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XU, Weiye; YU, Xiangbin; LEUNG, Shu-Hung; CHU, Junya

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Energy-Efficient Power Allocation Scheme for Distributed MISO System With OFDM Over Frequency-Selective Fading Channels

WEIYE XU1,2, XIANGBIN YU2,3, (Member, IEEE), SHU-HUNG LEUNG4, AND JUNYA CHU2
1School of Communication Engineering, Nanjing Institute of Technology, Nanjing 211167, China
2Department of Electronic Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
3National Mobile Communications Research Laboratory, Southeast University, Nangjing 210096, China
4Department of Electronic Engineering, City University of Hong Kong, Hong Kong

Corresponding author: Weiye Xu (xuweiye2014@hotmail.com)

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ABSTRACT Considering multiple transmit antennas in each distributed antenna unit (DAU), two power allocation (PA) schemes are proposed for energy efficiency (EE) maximization for downlink distributed multiple-input single-output (DMISO) systems with orthogonal frequency-division multiplexing (OFDM) over frequency-selective fading channels, where the power constraints for individual antenna units are addressed. The optimization problem for the maximization of the EE subject to per-antenna maximum power constraints is formulated. By means of linear programming, the optimization is simplified to as if for a DMISO system whose DAUs use a single antenna corresponding to the largest channel-gain-to-noise ratio (CGNR) for transmission. Using the block coordinate descent (BCD) method, an iterative optimal PA scheme to the simplified optimization problem is derived, where an efficient procedure for determining the number of effective subcarriers and the optimized PA is developed. Since the optimal scheme needs iterative calculations, a closed-form suboptimal PA scheme is also derived by sorting the total CGNR and using the principle of the BCD method. Interestingly, this suboptimal scheme has small performance loss in comparing with the optimal scheme, and its relative EE loss is decreased with the number of subcarriers. Moreover, these two schemes include the ones with single transmit antenna for distributed antenna systems as special cases. Computer simulations verify the effectiveness of the two proposed schemes, and the proposed optimal one can obtain the same performance as the existing optimal scheme for DMISO-OFDM but with lower complexity.

INDEX TERMS Distributed multiple-input single-output system, energy efficiency, power allocation, OFDM, frequency-selective fading channels.

I. INTRODUCTION
With increasing needs for various data and multimedia services in wireless communications, many techniques have been proposed to provide higher data rates. Distributed antenna systems (DASs) as a promising technique for increasing data rates have drawn much attention recently. Unlike traditional co-located antenna systems (CASs), the antennas of the DAS are remotely scattered within a cell and linked to a central processing unit via high speed connection links [1]–[3]. Therefore, transmit powers and access distances can be greatly reduced in DASs. Due to the obvious performance advantages of DAS, it has been regarded as one of the promising key technologies in 5G green communication because it can increase the energy efficiency (EE) greatly [4]–[6].

As a key technology, power allocation (PA) can effectively increase system performance [7]–[9]. Various PA schemes for obtaining higher EE have been reported in the literature [10]–[16]. In [10], the tradeoff between EE and spectral efficiency (SE) in downlink DAS was studied, and
a PA algorithm based on an exhaustive search method to attain maximum EE was presented. An optimal PA scheme for maximizing the EE in DAS was proposed in [11], and a closed-form expression for the optimal PA was derived by using the KKT conditions. In [12], a simplified optimal PA scheme that maximizes EE of DAS was presented, and it has lower complexity than the scheme in [11], but there exist small errors in the theoretical derivation. A low-complexity optimal PA scheme aiming at the EE maximization in DAS for different fading channels was developed in [13], which can obtain the same performance as the schemes in [11] and [12] with lower complexity. However, these PA schemes are designed for the DAS in flat fading channels, which cannot be applied to wideband systems that generally experience multipath fading. For this reason, the PAs of DAS over frequency selective fading channels were studied in [9] and [14]–[16]. The capacity of DAS with orthogonal frequency division multiplexing (OFDM) was analyzed in [9], where a suboptimal PA scheme was derived by maximizing an upper bound of the channel capacity. In [14], the EE of DAS with frequency selective fading was analyzed, and an iterative PA algorithm was presented to obtain an optimal energy-efficient PA. In [15] and [16], by exploiting the fractional programming theory, the EE for multiuser DAS with OFDM was studied, and iterative algorithms were proposed to achieve optimal and suboptimal power allocations.

With the discussions above, the EE optimization schemes in DAS have been studied well over Rayleigh fading channels, but these schemes basically consider single antenna only in each distributed antenna unit (DAU) for the sake of convenient analysis. Hence, the existing PA schemes and algorithms are lack of generality and their applications are limited in multi-antenna systems. Especially for distributed multiple-input single-output (DMISO) systems over frequency selective fading channels, the EE optimization is not yet addressed in the literature. For this reason, we study the EE performance of DMISO with OFDM (DMISO-OFDM) in composite fading channels including shadowing, path loss, and multipath Rayleigh fading, where each DAU is equipped with multiple transmit antennas, and derive optimal and suboptimal PA schemes for DMISO-OFDM in this paper.

The optimization of the transmit powers to the subcarriers of DAUs for the maximization of the EE subject to the maximum transmit power constraint of each DAU is formulated as a nonlinear fractional programming problem. By applying linear programming to the optimization, we simplify it to as an upper bound of the channel capacity. Using the block coordinate descent (BCD) method [17], an optimal PA scheme is developed to solve the optimization problem, where an efficient procedure for determining the number of effective subcarriers and the optimized PA for transmission is developed. Considering the higher computational complexity of the iterative algorithm of the optimal scheme, a simplified suboptimal PA scheme is developed by sorting the total CGNRs and determining the number of effective subcarriers in each DAU. This suboptimal scheme has a closed-form procedure and can obtain the EE close to the optimal scheme. Moreover, the proposed two schemes can include the ones with single transmit antenna in DAUs as special cases. Simulation results show that the proposed two schemes are valid and outperform the existing optimal scheme for DAS-OFDM with single transmit antenna, and the EE performance of the suboptimal scheme is close to that of the optimal scheme with lower complexity.

