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Published in:
IEEE/ACM Transactions on Networking

Published: 01/12/2022

Document Version:
Post-print, also known as Accepted Author Manuscript, Peer-reviewed or Author Final version

Publication record in CityU Scholars:
Go to record

Published version (DOI):
10.1109/TNET.2022.3171832

Publication details:

Citing this paper
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Submarine Cable Network Design for Regional Connectivity

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Abstract—This paper optimizes path planning for a trunk-and-branch topology network in an irregular 2-dimensional manifold embedded in 3-dimensional Euclidean space with application to submarine cable network planning. We go beyond our earlier focus on the weighted costs of cables (cable laying cost, resilience, design level and repair rate) to include the cost of branching units (BUs), including material and labor, as well as submarine cable landing stations (CLSs). This optimization also includes choices of locations of BUs and CLSs. These are important issues for the economics of cable laying and significantly change the model and the optimization process. We pose the problem as a variant of the Steiner tree problem, but one in which the Steiner nodes can vary in number, while incurring a penalty. We refer to it as the weighted Steiner node problem. It differs from the Euclidean Steiner tree problem, where Steiner points are forced to have degree three: this is no longer the case, in general, when nodes incur a cost. We are able to prove that our algorithm is applicable to Steiner nodes with degree greater than three, enabling optimization of network costs in this context. The optimal solution is achieved in polynomial-time using dynamic programming.

Index Terms—Steiner minimum tree, weighted Steiner nodes, submarine cable networks, manifold, branching units, cable landing stations.

I. INTRODUCTION

Submarine internet optical cables play an important and crucial role in global communications, transmitting more than 99% of global Internet data [1]. By early 2021, there are more than 1.3 million kilometers of submarine cables across the oceans and seas of the world [2] and, according to [1], global capacity of submarine cables is estimated to increase up to 100% by the end of 2024. This paper studies the optimal design of submarine cable systems. These systems typically have a trunk-and-branch tree topology as illustrated in Fig. 1. To be taken into account in their optimal design are cable laying costs (including terrain slope, materials, alternative protection level, labor, and cable survivability), installation costs of the submarine cable branching units (BUs), and the choice of locations of the submarine cable landing stations (CLSs). Our methods take account of the 3-dimensional topography of the earth, by modeling it as an irregular 2-dimensional manifold in a 3-dimensional Euclidean space. Unlike earlier work, the path planning optimization problem considered here extends to the more realistic provision of region-to-region connectivity, as shown in Fig. 1, rather than just point-to-point.

Fig. 1: Cable system interconnecting five regions.

A submarine optical cable system is conveniently divided into two parts: underwater equipment and onshore equipment. The underwater equipment includes submarine optical cables, repeaters, and BUs, whereas the onshore equipment includes CLSs. Submarine BUs are used to fork cables, allowing traffic to be routed to (or merged from) two or more different locations thus permitting a diversity of connections for the cable system. In most cases, a BU is a Y-shaped cable connector, connecting three terminals [3]. However, in this paper, we do not exclude the possibility that a terminal which is connected to a BU can also be another BU. More than one BU may be used in a real-life cable system. Indeed, many large scale cable systems use many BUs to allow multiple connections [4]; for example, 16 BUs are used in the Africa Coast To Europe submarine cable system and 9 BUs in the Asia Pacific Gateway system.

The cost of the BU is determined by its complexity and its number of ports [3]. In 2002, the cost of a BU, was estimated to be between $1 million and $1.5 million [5]. A more recent estimate gives the cost as $2 million [6].

There are two main reasons to use BUs in a submarine cable network.

1) In practice, the total cable length should be as short as possible. By using BUs, the cable system can be...
constructed as a trunk-and-branch topology, providing a relatively small total length of cables.

2) In some areas, there is a requirement to transmit data to various destinations, but there are limitations to potential routes related to, for example, legal/licensing and seabed space restrictions [7].

When planning submarine cable routes, the choice of location (or even existence) of BUs needs to take account, not only of total cable length, but of overall network cost. To this end, account needs to be taken of, for example, water pressure and depth, topography, etc., because under different seabed conditions, the type and cost of a BU varies [3]. This means the BU cost could be different in each grid node on the manifold; this is the case in the example shown in Section V-A2.

