50.56	76.83	92.09	160.37	143.99
cost (10 ⁶ dollars)	13.38	11.39	19.80	18.02	24.83	23.10
running time (s)	11.33	15.22	11.45	15.83	11.81	18.82
relative difference in cost	17.5%		9.9%		7.5%	

Dijkstra's algorithm is slightly faster than FMM but performs worse in the cost saving than FMM. Specifically, the cost reduction of FMM relative to the Dijkstra's algorithm is up to 17.5%. Considering the billions of dollars spent around the world each year on submarine cable projects, the cost saving of FMM is significant and attractive. In addition, by considering Scenario 1 based on FMM, we can obtain a lower bound on the cost for arbitrary placement of a branching unit, a matter of significant practical importance. The cost saving performance of FMM is better because the cable path planning problem described in this paper is modeled more accurately by considering a continuous manifold rather than a discrete graph. Many path planning options available under continuous manifold modeling are ignored by the discrete graph modeling. FMM can be optimally applied to a continuous manifold while the Dijkstra's algorithm requires discretization which adversely affects the results.

VI. CONCLUSION AND DISCUSSION

We have proposed a method to design a submarine cable route connecting a given site and an existing cable network on the Earth's surface aimed at minimization of the life-cycle cost of the cable. Our method is based on solving a variational optimization problem with a cost model that includes real-life design considerations for the cable route. Assignment of weights to the various design considerations leads to the design of a path planning method based on FMM to obtain the optimal route. We have presented a realistic three-dimensional study case, and compared the performance of our FMM-based method with that based on the Dijkstra's algorithm. The results demonstrate the applicability and significant cost saving of our path planning optimization approach for submarine cable network extension.

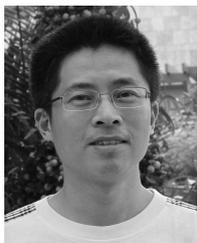
REFERENCES

- [1] A. McCurdy et al., "Submarine telecoms industry report," Submarine Telecoms Forum, Sterling, VA, USA, Tech. Rep., Oct. 2018, no. 7. Accessed: May 3, 2019. [Online]. Available: <https://subtelforum.com/products/submarine-telecoms-industry-report/>
- [2] N. Shalom. (Dec. 2008). *Amazon Found Every 100ms of Latency Cost Them 1% in Sales*. Accessed: May 3, 2019. [Online]. Available: https://natisshalom.typepad.com/nati_shaloms_blog/2008/12/latency-is-everywhere-and-it-costs-you-sales-how-to-crush-it-my-personal-take-away.html
- [3] S. Chattopadhyaya. (Sep. 2018). *New Submarine Cable for Digital Africa! How to Build! Who Will Build!* Accessed: May 3, 2019. [Online]. Available: <https://www.submarinenetworks.com/en/insights/new-submarine-cable-for-digital-africa>
- [4] The Economist. (Aug. 2018). *Tech Giants are Building Their Own Undersea Fibre-Optic Network*. Accessed: May 3, 2019. [Online]. Available: <https://www.economist.com/business/2017/10/07/tech-giants-are-building-their-own-undersea-fibre-optic-networks>
- [5] J. Hecht, "The bandwidth bottleneck," *Nature*, vol. 536, no. 7615, pp. 139–142, Aug. 2016.
- [6] NEC Corporation. (May 2017). *NEC Begins Construction of Submarine Cable Links to the Islands of Palau, Yap and Chuuk*. Accessed: May 3, 2019. [Online]. Available: https://www.palau.gov/wp-content/uploads/2017/05/NEC-International-Press-Release_May-4_FINAL.pdf
- [7] H. Sakuyama, M. Saitoh, Y. Kiuchi, T. Nishiyama, and M. Niino. (May 2014). *New Branch Construction of Third Party Existing Submarine Cable System in EACI Qingdao Landing Extension*. Accessed: May 3, 2019. [Online]. Available: <https://www.suboptic.org/wp-content/uploads/2014/10/We2.16.pdf>
- [8] Cinia Group. (Jul. 2015). *Cinia Builds New Branch to Hanko from the C-Lion1 Germany-Finland Submarine Cable System*. Accessed: May 3, 2019. [Online]. Available: <http://portofhanko.fi/en/2016/06/cinia-builds-new-branch-to-hanko-from-the-c-lion1-germany-finland-submarine-cable-system/>
- [9] J. Tisdale, Z. Kim, and J. K. Hedrick, "Autonomous UAV path planning and estimation," *IEEE Robot. Autom. Mag.*, vol. 16, no. 2, pp. 35–42, Jun. 2009. [Online]. Available: <http://dx.doi.org/10.1109/MRA.2009.932529>
- [10] V. Roberge, M. Tarbouchi, and G. Labonté, "Comparison of parallel genetic algorithm and particle swarm optimization for real-time UAV path planning," *IEEE Trans Ind. Informat.*, vol. 9, no. 1, pp. 132–141, Feb. 2013.
- [11] N. E. Du Toit and J. W. Burdick, "Robot motion planning in dynamic, uncertain environments," *IEEE Trans. Robot.*, vol. 28, no. 1, pp. 101–115, Feb. 2012.
- [12] M. Hinow and M. Mevissen, "Substation maintenance strategy adaptation for life-cycle cost reduction using genetic algorithm," *IEEE Trans. Power Del.*, vol. 26, no. 1, pp. 197–204, Jan. 2011.
- [13] J. Nilsson and L. Bertling, "Maintenance management of wind power systems using condition monitoring systems—Life cycle cost analysis for two case studies," *IEEE Trans. Energy Convers.*, vol. 22, no. 1, pp. 223–229, Mar. 2007.