The notations in this paper are shown as follows. The superscripts (·) and (·) denote the matrix or vector transposition and conjugate transposition, respectively. Bold uppercase letter and bold lowercase letter denote matrix and vector, respectively. $I_N$ denotes a $N \times N$ identity matrix. $\otimes$ and diag(·) denote the Kronecker product and diagonal matrix, respectively.

II. SYSTEM MODEL

We consider an OFDM based downlink distributed MISO system with $N_f$ distributed antenna units, each of which is equipped with $N_a$ antennas, and a mobile station (MS) with a single antenna as shown in Fig.1. The distributed antenna units are arbitrarily located and well separated in distance, and the MS is randomly located within the cell. The DAUs are connected to a central processing unit through dedicated links, and the $n$-th DAU is referred to as $DA_n (n = 1, \ldots, N_f)$. The OFDM with $N_c$ subcarriers is employed for the signal transmission of the DMISO due to its effectiveness against frequency-selective fading. The system operates in a multipath fading channel. Considering the implementation of MS as well as its limited size, only a single receive antenna is deployed in the MS [18].

At the receiver of the MS, the received signal $y(k)$ at the $k$-th subcarrier in the frequency domain is expressed as

$$y(k) = g(k)^T P^{1/2}(k) x(k) + z$$  \hspace{1cm} (1)
where \( \mathbf{g} (k) = \Omega \mathbf{h} (k), \mathbf{h} (k) = \begin{bmatrix} \mathbf{h}_{1}^T (k), \ldots, \mathbf{h}_{N_{c}}^T (k) \end{bmatrix} \) is an \( N_{t} N_{a} \times 1 \) vector whose \( n \)-th entry \( h_{n} (k) = [H_{n1} (k), \ldots, H_{n N_{c}} (k)]^T \) is an \( N_{a} \times 1 \) small-scale fading vector at the \( k \)-th subcarrier with the \( m \)-th entry \( H_{nm} (k) \) denoting the channel frequency response from the \( m \)-th antenna of the \( \text{DA}_{n} \) to the MS, \( \Omega = \mathbf{S} \otimes \Psi_{n}, \mathbf{S} = \text{diag} (s_{1}, \ldots, s_{N_{c}}) \) denotes the \( N_{c} \times N_{c} \) diagonal large-scale fading matrix, \( s_{n} = \sqrt{d_{a}^{-\alpha_{n}}} \Psi_{n} \), \( d_{a} \) is the distance between the \( \text{DA}_{n} \) and the MS, \( \alpha_{n} \) and \( \Psi_{n} \) represent the path loss exponent and shadowing, respectively, \( \mathbf{P} (k) = \text{diag} (p_{1} (k), \ldots, p_{N_{a}} (k)) \) is a diagonal transmit power matrix with \( p_{n} (k) = \text{diag} (p_{n1} (k), \ldots, p_{n N_{c}} (k)) \) whose \( m \)-th entry \( p_{nm} (k) \) denotes the transmit power of the \( m \)-th antenna of the \( \text{DA}_{n} \), and \( \mathbf{x} = [x_{1}^T, \ldots, x_{N_{a}}^T]^T \) is an \( N_{t} N_{a} \times 1 \) transmit signal vector whose \( n \)-th entry \( x_{n} = [x_{n1}, \ldots, x_{n N_{c}}]^T \) represents the \( N_{c} \times 1 \) transmit signal vector of the \( \text{DA}_{n} \) with variance \( \mathbf{E} \{ |x_{n}|^2 \} = 1 \), and \( \mathbf{z} \) is the Gaussian channel noise with zero mean and variance \( \sigma_{z}^2 \).

With (1), the achievable rate of the DMISO-OFDM can be expressed as

\[
R = \sum_{k=1}^{N_{t}} \log_{2} \det \left( \mathbf{I}_{N_{t} N_{a}} + \frac{\mathbf{P}^{1/2} (k) \mathbf{g} (k) \mathbf{g}^H (k) \mathbf{P}^{1/2} (k)}{\sigma_{z}^2} \right) = \sum_{k=1}^{N_{t}} \log_{2} \left( 1 + \frac{\mathbf{g}^H (k) \mathbf{P} (k) \mathbf{g} (k)}{\sigma_{z}^2} \right). \tag{2}
\]

Equation (2) can be further expressed as

\[
R = \sum_{k=1}^{N_{t}} \log_{2} \left( 1 + \frac{\mathbf{g}^H (k) \mathbf{P} (k) \mathbf{g} (k)}{\sigma_{z}^2} \right). \tag{3}
\]

where \( y_{nm} (k) = d_{a}^{-\alpha_{n}} \Psi_{n} |H_{nm} (k)|^2 / \sigma_{z}^2 \) is the CGNR (channel-gain-to-noise ratio).