CLSs selection is another important factor in the construction of a submarine optical cable system connecting several regions. Most CLSs are very close to the coast. In multi-point coastal systems, cost associated with CLSs can become a significant component of total network cost. Large CLSs cost $10-15 million while small CLSs are around $5 million [3, 8]. The location of CLSs can also affect the total length of a cable network. The specific locations of CLSs that serve regions/areas are usually carefully chosen, taking into account the following issues [8, 9].

1) Marine zoning and compatibility with different development and planning activities;
2) Easy access to landing points by the submarine optical cable installation vessel for installation and maintenance;
3) Terrain risks for cables at landing points, for example, avoidance of steep rocky coasts and areas of subsidence as well as steep, sandy or silty sea-floors;
4) Environmental risks for cables such as storm surges and other marine disasters;
5) Avoidance of areas where corrosion (chemical pollution) of optical cables is likely;
6) Avoidance of areas of risk of cable damage by anchors and trawling nets.

The key novelty of this paper is the extension of the optimization problem of the submarine cable system from minimizing the total cable construction cost in a cable system to minimizing the construction costs of the entire cable system, including, in addition to the cable construction costs, the installation costs of BUs and CLSs. We pose the optimization problem as a variant of the Steiner minimum tree problem, on an irregular 2-dimensional manifold in a 3-dimensional Euclidean space and propose polynomial-time dynamic programming for its solution. We call this the weighted Steiner node problem.

The main technical advances and contributions of this work beyond [10] are as follows.

1) In the cable system construction optimization, the installation costs of BUs are included in addition to minimization of the total cable length. By optimizing the cable network structure, we can determine the number of BUs that should be used in the cable network taking account of the unit installation cost of a BU that, in turn, depends on the environmental conditions in its location.

2) In addition to optimizing the network based on the limited assumption that all BUs are Y-shaped (have three branches), our algorithm also solves the cable network optimization problem with more than three branch BUs. Moreover, we prove a new theorem in Section IV to the effect that our algorithm is applicable to Steiner nodes with degree 3 or more.

3) Cable connections between regions, rather than between locations become the focus of the optimization, so that our destinations are many sets of points. Such a cable network connects regions rather than specified coastal locations. We consider a range of CLS construction alternatives within a region. Our algorithm can select the best CLS in each region according to their locations and construction costs, so that the total construction cost of the cable network is minimized.

4) We consider the trade-off between the laying cost, related to different cable protection levels and the depth of the seabed, and the total number of repairs that are associated with kinds of risk, such as earthquake-related cable damage risk, human activity related risk, and different cable protection levels.

The remainder of this paper is organized as follows. In Section II, we review related path planning research in cable networks, including point-to-point path planning and cable network path planning algorithms. Section III focuses on the modeling of our cable network optimization problem. In Section IV, we propose a method to solve the problem. The performance of our method is demonstrated by several numerical simulations based on realistic examples in Section V. Finally, we conclude the paper in Section VI.

II. RELATED RESEARCH

In the current extensive literature on submarine cable path planning, we concentrate on optimization of the following three relevant network topologies in the context of submarine cable path planning: (1) point-to-point [11–14]; (2) point-to-area/existing cables [15]; and (3) cable networks connecting more than two points [10, 16, 17].

In practice, the industry generally uses a manual approach, based on expert experience, for the path planning of submarine cables [16]. Based on data available on the relevant region, planners determine the cable path by checking carefully all the relevant details along the path and comparing between various alternatives. This approach cannot guarantee an optimal path and can be expensive in human resources.

Currently, MakaiPlan, an industry-leading submarine cable route planning software is available on the market. For a pair of nodes to be connected, MakaiPlan chooses the shortest distance along a great circle route in a series of rhumb lines, providing a rough estimate of the geodesic. MakaiPlan only provides path planning of submarine cables connecting two nodes, and provides no means to optimize, taking account of risk factors, submarine cable networks with trunk-and-branch and mesh architectures; commonly used topologies in submarine cable systems.