- [14] R. W. Neal. (Mar. 2014). *Underwater Internet Cables: 'Submarine Cable Map,' Shows How the World Gets Online*. Accessed: May 3, 2019. [Online]. Available: <https://www.ibtimes.com/underwater-internet-cables-submarine-cable-map-shows-how-world-gets-online-1559604>
- [15] ITU. (Dec. 2016). *Maximising Availability of International Connectivity in Developing Countries: Strategies to Ensure Global Digital Inclusion*. Accessed: May 3, 2019. [Online]. Available: <http://handle.itu.int/11.1002/pub/80f5eade-en>
- [16] T. L. Saaty, *The Analytical Hierarchical Process: Planning, Priority Setting, Resource Allocation*. New York, NY, USA: McGraw-Hill, 1980.
- [17] J. A. Sethian, *Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science ... on Applied and Computational Mathematics*. Cambridge, U.K.: Cambridge Univ. Press, 1999.
- [18] J. A. Sethian, "Fast marching methods," *SIAM Rev.*, vol. 41, no. 2, pp. 199–235, 1999.
- [19] R. Kimmel and J. A. Sethian, "Optimal algorithm for shape from shading and path planning," *J. Math. Imag. Vis.*, vol. 14, no. 3, pp. 237–244, May 2001.
- [20] D. R. Burnett, R. Beckman, and T. M. Davenport, *Submarine Cables: The Handbook Law Policy*. Leiden, The Netherlands: Martinus Nijhoff, 2013.
- [21] D. L. Msongaleli, F. Dikbiyik, M. Zukerman, and B. Mukherjee, "Disaster-aware submarine fiber-optic cable deployment for mesh networks," *J. Lightw. Technol.*, vol. 34, no. 18, pp. 4293–4303, Sep. 15, 2016.
- [22] P. N. Tran and H. Saito, "Geographical route design of physical networks using earthquake risk information," *IEEE Commun. Mag.*, vol. 54, no. 7, pp. 131–137, Jul. 2016.
- [23] P. N. Tan and H. Saito, "Enhancing physical network robustness against earthquake disasters with additional links," *J. Lightw. Technol.*, vol. 34, no. 22, pp. 5226–5238, Nov. 15, 2016.
- [24] H. Saito, "Analysis of geometric disaster evaluation model for physical networks," *IEEE/ACM Trans. Netw.*, vol. 23, no. 6, pp. 1777–1789, Dec. 2015.
- [25] S. Neumayer, G. Zussman, R. Cohen, and E. Modiano, "Assessing the impact of geographically correlated network failures," in *Proc. IEEE MILCOM*, San Diego, CA, USA, Nov. 2008, pp. 1–6.
- [26] S. Neumayer, G. Zussman, R. Cohen, and E. Modiano, "Assessing the vulnerability of the fiber infrastructure to disasters," *IEEE/ACM Trans. Netw.*, vol. 19, no. 6, pp. 1610–1623, Dec. 2011.
- [27] P. K. Agarwal, A. Efrat, S. K. Ganjugunte, D. Hay, S. Sankararaman, and G. Zussman, "The resilience of WDM networks to probabilistic geographical failures," *IEEE/ACM Trans. Netw.*, vol. 21, no. 5, pp. 1525–1538, Oct. 2013.
- [28] S. Neumayer and E. Modiano, "Network reliability under random circular cuts," in *Proc. IEEE GLOBECOM*, Kathmandu, Nepal, Dec. 2011, pp. 1–6.
- [29] S. Neumayer, A. Efrat, and E. Modiano, "Geographic max-flow and min-cut under a circular disk failure model," *Comput. Netw.*, vol. 77, pp. 117–127, Feb. 2015.
- [30] C. Cao, M. Zukerman, W. Wu, J. H. Manton, and B. Moran, "Survivable topology design of submarine networks," *J. Lightw. Technol.*, vol. 31, no. 5, pp. 715–730, Mar. 2013.