The total power consumption of the DMISO-OFDM is expressed as

\[
P_{\text{con}} = \sum_{i=1}^{N_{t}} \sum_{k=1}^{N_{a}} \sum_{m=1}^{N_{c}} p_{im} (k) + P_{c} \tag{4}
\]

where \( P_{c} \) denotes the circuit power and is a constant. Hence, the EE of the DMISO-OFDM is given by

\[
\eta = \frac{\sum_{k=1}^{N_{t}} \log_{2} \left( 1 + \frac{\sum_{i=1}^{N_{t}} \sum_{m=1}^{N_{c}} p_{im} (k) y_{im} (k)}{\sum_{i=1}^{N_{t}} \sum_{m=1}^{N_{c}} p_{im} (k) + P_{c}} \right)}{\sum_{i=1}^{N_{t}} \sum_{m=1}^{N_{c}} p_{im} (k) + P_{c}}. \tag{5}
\]

Considering the power limitations of individual antenna ports, each \( \text{DA} \) unit has a maximum power constraint, which is different from the conventional CAS that has a total power constraint. So subject to the maximum transmit power \( P_{\text{max}} \) for each \( \text{DA} \), the transmit powers \( \{ p_{im} (k) \} \) need to meet the following power constraints:

\[
0 < \sum_{m=1}^{N_{c}} p_{im} (k) \leq P_{\text{max}} \tag{6}
\]

and

\[
p_{im} (k) > 0 \quad \text{for} \quad i = 1, \ldots, N_{t}, \quad m = 1, \ldots, N_{a}, \quad k = 1, \ldots, N_{c}. \tag{7}
\]

### III. OPTIMAL PA SCHEME FOR EE MAXIMIZATION IN DMISO-OFDM

In this section, we present an optimal power allocation scheme for the DMISO-OFDM system by maximizing the EE in (5) subject to the per-antenna maximum power constraints in (6) and (7). Correspondingly, the optimization problem can be formulated as

\[
\max_{\{ p_{im} (k) \}} J = \frac{\sum_{k=1}^{N_{t}} \log_{2} \left( 1 + \frac{\sum_{i=1}^{N_{t}} \sum_{m=1}^{N_{c}} p_{im} (k) y_{im} (k)}{\sum_{i=1}^{N_{t}} \sum_{m=1}^{N_{c}} p_{im} (k) + P_{c}} \right)}{\sum_{i=1}^{N_{t}} \sum_{m=1}^{N_{c}} p_{im} (k) + P_{c}} \tag{8a}
\]

subject to

\[
\sum_{m=1}^{N_{c}} p_{im} (k) \leq P_{\text{max}} \quad \text{for} \quad i = 1, \ldots, N_{t}, \quad m = 1, \ldots, N_{a}, \quad k = 1, \ldots, N_{c}. \tag{8b}
\]

The objective function in (8a) is pseudo-concave and the constraint functions are linear. Thus, the maximum is unique if the solution is in the feasible set. For this reason, we can solve the problem numerically by means of the function fmincon in Matlab software to obtain the global optimum. However, the computational complexity is very high due to its poor efficiency. For this reason, we firstly simply the optimization objective (8a), and then use the BCD method [17] to find the solution to the simplified optimization problem.

In the following, we introduce Lemma 1 for simplifying the design.

**Lemma 1:** The optimization problem (8) is equivalent to the following optimization problem (9).

\[
\max_{\{ P_{i} (k) \}} J_{s} = \frac{\sum_{k=1}^{N_{t}} \log_{2} \left( 1 + \frac{\sum_{i=1}^{N_{t}} P_{i} (k) y_{im} (k)}{\sum_{i=1}^{N_{t}} P_{i} (k) + P_{c}} \right)}{\sum_{i=1}^{N_{t}} P_{i} (k) + P_{c}} \tag{9a}
\]

subject to

\[
\sum_{k=1}^{N_{t}} P_{i} (k) \leq P_{\text{max}} \quad \text{for} \quad i = 1, \ldots, N_{t}, \quad k = 1, \ldots, N_{c}. \tag{9b}
\]

\[
\text{and} \quad P_{i} (k) > 0 \quad \text{for} \quad i = 1, \ldots, N_{t}, \quad k = 1, \ldots, N_{c}. \tag{9c}
\]

where \( M_{i,k} = \arg \max \{ y_{im} (k) \} \) for given subcarrier \( k \) and \( \text{DA}_{i} \), and \( P_{i} (k) = \sum_{m=1}^{N_{c}} p_{im} (k) \).

**Proof:** For given \( \{ P_{i} (k) \} \) that satisfy (9b) and (9c), the maximization problem (8a) is equivalent to

\[
\max_{J_{1}} = \sum_{k=1}^{N_{t}} \log_{2} \left( 1 + \frac{\sum_{i=1}^{N_{t}} P_{i} (k) y_{im} (k)}{\sum_{i=1}^{N_{t}} P_{i} (k) + P_{c}} \right) \tag{10a}
\]

subject to

\[
\sum_{m=1}^{N_{c}} p_{im} (k) = P_{i} (k) \quad \text{for} \quad k = 1, \ldots, N_{c}. \tag{10b}
\]

Given \( \{ P_{i} (k) \} \), since the logarithmic function is a monotonically increasing function, the maximization

\[
\} \quad \text{for} \quad k = 1, \ldots, N_{c}. \tag{10b}
\]

Given \( \{ P_{i} (k) \} \), since the logarithmic function is a monotonically increasing function, the maximization
problem (10a) is equivalent to

\[
\max_{\{p_{im}(k)\}} J_2 = \sum_{i=1}^{N_t} \sum_{m=1}^{N_u} p_{im}(k) y_{im}(k) \quad (11a)
\]

subject to \(\sum_{m=1}^{N_u} p_{im}(k) = P_i(k)\). \( (11b)\)

This is a linear programming problem. The maximum can be obtained by assigning all the power \(P_i(k)\) to the \(M_{i,k}\)-th antenna of \(DA_i\), where \(M_{i,k} = \arg\max_{1 \leq m \leq N_u} y_{im}(k)\). That is, \(p_{im}(k) = P_i(k)\), and \(p_{im}(m) = 0\) for \(m \neq M_{i,k}\). Substituting the results above to (8), the optimization problem (8) can be equivalently transformed to the one in (9).

It is hard to obtain a closed-form optimal \(\{P_i(k)\}\) to the optimization problem (9). Considering the computational efficiency and convergence property of the BCD method for convex problems [17], it is adopted to find the solution to the equivalent optimization problem (9).