In [14], using the Dijkstra’s algorithm, the authors proposed a raster-based method for path planning to obtain the cable
path with least cost, specifically, minimizing the weighted sum of the seismic risk and the cable length (cost). A disadvantage of this method is that a path must traverse either a diagonal or a lateral edge between neighboring cells. Additionally, they only considered point-to-point cable path planning.

Zhang et al. [19] addressed the problem of shielding telecommunication cables in high-risk areas. In particular, they studied a given telecommunication network with a focus on the problem of how to decide which cables in the network require shielding to guarantee network connectivity at minimum cost. They assumed a given shielding cost for each cable and considered certain failure models to formulate and solve their optimization problem. Wang et al. [20] considered a triangulated irregular network in 3-dimensional Euclidean space; at any point on this triangulate network the measure of risk was specified in terms of the expected number of repairs. They also considered multiple protection-level models based on this risk measure and laying cost for optimizing the path of a cable with different design levels. The optimal cable path was derived by running Dijkstra’s algorithm with an interval-partition-based label-setting approach.

In [13, 21], Wang et al. presented a method based on the fast marching method (FMM) for cable path planning on the earth’s surface. FMM is a path planning approach that avoids the weaknesses of Dijkstra’s algorithm [22] by solving the Eikonal equation as the grid size of the triangulated manifold approaches zero. Wang et al. [21] took account of multiple design levels of cable shielding and applied FMM to design optimal cable paths. They addressed the cable path planning problem as a multi-objective optimization problem, taking account of cable breaking risk, cable laying cost, and multiple design levels for cable shielding. They converted it into a single objective optimization problem by assigning different weights to each objective.

More recently, in [10, 15, 16], FMM was applied to cable path planning, designing a cable path from a start node to an existing cable network. Their objective was to lay a cable from node A to a certain end location B on an existing cable at a minimal cost. They considered three scenarios for the destination: an arbitrary location on an existing cable with the addition of a new BU, an installed BU or a landing station; a landing station. The proposed method can be applied to cable path planning from one node to a set of nodes. Wang et al. [10] proposed an FMM-based method to achieve a trunk-and-branch tree network topology for submarine cable systems that connect multiple landing stations. That work considered the optimization of the trunk-and-branch tree network problem as a Steiner minimum tree (SMT) problem, and based on dynamic programming, obtained the minimum tree on an irregular 2-dimensional manifold in a 3-dimensional Euclidean space for a given topology in polynomial time. However, it did not take account of installation costs of BUs and, in that paper, the CLS location in every region was uniquely specified.

In [16], we proposed a method based on FMM and Integer Linear Programming to optimize a tree-topology network design for a submarine cable system that connects multiple stations. We also proposed a lightweight heuristic algorithm to solve this problem. We considered the optimization of the cable network problem as a minimum spanning tree (MST) problem. The objective was to construct a tree-topology cable network without additional Steiner nodes at minimal cost. In that work, the installation costs of BUs, and location choices of CLSs were not considered.