- [31] C. Cao, Z. Wang, M. Zukerman, J. H. Manton, A. Bensoussan, and Y. Wang, "Optimal cable laying across an earthquake fault line considering elliptical failures," *IEEE Trans. Rel.*, vol. 65, no. 3, pp. 1536–1550, Sep. 2016.
- [32] A. Agrawal, V. Bhatia, and S. Prakash, "Network and risk modeling for disaster survivability analysis of backbone optical communication networks," *J. Lightw. Technol.*, vol. 37, no. 10, pp. 2352–2362, May 2019.
- [33] J. Zhang, E. Modiano, and D. Hay, "Enhancing network robustness via shielding," *IEEE/ACM Trans. Netw.*, vol. 25, no. 4, pp. 2209–2222, Aug. 2017.
- [34] J. Zhang and E. Modiano. (Sep. 2017). "Connectivity in interdependent networks." [Online]. Available: <https://arxiv.org/abs/1709.03034>
- [35] D. Kilper, K. Bergman, G. Zussman, and B. Birand, "Resilient optical networking," U.S. Patent 2017 0279557, Sep. 1, 2017.
- [36] M. Zhao, T. W. S. Chow, P. Tang, Z. Wang, J. Guo, and M. Zukerman, "Route selection for cabling considering cost minimization and earthquake survivability via a semi-supervised probabilistic model," *IEEE Trans Ind. Informat.*, vol. 13, no. 2, pp. 502–511, Apr. 2017.
- [37] Z. Wang et al., "Multiobjective path optimization for critical infrastructure links with consideration to seismic resilience," *Comput.-Aided Civil Infrastruct. Eng.*, vol. 32, no. 10, pp. 836–855, Oct. 2017.
- [38] Z. Wang, Q. Wang, M. Zukerman, and B. Moran, "A seismic resistant design algorithm for laying and shielding of optical fiber cables," *J. Lightw. Technol.*, vol. 35, no. 14, pp. 3060–3074, Jul. 15, 2017.
- [39] Z. Wang, Q. Wang, B. Moran, and M. Zukerman, "Application of the fast marching method for path planning of long-haul optical fiber cables with shielding," *IEEE Access*, vol. 6, no. 1, pp. 41367–41378, 2018.
- [40] Z. Wang, Q. Wang, B. Moran, and M. Zukerman, "Terrain constrained path planning for long-haul cables," *Opt. Express*, vol. 27, no. 6, pp. 8221–8235, Mar. 2019.
- [41] I. Florinsky, *Digital Terrain Analysis in Soil Science and Geology*. New York, NY, USA: Academic, 2016.
- [42] J. Lee, "Comparison of existing methods for building triangular irregular network, models of terrain from grid digital elevation models," *Int. J. Geogr. Inf. Syst.*, vol. 5, no. 3, pp. 267–285, Jul. 1991.
- [43] G. W. McDonald, N. J. Smith, J.-H. Kim, S. J. Cronin, and J. N. Proctor, "The spatial and temporal 'cost' of volcanic eruptions: Assessing economic impact, business inoperability, and spatial distribution of risk in the Auckland region, New Zealand," *Bull. Volcanol.*, vol. 79, no. 7, pp. 48:1–48:13, Jul. 2017.
- [44] L. Carter et al., "Submarine cables and the oceans: Connecting the world," ICPC/UNEP/UNEP-WCMC, Cambridge, U.K., Tech. Rep. 30, 2009. Accessed: May 3, 2019. [Online]. Available: <https://www.unep-wcmc.org/resources-and-data/submarine-cables-and-the-oceans-connecting-the-world>