Given \(P_i\) and \(\{P_j(k)\}\) for \(j = 1, \ldots, N_t\) and \(j \neq i\), the optimization problem (12), as shown at the top of the next page, can be written as

\[
\max_{\{P_i(k)\}} J_3 = \sum_{k=1}^{N_c} \log_2(1 + P_i(k)y_{im,k}(k)\sum_{j=1, j \neq i}^{N_c} P_j(k)y_{im,k}(k))
\]

subject to \(P_i(k) = P_i\) and \(P_i(k) > 0\) \( (13)\)

where \(Y_i(k) = \sum_{j=1, j \neq i}^{N_c} P_j(k) y_{im,k}(k)\).

Using the Lagrange multiplier method, we can obtain the solution of \(\{P_i(k)\}\) in a form of water-filling as

\[
P_i(k) = \max \left\{ 0, \frac{P_i}{N_c} \sum_{l=1}^{N_c} \frac{1 + \phi_i(l)}{y_{im,l}(l)} - \frac{1}{N_c} \phi_i(k) \right\}
\]

The derivation of (14) can be found in Appendix A.

For the PA design in (14), \(\left\{\frac{1 + \phi_i(k)}{y_{im,l}(k)}\right\}\) are sorted in ascending order over \(k\). Considering the positive power constraint, (14) is expressed as

\[
P_i(k) = \frac{P_i}{N_c} + \frac{1}{N_c} \sum_{l=1}^{N_c} \frac{1 + \phi_i(l)}{y_{im,l}(l)} - \frac{1}{N_c} \phi_i(k),
\]

for \(k = 1, \ldots, N_c\), and \(P_i(k) = 0\), for \(k = N_c + 1, \ldots, N_c\) \( (15)\)

where \(N_{c,i}^0\) is the number of subcarriers of the \(i\)-th DAU having positive power, and it can be obtained as

\[
N_{c,i}^0 = \max_{1 \leq n \leq N_c} \left\{ n : \frac{1 + \phi_i(n)}{y_{im,n}(n)} < \frac{P_i}{n} + \frac{1}{n} \sum_{l=1}^{N_c} \frac{1 + \phi_i(l)}{y_{im,l}(l)} \right\}
\]

With (16), \(N_{c,i}^0\) can be related to \(P_i\) as

\[
N_{c,i}^0 = \begin{cases} 1 & t_1 \leq P_i \leq t_2 \\ 2 & t_2 \leq P_i \leq t_3 \\ \vdots & \vdots \\ N_c & t_N \leq P_i \end{cases}
\]

where \(t_n = n \frac{1 + \phi_i(n)}{y_{im,n}(n)} - \sum_{l=1}^{n} \frac{1 + \phi_i(l)}{y_{im,l}(l)}\). Since \(\left\{\frac{1 + \phi_i(n)}{y_{im,n}(n)}\right\}\) are sorted in ascending order of \(n\), \(\{t_n\}\) are positive and also in ascending order of \(n\).

Substituting (15) into (12) yields

\[
\eta = \left( \sum_{i=1}^{N_c} \log_2 \left( P_i + \sum_{l=1}^{N_c} \frac{1 + \phi_i(l)}{y_{im,l}(l)} \right) 
+ \sum_{k=1}^{N_c} \log_2 \left( \frac{y_{im,k}(k)}{N_{c,i}^0} \right) 
+ \sum_{k=N_{c}+1}^{N_c} \log_2 (1 + \phi_i(k)) \right) 
/ \left( \frac{P_i + \sum_{j=1, j \neq i}^{N_c} \sum_{k=1}^{N_c} P_j(k) + P_e} {1 + \phi_i(k)} \right)
\]

With (17), the EE in (18) can be expressed as a piecewise function:

\[
\eta = \begin{cases} V_1 / \ln 2 & t_1 \leq P_i \leq t_2 \\ V_2 / \ln 2 & t_2 \leq P_i \leq t_3 \\ \vdots & \vdots \\ V_N / \ln 2 & t_N \leq P_i \end{cases}
\]

where \(V_n = \frac{n \ln (P_i + a_n) + b_n}{P_i + c_i}, a_n = \sum_{l=1}^{n} \frac{1 + \phi_i(l)}{y_{im,l}(l)}, b_n = \sum_{l=1}^{n} \ln \left( \frac{y_{im,l}(k)}{y_{im,l}(l)} \right) + \sum_{k=1}^{n} \ln (1 + \phi_i(k))\), \(c_i = \sum_{k=1}^{n} \sum_{j=1, j \neq i}^{N_c} P_j(k) + P_e\). It is found that the EE in (19) is a continuous function of \(P_i\). This is because \(V_n|_{P_e=0} = \frac{n \ln (P_i + a_n) + b_n}{P_i + c_i}\), and the related details can refer to Appendix B. Moreover, conditioned on \(\{P_i(k)\}, j = 1, \ldots, N_t\) and \(j \neq i\), the EE in (18) is a pseudo concave function of \(P_i\). So for the \(n\)-th segment of \(\eta\), we can find the corresponding optimal \(P_i\) by maximizing \(V_n\) in (19) subject to the maximum power constraint \(P_i \leq P_{\max}\), which can be attained by evaluating the derivative of \(V_n\) with respect to (w.r.t) \(P_i\). With (19), \(\frac{\partial V_n}{\partial P_i}\) can be expressed as

\[
V_n'(P_i) = \frac{\partial V_n}{\partial P_i} = \frac{n (P_i + c_i) - (P_i + a_n) [n \ln (P_i + a_n) + b_n]}{(P_i + a_n) (P_i + c_i)^2}
\]

where \(T_n = n (P_i + c_i) - (P_i + a_n) [n \ln (P_i + a_n) + b_n]\). Since the denominator in (20) is positive, the sign of \(V_n'\) is the same as that of \(T_n\) in (20). Using (20), the derivative of \(T_n\) w.r.t \(P_i\) is given by

\[
T_n' = \frac{\partial T_n}{\partial P_i} = - [n \ln (P_i + a_n) + b_n] < 0
\]
Since $T_n'$ is always negative, it means that $T_n$ is a decreasing function of $P_i$. Thus, $V_n'(P_i)$ is also a decreasing function of $P_i$ according to (20). Based on this result, the optimal $P_i$ of the $n$-th segment for the three cases of $V_n'$ can be obtained as follows:

(i) If $V_n'(t_n) \leq 0$, then the optimal $P_i$ of the $n$-th segment is $P_{i,n}^* = t_n$ since $V_n$ is a decreasing function.