The SMT problem in $d$-dimensional Euclidean space is a well-known NP-hard (non-deterministic polynomial-time hard) combinatorial optimization problem. Many authors have contributed ample research on this problem and proposed many solutions [23-27]. Gilbert and Pollak [23] proposed a method to solve the SMT problem in $d$-dimensional Euclidean space. In their algorithm, the first step is to enumerate all Steiner topologies and then calculate the relative minimal tree (RMT) corresponding to each topology. Their algorithm is very computationally intensive, as the number of Steiner topologies grows super-exponentially with the number of terminals. As a result, it can only be applied to very small instances of the SMT problem. The GeoSteiner algorithm [28] can solve a typical SMT problem with a large number of terminals in the 2-dimensional Euclidean plane by using geometrical properties (such as node degree and the angle conditions). However, GeoSteiner heavily utilizes the properties of the Euclidean plane, and these geometrical properties do not translate to higher dimensions and irregular 2-dimensional manifolds in $\mathbb{R}^3$. Smith [29] proposed an implicit enumeration scheme to generate full Steiner topologies by using branch and bound (B&B). Smith’s B&B algorithm consists of a tree generation step and a numerical optimization step. In the B&B algorithm, there is a continuously updated maximum value. Spanning trees that exceed the maximum value are eliminated, thus reducing the computational complexity. The metric in [29] is Euclidean (albeit $d$ dimensional), so it is not applied directly to our case of an irregular 2-dimensional manifold $\mathbb{M}$ in $\mathbb{R}^3$. Fampa et al. [30] improved Smith’s B&B algorithm by using a conic formulation for the subproblem of locating the Steiner nodes, but the general Euclidean SMT problem in cases other than the 2-dimensional plane remains challenging. The work discussed above on the SMT problem in general Euclidean space cannot be directly used in our problem of finding a minimal cost tree in a manifold. Approximate solutions to the SMT problems have been applied to communication networks design in [26, 31]. Caleff et al. [26] used an algorithm based on a BioNetwork of a unicellular organisms for the SMT problem. Sun et al. [31] proposed an algorithm based on Physarum-inspired algorithm for the SMT problem.

Unlike the available approximate solutions, we seek an optimal solution for the problem under consideration.

The closest study to our work is the above-mentioned work [10], which provided a polynomial time complexity numerical algorithm, based on dynamic programming, to find the RMT on the manifold. But the work in [10] did not consider the cost of Steiner nodes and terminal nodes in the network which is what is achieved in the present paper. Moreover, beyond the work in [10], this paper considers the different cable protection levels and the total number of repairs associated with various risks to the cable, such as earthquake-related cable damage risk and human activity related risks. One way to reduce human related risks, such as those associated
with fishing areas, is to lay cables in deep water. Hence, our algorithm tends to avoid earthquake prone areas while giving preference to deep water. Typical cable life-times are around 25 years [32]. It is important, then, to be aware that risk related areas do not change rapidly over time. This is clearly true for earthquake prone areas and the depth of the seabed. Though fishing areas may change over time, they do not change rapidly.

III. PROBLEM DESCRIPTION AND MODELING

In this section, we pose a new optimal cable network design problem, which we call the weighted Steiner node problem. To address this problem, a 2-dimensional manifold in 3-dimensional space is used to model the surface of the earth on which we aim to minimize the total cost of a submarine cable system. Let \( \mathbb{D} \) denote a bounded closed region on the earth’s surface modeled as a 2-dimensional manifold, where any pair of nodes in this region can be connected by a path on the 2-dimensional manifold. As in [13, 17, 21], to represent the earth’s surface with a reasonable level of fidelity, we use a triangulated piecewise-linear 2-dimensional manifold \( \mathbb{M} \) to model the region \( \mathbb{D} \). The “smoothly” continuous area \( \mathbb{D} \) is rendered as a triangulated manifold \( \mathbb{M} \), each point \( x \) on \( \mathbb{M} \) being represented by 3-dimensional coordinates \( (x, y, z) \), where \( z = \xi(x, y) \) is the altitude of \( (x, y) \). For a cable represented by a Lipschitz continuous [33] curve \( y \) connecting two nodes in \( \mathbb{M} \), the total laying cost (based on the length and depth) of the cable \( y \) with design levels \( u(\cdot) \in \mathbb{U} \) is denoted by \( \mathbb{H}(y, u(\cdot)) \), as shown in Eq. (1), where \( h(x, u(x)) \) is the laying cost per unit length of the cable passing through point \( x \) on the cable that may include soil type, elevation, labor, licenses, and protection level [15].

\[
\mathbb{H}(y, u(\cdot)) = \int_y h(x, u(x)) ds. \tag{1}
\]

As in [13], the repair rate at location \( x = (x, y, z) \in \mathbb{M}, z = \xi(x, y) \) is defined to be \( g(x, u(x)) \). Let \( \mathcal{G}(y, u(x)) \) denote the total number of repairs of a cable \( y \). Again, we assume that \( \mathcal{G}(y, u(x)) \) is additive. That is, \( \mathcal{G}(y, u(x)) \) can be written as

\[
\mathcal{G}(y, u(\cdot)) = \int_y g(x, u(x)) ds. \tag{2}
\]

where \( g(x, u(x)) \in \mathbb{R}^1_+ \) is the repair rate with a particular design level \( u \) at location \( x \).