- [45] ICPC. (Oct. 2017). *ICPC Recommendations: Minimum Technical Requirements for a Desktop Study*. Accessed: May 3, 2019. [Online]. Available: <https://www.iscpc.org/publications/recommendations/>
- [46] S.-S. Jeon and T. D. O'Rourke, "Northridge earthquake effects on pipelines and residential buildings," *Bull. Seismological Soc. Amer.*, vol. 95, no. 1, pp. 294–318, Feb. 2005.
- [47] M. E. Kordahi et al., "Global trends in submarine cable system faults," in *Proc. SubOptic*, Apr. 2016, pp. 1–7.
- [48] S.-W. Huang, E. Chen, and J. Guo, "Efficient seafloor classification and submarine cable route design using an autonomous underwater vehicle," *IEEE J. Ocean. Eng.*, vol. 43, no. 1, pp. 7–18, Jan. 2018.
- [49] ICPC. (2018). *Fishing and Submarine Cables*. Accessed: May 3, 2019. [Online]. Available: <https://www.iscpc.org/publications/>
- [50] R. Rapp, M. Lawrence, D. Borwick, and T. Kuwabara, "Marine survey cable routing," in *Proc. SubOptic*, Apr. 2004.
- [51] ICPC. (Oct. 2017). *ICPC Recommendations: The Proximity of Offshore Renewable Wind Energy Installations and Submarine Cable Infrastructure in National Waters*. Accessed: May 3, 2019. [Online]. Available: <https://www.iscpc.org/publications/recommendations/>
- [52] C. Gerlach and R. Seitz, "Economic impact of submarine cable disruptions," APEC Policy Support Unit, Tech. Rep. APEC#213-SE-01.2, Dec. 2012. Accessed: May 3, 2019. [Online]. Available: <http://publications.apec.org/Publications/2013/02/Economic-Impact-of-Submarine-Cable-Disruptions>
- [53] K. Eriksson, D. Estep, and C. Johnson, *Applied Mathematics, Body and Soul*. New York, NY, USA: Springer, 2013.
- [54] D. Burago, Y. Burago, and S. Ivanov, *A Course Metric Geometry*, vol. 33. Providence, RI, USA: AMS, 2001.
- [55] G. M. Atkinson and D. M. Boore, "Ground-motion relations for eastern north america," *Bull. Seismol. Soc. Amic.*, vol. 85, no. 1, pp. 17–30, Feb. 1995.
- [56] RPS Group. (Apr. 2012). *Irish-Scottish Links on Energy Study (ISLES): Construction and Deployment Report*. Accessed: May 3, 2019. [Online]. Available: <http://www.islesproject.eu/wp-content/uploads/2014/09/8.0-Construction-and-Deployment.pdf>
- [57] F. Sharkey, *Economic Challenges and Optimisation of Ocean Energy Electrical Systems*. Dublin, Ireland: Dublin Institute of Technology, 2013.
- [58] Y. Ye, X. Jiang, G. Pan, and W. Jiang, *Submarine Optical Cable Engineering*. New York, NY, USA: Academic, 2018.
- [59] D. R. Burnett, "Recovery of cable repair ship cost damages from third parties that injure submarine cables," *Tulane Marit. Law J.*, vol. 35, no. 1, pp. 103–124, Aug. 2010.
- [60] A. Al-Lawati, "Balancing cost and protection of submarine fiber optic cables," in *Proc. IEEE GCCCE*, Muscat, Oman, Feb. 2015, pp. 1–5.
- [61] P. F. Williams, "Is cable congestion a threat to cable security? What is the solution?" in *Proc. SubOptic*, Yokohama, Japan, May 2010.
- [62] E. W. Dijkstra, "A note on two problems in connexion with graphs," *Numer. Math.*, vol. 1, no. 1, pp. 269–271, Dec. 1959.



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