(ii) If $V_n'(t_{n+1}) \geq 0$, then the optimal $P_i$ of the $n$-th segment is $P_{i,n}^* = t_{n+1}$ since $V_n$ is an increasing function.

(iii) If $V_n'(t_n) > 0$ and $V_n'(t_{n+1}) < 0$, then by setting $T_n = 0$ in (20) to zero, $P_{i,n}^*$ of the $n$-th segment can be obtained as 

$$P_{i,n}^* = \exp \left[ W \left( \frac{c_i - a_n}{e^{1-b_n/n}} \right) + 1 - \frac{b_n}{n} \right] - a_n, \quad \text{where } W(\cdot) \text{ is the Lambert W Function.}$$

As a summary, the optimal PA of the $n$-th segment is calculated as

$$P_{i,n}^* = \left\{ \begin{array}{ll}
 t_n & V_n'(t_n) \leq 0 \\
 t_{n+1} & V_n'(t_{n+1}) \geq 0 \\
 \exp \left[ W \left( \frac{c_i - a_n}{e^{1-b_n/n}} \right) + 1 - \frac{b_n}{n} \right] - a_n & V_n'(t_n) > 0 \quad \text{and } V_n'(t_{n+1}) < 0
\end{array} \right.$$  

Considering the power constraint, the optimal PA of the $n$-th segment is $P_{i,n}^* \in \arg \max \{ P_{i,n}^*, P_{\max} \}$.

By (22) and (19), we can compute the optimal $P_{i,n}^*$ and the corresponding $V_n^*$ for the $n$-th segment of $J_i$, respectively. The optimal number of subcarriers can be obtained as $N_i^0 = \arg \max V_n^*$, and the optimal PA is $P_i^* = P_{i,n}^*$. With $N_i^0$ and $P_i^*$, we can calculate the optimal $\{ P_i^* (k) \}$ by (15).

The above optimal power allocation design procedure for $P_i$ needs to compute $\{ P_{i,n}^*, V_n^* \}$ for $n = 1, \ldots, N_i$ for the determination of the optimal power. In the following, we introduce an efficient method to find a set of effective segments for the determination of the optimal power in order to save the computations.

Firstly, taking the maximum power constraint into account, the effective segments should have their lower bounds of the power in (19), $\{ t_n \}$, smaller than or equal to $P_{\max}$. Let $N_e$ denote the number of effective segments in the set. Using the increasing property of $\{ t_n \}$, we have

$$N_e \leq N_i^0 = \arg \max_n \{ t_n \leq P_{\max} \}  \quad \text{(23)}$$

Secondly, by exploring upper and lower bounds of $V_n$ in (19), ineffective segments can be further removed. With (19), considering the ascending order of $\{ t_n \}$, the upper and lower bounds of $V_n$ for $n = 1, \ldots, N_i^0$ can be obtained as

$$V_{n,t} = \frac{n \ln (t_n + a_n) + b_n}{t_{n+1} + c_i}  \quad \text{(24)}$$

$$V_{n,u} = \frac{n \ln (t_{n+1} + a_n) + b_n}{t_n + c_i}  \quad \text{(25)}$$

We compare the upper bounds $\{ V_{n,u} \}$ with the greatest lower bound of $\{ V_{n,t} \}$ to identify ineffective segments. The segments whose upper bounds smaller than the greatest lower bound are eliminated from the set because their EE are smaller than that of the segment corresponding to the greatest lower bound. With (24) and (25), we can obtain the set of effective segments

$$I_e = \{ n : V_{n,u} > \max \{ V_{n,t} \}, 1 \leq n \leq N_i^0 \}  \quad \text{(26)}$$

As a result, the number of searches for the optimal PA is greatly reduced. With the reduced cardinality of $I_e$, the optimal number of subcarriers can be efficiently obtained as $N_{c,e}^0 = \arg \max_{n \in I_e} V_n^*.$

The above PA method is used for the calculation of the power $\{ P_i \}$ of the DAUs at each iteration of the BCD method. In each iteration, the optimal $\{ P_i^* \}$ and $\{ N_{c,i}^0 \}$ as well as their corresponding $\{ P_i^* (k) \}$ are computed until the PA converges. Since the PA to the subcarriers of each DAU is optimized to increase the EE, thus, the unique global optimum can be attained by using the iterative BCD procedure.

As a summary, the procedure of the optimal energy efficient PA scheme with the BCD method is summarized in Algorithm 1, where $P_i^{(m)}(k)$ denotes the optimized PA to the $k$-th subcarrier of $DA_i$ at the $m$-th iteration.

We notice that [14] presented an optimal energy efficient PA scheme for DAS with OFDM over frequency selective fading channels subject to per-antenna power constraint, but this scheme is designed for DAS, where each DAU is only equipped with a single antenna. Thus, the EE optimization and PA scheme can be viewed as special cases of our scheme. Moreover, to obtain the optimal solution, it needs to search the optimal set $\Phi$ that satisfies $\sum_{i=1}^{N_i} P_i (k) \leq P_{\max}$ for $N_i$ DAUs, resulting in $2^{N_i}$ searches. Furthermore, for each search, iterative calculation needs to be performed. Thus, the complexity is much higher, especially for large $N_i$.