As in [12, 13, 15, 17, 21], the repair rate \( g \) is related to ground motion intensities, such as Peak Ground Velocity (PGV) and Peak Ground Acceleration (PGA) which is, in turn, associated with earthquakes and other natural hazards (e.g. landslides, debris flows, volcanoes, storms), as well as human activities.

To consider both the cable laying cost and the cable repair rate in cable system design, a weighted sum approach is used. Specifically, for any grid point, \( x = (x, y, z) \in \mathbb{M}, z = \xi(x, y) \), let \( f(x, u(x)) = \alpha \cdot h(x, u(x)) + g(x, u(x)) \) be a weighted cost at \( x \), where \( \alpha \in \mathbb{R}^1_+ \cup \{0\} \) can be regarded as the exchange rate between the laying cost and the risk. Then the total weighted cost of a cable \( y \) is

\[
c(y, u(x)) = \int_0^{l(y)} f(y(s), u(x)) ds. \tag{3}
\]

The problem of submarine cable system design with a trunk-and-branch tree topology is restated as follows. Given \( n \) terminals \( x_1, x_2, \ldots, x_n \), on the 2-dimensional triangulated manifold \( \mathbb{M} \in \mathbb{R}^3 \) to be connected as a network, find the Steiner tree with minimal weighted sum cost.

Let \( \mathbf{R}_1, \mathbf{R}_2, \ldots, \mathbf{R}_r \subseteq \mathbb{D} (R \) is the number of areas\) be the destination areas to be connected via a cable network with a trunk and branch topology. Note that, although in this paper each \( \mathbf{R}_i \) represents an area, our solutions are applicable also to the case where some or all of the \( \mathbf{R}_i \)s correspond to a specific point.

The network may include new nodes called Steiner nodes (representing BUs in the cable network) to reduce total cable length of the network and influence the overall construction cost. As discussed in Introduction, the construction costs of the cable network, in our situation, includes the costs of submarine cables, BUs and CLSs, so decisions on the number and locations of BUs in the network and locations of CLSs (if there is a choice) are needed.

We denote by \( b_i \) the BU installation cost at the grid node \( x_i \in \mathbb{M} \). If a Steiner node has the same location as a terminal node, \( b_i = 0 \). This means that, since a Steiner node represents a BU, if a Steiner node is included in a terminal node, the elimination of the Steiner node implies a potential savings as the BU is not required, and the number of BUs is reduced by one.

If BUs in the cable systems have no branch number constraints, there is another case in which the cost of Steiner nodes are ignored: two or more Steiner nodes have the same location. Then these Steiner nodes can be taken to represent a single BU which the number of branches is the addition of the number of branches of each combined BU and subtract 2, so we count the BU cost (may depend on the number of branches) once.

Because of the widespread usage of 3-branch BUs in the industry, we demonstrate in realistic cable networking scenarios that our algorithm can restrict the solutions to just 3-branch BUs. Additionally, two realistic examples of a cable system with \( K \)-branch (\( K \geq 3, K \in \mathbb{N}^+ \)) BUs is demonstrated in Section V-A3 and Section V-B.

Another consideration in cable system optimization is the choice of CLS locations. Fig. 2 gives a simple and clear explanation of this problem. For this problem, more than one potential site can be the location of a CLS in each area \( \mathbf{R}_i \). Let \( \mathbf{r}_{i1}, \mathbf{r}_{i2}, \ldots, \mathbf{r}_{iL_i} \in \mathbf{R}_i \) be the potential locations for CLSs in area \( \mathbf{R}_i \). As discussed in Introduction, the construction costs of the CLSs depend on many factors. In each area, several potential locations are available for CLS. Different locations have different construction cost and will also result in different lengths of cables in the network.
IV. METHODOLOGY AND ALGORITHM

Our cable network optimization problem on the triangulated manifold $\mathbb{M}$ is clearly NP-hard since it generalizes the SMT problem in the 2-dimensional Euclidean plane.