IV. SUBOPTIMAL PA SCHEME FOR DMISO-OFDM SYSTEM

As analyzed in Section III, the optimal scheme with the BCD method needs iterative calculations to update $\{ P_i \}$ and the
Algorithm 1 Optimal PA Scheme

1: Initialization: Set iteration index \( u = 0 \), tolerance \( \varepsilon \),
   \[ P_i^{(u)}(k) = P_{\text{max}} / N_c \text{ for } k = 1, \ldots, N_c, \; i = 1, \ldots, N_t. \]

2: for \( i = 1 : N_t \) do
3: \[ M_{i,k} = \arg \max \{ y_{\text{im}}(k) \}, \]
4: \[ P_{\text{im}}^{(u)}(k) = P_i^{(u)}(k) \text{ and other } P_{\text{im}}^{(u)}(k) = 0 \text{ for } m = 1, \ldots, N_a \text{ and } m \neq M_{i,k}. \]
5: Calculate \( S_T = \sum_{j=1}^{N_t} P_j^{(u)}(k) y_{mj,k}(k). \)
6: for \( k = 1 : N_t \) do
7: \[ \phi_i(k) = S_T - P_i^{(u)}(k) y_{mj,k}(k). \]
8: end for
9: \[ c_{\text{im}} = \sum_{j=1}^{N_t} P_j^{(u)}(k) y_{mj,k}(k). \]
10: Compute \( P_i^{(u+1)}(k) = \left\{ \begin{array}{ll}
   t_{\text{im}} & \text{if } V_{\text{im}}(t_{\text{im}}) \leq 0 \\
   V_{\text{im}}(t_{\text{im}}+1) & \text{if } V_{\text{im}}(t_{\text{im}}+1) > 0 \\
   \exp \left[ W \left( \frac{c_{\text{im}} - a_{\text{im}}}{1 - b_{\text{im}}/n_{\text{im}}} \right) \right] & \text{if } V_{\text{im}}(t_{\text{im}}+1) < 0
   \end{array} \right. \]
   where \( V_{\text{im}}(t_{\text{im}}) \) is calculated by (20),
   \[ a_{\text{im}} = \sum_{l=1}^{n_{\text{im}}} \frac{1 + \phi_l(l)}{y_{jm,l}(l)}, \]
   \[ b_{\text{im}} = \sum_{k=1}^{n_{\text{im}}} \ln \left( \frac{y_{jm,k}(l)}{n_{\text{im}}} \right) + \sum_{k=n_{\text{im}}+1}^{n_{\text{im}}+1} \ln (1 + \phi_l(l)), \]
   and \( c_{\text{im}} = \sum_{k=1}^{n_{\text{im}}} \sum_{l=1}^{n_{\text{im}}} P_j^{(u)}(k) + P_c. \]
11: Compute \( \phi_i(k) = \frac{n_{\text{im}} P_j^{(u)}(k) + P_{\text{max}} \sum_{j=1}^{N_t} y_{mj,k}(k)}{N_c} \text{ for } i = 1, \ldots, N_t \)
12: end if
13: Compute \( N_{c,i}^0 = \arg \max_{n \in I_c} V_n. \)
14: Set \( P_i^0 = P_{\text{im}}^0. \)
15: Compute \( P_i^{(u+1)}(k) \) for \( k = 1, \ldots, N_{c,i}^0 \) by (15), and \( P_i^{(u+1)}(k) = 0 \), for \( k = N_{c,i}^0 + 1, \ldots, N_c. \)
16: end for
17: if \( \sum_{k=1}^{N_t} \sum_{k=1}^{N_{c,i}} \left| P_i^{(u+1)}(k) - P_i^{(u)}(k) \right|^2 > \varepsilon \) then
18: \( u = u + 1 \), Go to Step 2
19: end if
20: return \( \{ P_i^{u}(k) \}. \)

For the DAU with large CGNRs (say \( DA_1 \) has large \( y_{mj,k}(k) \)) in comparing with those of other DAUs, the term \( \frac{1}{N_c} \sum_{i=1}^{N_t} \frac{1 + \phi_i(l)}{y_{mj,i}(l)} - \frac{1 + \phi_i(l)}{y_{mj,k}(k)} \) in (14) becomes small, where \( \phi_i(k) = \sum_{j=1}^{N_t} P_j(k) y_{mj,k}(k) \). Thus, the power allocation in (14) to the subcarrier \( k \) of \( DA_1 \) is approximately equal to \( P_i/N_c. \) Based on this result, the equal power scheme is more suitable for the initialization of the PA of the DAUs with larger CGNRs. The analysis suggests a sequential procedure of computing the PA of DAUs one by one as follows. The power allocation to the subcarriers of the DAU (labelled as \( DA_1 \)) with the smallest total CGNR is firstly computed with other DAUs using the equal power allocation scheme. The obtained PA of \( DA_1 \) is then fixed, and the procedure is repeated to calculate the PA of next DAU in ascending order of total CGNR (\( y_i = \sum_{j=1}^{N_t} y_{mj,i}(l) \)).

According to the proposed procedure above, \( \{ P_i(k) \} \) are initially set as \( P_i(k) = P_{\text{max}} / N_c \), and the total CGNRs \( \{ y_i \} \) are sorted in ascending order of \( i \). Following the principle of the BCD method, the PA of \( DA_1 \) at the \( i \)-th stage can be computed by Step 6 to Step 10 of Algorithm 1 with \( \phi_i(k) \) in (13) modified as
\[ \phi_i(k) = \sum_{j=1}^{i-1} y_{mj,k}(k) P_j^{2}(k) + \frac{P_{\text{max}} \sum_{j=1}^{N_t} y_{mj,k}(k)}{N_c} \text{ for } i = 1, \ldots, N_t \]
(27)
where \( P_{j}^{2}(k) \) is the optimized power allocation to the \( k \)-th subcarrier of \( DA_j \) designed at the previous \( j \)-th stage of the PA procedure, and the equal power scheme is applied to the ensuing stages. At the first and the last stages, the \( \phi_i(k) \)'s are respectively given as
\[ \phi_i(k) = \frac{P_{\text{max}} \sum_{j=1}^{N_t} y_{mj,k}(k)}{N_c} \]
(28)
\[ \phi_{N_t}^{k}(k) = \sum_{j=1}^{N_t-1} y_{mj,k}(k) P_j^{2}(k) \]
(29)
As a summary, the procedure of the suboptimal PA scheme is summarized in Algorithm 2.
TABLE 1. Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of DAU ($N_t$)</td>
<td>5, 7</td>
</tr>
<tr>
<td>Number of antennas in each DAU ($N_a$)</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>Number of subcarriers ($N_c$)</td>
<td>32, 64, 128</td>
</tr>
<tr>
<td>Path loss exponent ($\alpha$)</td>
<td>3.5</td>
</tr>
<tr>
<td>Shadow fading standard deviation</td>
<td>8dB</td>
</tr>
<tr>
<td>Cell radius ($r$)</td>
<td>$10^3$ m</td>
</tr>
<tr>
<td>Circuit power ($P_c$)</td>
<td>5W</td>
</tr>
<tr>
<td>MS distribution</td>
<td>uniform</td>
</tr>
<tr>
<td>Number of channel realizations</td>
<td>$10^4$</td>
</tr>
</tbody>
</table>

Based on the above algorithm, we can obtain all the \{$P_{\text{im}}^n(k)$\} and the corresponding suboptimal energy efficiency. Interestingly, the suboptimal algorithm has the EE value close to the optimal one, and only small performance loss is found. By simulation, it is observed that the relative EE loss of larger number of subcarriers is smaller than that of smaller $N_c$ (the relative EE loss is defined as the absolute difference of the EE of the suboptimal scheme and the optimal value divided by the optimal one). Unlike the optimal scheme, the suboptimal scheme avoids iterative calculations and can provide the closed-form calculation of PA. Hence, the computational complexity is greatly reduced.

V. SIMULATION RESULT AND DISCUSSIONS

In this section, the performances of the proposed two schemes are evaluated by computer simulations. Unless otherwise specified, the simulation parameters are mainly listed in Table 1. The DAUs are symmetrically placed inside the cell. Specifically, the polar coordinates of the DAU are $(2r/3, 2\pi n/N_t)$, $n = 1, \ldots, N_t$, where $r$ is the radius of the cell. The gain of the multipath channel between the $n$-th DAU and the MS is normalized as one. The simulation results are shown in Figs. 2-4, respectively. In these figures, the optimal scheme, suboptimal scheme, and the scheme based on the function fmincon in Matlab are referred to as ‘op-scheme’, ‘sub-scheme’ and ‘fmin-scheme’, respectively.

In Fig. 2, we plot the EE of DMISO-OFDM versus maximum power $P_{\text{max}}$ for different PA schemes, where the path loss exponent $\alpha_n = \alpha$ ($n = 1, \ldots, N_t$), $N_{\alpha} = 1$ (which corresponds to the conventional DAS). The numbers of subcarrier for Figs. 2(a) and 2(b) are $N_c = 32$ and $N_c = 128$, respectively. The existing optimal PA scheme for DAS in [14] is included for comparison in Fig.2. The simulation results are shown in Figs. 2-4, respectively. In these figures, the optimal scheme, suboptimal scheme, and the scheme based on the function fmincon in Matlab are referred to as ‘op-scheme’, ‘sub-scheme’ and ‘fmin-scheme’, respectively.

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In Table 2, we compare the average run times of the four schemes. As indicated in Table 2, the run time of the suboptimal scheme is much less than that of the other three optimal schemes because of its closed-form PA procedure. Furthermore, the run time of the proposed optimal scheme is much less than that of the fmincon scheme and the scheme in [14] due to their poor computation efficiency. For the above simulations, the computer we used is equipped with Intel Core™ i5-4590 CPU @3.3GHz and 8G RAM.
TABLE 2. Complexity comparison of three schemes.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Running time</th>
<th>Power allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal scheme</td>
<td>10937.48s</td>
<td>Iterative calculation</td>
</tr>
<tr>
<td>Sub-scheme</td>
<td>2620.9s</td>
<td>Closed form</td>
</tr>
<tr>
<td>The scheme in [14]</td>
<td>72165.68s</td>
<td>Iterative calculation</td>
</tr>
<tr>
<td>fmincon scheme</td>
<td>127879.29s</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3 shows the energy efficiency of DMISO-OFDM versus maximum power $P_{\text{max}}$ for different power allocation schemes with $N_t = 5, 7, N_c = 64, \text{ and } N_a = 2$. As shown in Fig.3, the results similar to Fig. 2 can be observed, namely, the EE is increased $P_{\text{max}}$ as increases, and the optimal scheme and fmincon schemes outperform upon the suboptimal scheme, but the EE loss of the suboptimal scheme is small. Besides, the EE increases with the number of DAUs. Specifically, the system with $N_t = 7$ has higher EE than that with $N_t = 5$. This is because the DMISO-OFDM with more distributed antennas has higher spatial diversity. The results above indicate that the proposed optimal and suboptimal schemes are both valid.