We consider a tree topology $T$ to be a combined structure of points and edges in terms of topological rather than metric entities, where we include the labelling of some nodes as Steiner nodes and the others as terminal nodes, as well as specifying which terminal/Steiner node pairs are connected by edges. Such a topology $T$ is called a Steiner topology if the degree of each Steiner node is equal to three and the degree of each terminal node is three or less [30]. We use the generic term topology of a tree to mean that there are no restrictions on the degrees of Steiner nodes and terminal nodes, though again with a labelling of nodes as Steiner nodes or terminal nodes. General topologies may include, in particular, some Steiner nodes with more than three branches. For given $N$ terminal nodes, Steiner topologies form a subset of the set of all tree topologies.

We extend these ideas to cover topologies in which there are $K$-degree $(K \geq 4, K \in \mathbb{N}^+)$ Steiner nodes by regarding such nodes as combinations of linked 3-degree Steiner nodes, with zero-cost (and, thereby, zero length) edges joining them. In this way, a combination of two 3-degree Steiner nodes will produce a 4-degree Steiner node, three linked 3-degree Steiner nodes will produce a 5-degree Steiner node, and so on. As Fig. 3 shows, the new Steiner node $s_{23}$ is a combination of the Steiner nodes $s_2$ and $s_3$; in Fig. 4, the new Steiner node $s_{123}$ is a combination of the Steiner nodes $s_1$, $s_2$, and $s_3$. If a tree involves a mix of $K$-degree $(K \geq 4, K \in \mathbb{N}^+)$ Steiner nodes and 3-degree Steiner nodes (there may be no 3-degree Steiner nodes so it could consist of only $K$-degree Steiner nodes), we can take any $K$-degree Steiner node and split it into $(K - 2)$ 3-degree Steiner nodes. Note that the order in which we do this is irrelevant. For terminal nodes with more than three branches, we can add more virtual BUs and zero-cost edges [29]. The topology of this tree then becomes a Steiner topology. The reverse conversion is also valid. Therefore, if we know all Steiner topologies then we know all topologies.

From the above discussion, we are able to articulate the following theorem.

Theorem 1. For a given graph with $N$ nodes, a general topology with $K$-degree $(K \geq 3, K \in \mathbb{N}^+)$ Steiner nodes can be derived from a Steiner topology with only degree-3 Steiner nodes by the process described above.

Our new algorithm for the minimal cost network problem is based on solving the Steiner tree problem with Steiner nodes having three branches, that is, the SMT problem. But our algorithm can also be applied to the case where the tree topology includes $K$-degree $(K \geq 3, K \in \mathbb{N}^+)$ Steiner nodes.

A Steiner topology with $N$ terminal nodes is a full Steiner topology if there are $N - 2$ Steiner nodes, and the degree of each terminal node is equal to one. The number of Steiner nodes in a Steiner topology can only be reduced by coalescing Steiner nodes with terminal nodes, while for general topologies, the number of Steiner nodes can also be reduced by coalescing Steiner nodes with other Steiner nodes to create higher branched Steiner nodes.

Given $N$ terminals $x_1, x_2, \ldots, x_N \in \mathbb{M}$, let $N = \{1, 2, \ldots, N\}$ be the index set of terminals, $S = \{N + 1, N + 2, \ldots, N + M\}$ be the index set of Steiner nodes ($M \leq N - 2$), and $V = N \cup S$. Given a topology $T$ derived by Smith’s B&B algorithm, let $E = E_1 \cup E_2$ be the set of all edges, i.e., $T = (V, E)$, where $E_1 = \{(i, j) | i \in N, j \in S\}$ and $E_2 = \{(i, j) | i \in S, j \in S\}$. We aim to find the coordinates of Steiner nodes: $x_{N+1}, x_{N+2}, \ldots, x_{N+M} \in \mathbb{M}$, the paths $\Gamma = \{\gamma(e) | e \in E\}$ (i.e., the geodesics corresponding to the edges in $E$) and the specific cable design level through the paths.

We impose the constraint that all terminals and all potential Steiner nodes are grid nodes $x_i$ of $\mathbb{M}$, $x_i \in \mathbb{M}$ (we are only given the coordinates of grid nodes of $\mathbb{M}$ in practice). For each grid point $x$ in $\mathbb{D}$, let $f^\prime(x, u(x)) = \min_u (h(x, u) + \alpha \cdot g(x, u))$, where $h(x, u)$ and $g(x, u)$ are the laying cost and the repair rate at grid point $x$, respectively, with design level $u$. By applying
FMM, we are able to find the optimal path between every terminal node and grid node in $\mathbb{M}$, as proved in [21].

Similar to [10], for a cable segment $\gamma_{ij}$ that connects two grid nodes $x_i, x_j \in \mathbb{M}$, let $c_u(x) = \int_{x_i} f'(x(s), u(x(s)))ds$ denote the cumulative weighted sum cost (here after called “cost” for brevity) over the cable path $\gamma_{ij}$ with design levels $u(\cdot) \in U$. Note that the cable is not required to traverse edges of triangles of $\mathbb{M}$; we assume that the cable can pass through the interior of triangles of $\mathbb{M}$ and FMM optimizes the path accordingly.

A. Minimization of weighted Steiner trees with BUs

The mathematical description $T$ of a physical cable network comprises the locations of all Steiner nodes, all terminal nodes, and all paths of cables. We denote the total cost of the physical cable network $T$ by $\Psi(T)$. At this stage, we do not include the cost of CLSs. Eq. (4) shows the problem of optimizing the cost of a Steiner tree ignoring the cost of Steiner nodes.

$$\min_{X, u(\cdot), \Gamma} \Psi(X, u(X), \Gamma) = \min \left\{ \sum_{j \in S} \bar{c}_u(x_j) + \sum_{(i,j) \in E} c_{ij}(x_i, x_j) \right\}$$

where $X = \{x_{N+1}, x_{N+2}, \ldots, x_{N+M}\}, x_{N+1}, x_{N+2}, \ldots, x_{N+M} \in \mathbb{M}$ are coordinates of the Steiner nodes. $\bar{c}_u(x_j)$ is the weighted cost from each terminal that is a neighbour of $x_j$ to $x_j$ itself, which can be calculated using FMM with the terminal being the source node, while considering the cable design level $u(\cdot) \in U$. Let $\bar{c}_u(x_j) = 0$ if no terminal is adjacent to the Steiner node $x_j$.

We define the skeleton tree of the topology $T$ to be the subtree $\mathbb{T} = (S, E_2)$ composed of only Steiner nodes and the edges connect them to each other. This is illustrated in Fig. 5.

Fig. 5: A Steiner tree and its skeleton tree.

For a tree with a Steiner topology $T$; that is, with each Steiner node having three branches and each terminal node three or less branches, we choose an arbitrary Steiner node $s_1$, as the root of its skeleton tree $\mathbb{S}$, $s_1 \in \mathbb{T}$. Next, we assign to the edges of $\mathbb{T}$ an orientation towards the root. Then, $\mathbb{T}$ becomes a directed rooted tree (i.e., anti-arborescence). See Fig. 6. For more details on directed rooted trees, the reader is referred to [10]. We order the nodes of the skeleton tree so that children of a given node $s_i$ appear in the list earlier than the given node (that is, with arrows from them to $s_i$). An order with this property is referred to as a topological order.