Figure 4 gives the EE of DMISO-OFDM versus maximum power $P_{\text{max}}$ for different power allocation schemes with $N_t = 7, N_c = 64, \text{ and } N_a = 1, 3$. As shown in Fig.4, the suboptimal scheme still achieves the EE performance close to that of optimal scheme for different numbers of transmit antenna in each DAU. Moreover, with the increase of $N_a$, the EE is improved accordingly. Compared to Fig.3, the system with $N_a = 3$ has higher EE than that with $N_a = 2$ because of greater space diversity. For the same reason, the system with $N_a = 2$ has higher EE than that with $N_a = 1$. Hence, the DMISO-OFDM outperforms the conventional DAS-OFDM ($N_a = 1$) and can obtain higher EE than the latter because more transmit antennas are employed. The results above further illustrate the effectiveness of the proposed two schemes.

VI. CONCLUSIONS

We have developed two power allocation schemes for EE maximization in DMISO with OFDM in frequency-selective fading channels subject to the per-antenna maximum power constraint. By using the linear programming and the BCD method, the original optimized problem is simplified, and an efficient iterative algorithm is proposed to solve the simplified problem to obtain an optimal solution. To avoid iterative calculation, a closed-form suboptimal PA scheme is developed by sorting the total CGNR in ascending order and determining the number of effective subcarriers in each DAU. This suboptimal scheme has lower computational complexity than that of the optimal scheme and can obtain the EE close to that of the latter. It provides a good tradeoff between the performance and computational complexity. The proposed optimal and suboptimal schemes include the ones with single transmit antenna in DAS as special cases. Simulation results show that the proposed optimal scheme has the same EE as the scheme based on function fmincon in Matlab because they are both optimal, and identical to the existing optimal scheme in DAS but with lower complexity.

APPENDIX A

In this appendix, we give the derivation of (14). The Lagrangian function $L_1$ for (13) is constructed as

$$L_1 = \sum_{k=1}^{N_c} \ln (1 + P_i(k) y_{\text{IM},k}(k) + \phi_i(k)) + \lambda \left[ P_i - \sum_{k=1}^{N_c} P_i(k) \right]$$

where $\lambda$ is a Lagrange multiplier.

Taking the derivative of $L_1$ w.r.t. $P_i(k)$ gives

$$\frac{\partial L_1}{\partial P_i(k)} = \frac{y_{\text{IM},k}(k)}{1 + P_i(k) y_{\text{IM},k}(k) + \phi_i(k)} - \lambda$$

By setting $\frac{\partial L_1}{\partial P_i(k)}$ to zero yields

$$P_i(k) = \frac{1}{\lambda} \left( 1 + \frac{1 + \phi_i(k)}{y_{\text{IM},k}(k)} \right)$$
Substituting (A3) into the power constraint in (13) gives
\[ P_i(k) = \frac{P_i}{N_c} + \frac{1}{N_c} \sum_{k' = 1}^{N} \frac{1 + \phi_i(k')}{\gamma_iM_k(k')} - \frac{1 + \phi_i(k)}{\gamma_iM_k(k)} \quad (A4) \]
According to (A3) and (A4), the larger the \( \gamma_iM_k(k) \), the more is the power allocation \( P_i(k) \) to the \( k \)-th subcarrier. When \( \gamma_iM_k(k) \) is too small, no power may be allocated in order to avoid negative power allocation.

**APPENDIX B**

In this appendix, we give the derivation of \( V_{n+1}|p_i = p_{n+1} \) for \( n = 1, \ldots, N_c - 1 \). With (19), we have:
\[
V_{n+1}|p_i = p_{n+1} = V_n|p_i = p_{n+1} - n \ln(t_{n+1} + a_n) + b_{n+1} - b_n
\]
\[
= (n + 1) \ln(t_{n+1} + a_{n+1}) - n \ln(t_{n+1} + a_n) + b_{n+1} - b_n
\]
\[
= (n + 1) \ln(n + 1) - n \ln(n)
\]
\[(B1)\]
According to definitions of \( t_n \) (below (17)) and \( a_n \) (below (19)), we can obtain the following equation:
\[
(n + 1) \ln(t_{n+1} + a_{n+1}) - n \ln(t_{n+1} + a_n) = \ln(1 + \phi(n + 1)) - \ln(y_iM_{n+1}) + (n + 1) \ln(n + 1) - n \ln(n)
\]
\[(B2)\]
With the definition of \( b_n \) below (19), the following equation can be derived as
\[
b_{n+1} - b_n = \ln(y_iM_{n+1}) - (n + 1) \ln(n + 1)
\]
\[
= n \ln(n) - (1 + \phi(n + 1))
\]
\[(B3)\]
Substituting B(2) and B(3) into B(1) yields
\[
V_{n+1}|p_i = p_{n+1} = V_n|p_i = p_{n+1} = 0
\]
\[(B4)\]
Hence, \( V_{n+1}|p_i = p_{n+1} = V_n|p_i = p_{n+1} \).

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SHU-HUNG LEUNG received the B.Sc. degree (Hons.) in electronics from The Chinese University of Hong Kong in 1978 and the M.Sc. and Ph.D. degrees in electrical engineering from the University of California at Irvine in 1979 and 1982, respectively. From 1982 to 1987, he was an Assistant Professor with the University of Colorado at Boulder. Since 1987, he has been with the Department of Electronic Engineering, City University of Hong Kong, where he is currently an Associate Professor. His current research interest is in digital communications, speech signal processing, image processing, and adaptive signal processing. He has received more than 20 research grants from CERG, Croucher Foundation, and City University strategic grants and published over 200 technical papers in journals and international conference proceedings. He served as the Chairman of the Signal Processing Chapter of the IEEE Hong Kong Section from 2003 to 2004 and an Organizing Committee Member for a number of international conferences. He is listed in the Marquis Who’s Who in Science and Engineering and Marquis Who’s Who in the World. He is currently an Associate Editor of the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY.

JUNYA CHU received the B.S. degree in electrical engineering from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2016, where she is currently pursuing the M.S. degree.