We denote, without loss of generality, the reordered Steiner node sequence as $1, 2, \ldots, M$, where $M$ corresponds to the root of $\mathbb{T}$, and denote $x_i = x_{N+i}$, $i = 1, 2, \ldots, M$. Altering the order of Steiner nodes $x_{N+i} \in \mathbb{M}$ in Eq. (4) does not change the optimization problem, so we can rewrite Eq. (4) as Eq. (5). Note that once the location of Steiner nodes is determined, the cable path $\Gamma$ and the cable design levels through the path are determined.

$$\min_{X, u(\cdot), \Gamma} \Psi(X, u(X), \Gamma) = \min_{x_M, x_{M-1}, \ldots, x_1} \Phi(x_M, x_{M-1}, \ldots, x_1)$$

where

$$\Phi(x_M, x_{M-1}, \ldots, x_1) = \tilde{c}_u(x_M) + \sum_{(i,j) \in E_2 \cap C(M)} c_{ij}(x_j, x_M) + \Phi(x_{M-1}, \ldots, x_1).$$

Then, the problem shown in Eq. (4) is converted to a dynamic programming problem with Bellman equation:

$$\Phi^*(x_M, x_{M-1}, \ldots, x_1) = \min_{x_M, x_{M-1}, \ldots, x_1} \Phi(x_M, x_{M-1}, \ldots, x_1).$$

To solve this dynamic programming problem, we construct a new directed acyclic graph (DAG) $G = (V', E')$ based on $T$, as shown in Fig. 7. Each Steiner node in $\mathbb{T}$ is associated with a subset $A_i$ of $V'$, where $A_i$ are grid nodes of $\mathbb{M}$ and the weight on each node in $\mathbb{A}_i$ is $\tilde{c}_u(x_i)$. It follows that $V' = \cup_{i \in S} A_i$; that is, $V'$ is composed of $m$ copies of the grid nodes of $\mathbb{M}$. For an arc $e = (i, j) \in E_2$, where $s_j$ is the parent of $s_i$, we construct a complete connection from $A_i$ to $A_j$ for $G$; that is, an arc $e = (p, q)$ from every $x_p \in A_i$ to every $x_q \in A_j$ in $G$. The cost of the arc $e$ is defined as the minimum cost from node $x_p$ to node $x_q$, calculated by FMM, i.e., $w(e) = w(p, q) = \min c_{u}(x_p, x_q)$. We define $\phi^i_p$ as the minimum cumulative cost (MCC) for a node $x_p \in A_i$.

$$\phi^i_p = \tilde{c}_u(x_p) + \min_{q \in A_j} w(x_q, x_p) + \phi^j_q.$$
system), we have a new Bellman equation as shown in Eq. (9), which is an extension of Eq. (7).

We refer to the DAG-Least-Cost-Tree algorithm in [10] that finds the tree on DAG with the minimum cost and return the coordinates of the Steiner nodes. We propose a new algorithm— the DAG-Least-Cost-System algorithm— listed as Algorithm 1 that finds the tree on DAG with the minimum total cost (length and number of Steiner nodes) and return the coordinates of the Steiner nodes. The extension of the DAG-Least-Cost-System algorithm over our earlier DAG-Least-Cost-Tree [10] enables the inclusion of weights (costs) of the Steiner nodes. This extension has to take account of Steiner nodes with degree in excess of three. The extension can also be used to optimize connections between regions, as opposed to points, as we will demonstrate here.

In Algorithm 1, the statements in Lines 1-13 form the initialization. Implementation of Eq. (9) is in Lines 14-32. Note that $T$ is the skeleton of a Steiner tree with full Steiner topology. If a leaf node in $T$ has the same coordinates as a terminal node then the BU cost $b_i$ at this location is zero. If two Steiner nodes have the same location, we need to decide whether to count the cost of the Steiner node once or twice depending on the BU branch constraints (Lines 19-23 achieve this). As in [10], once the iterations reach the root, in Line 32, we choose the grid node $p$ (with MCC $\phi^M_p$) in $A_M$. The node $p$ is the physical Steiner node corresponding to the root. To derive the coordinates of the remaining Steiner nodes, we track back on $G$ starting from $p$. In the meantime, we can get the path $\gamma$ of the tree with specific cable design level and total cost as well as total length. From the locations of the Steiner nodes, we infer the number of BUs (Steiner nodes).

To make Algorithm 1 easier to understand, we propose an example with a given topology $T$ as shown in Fig. 8. The five terminal nodes and three Steiner nodes are denoted by $x_1, x_2, x_3, x_4, x_5$ and $s_1, s_2, s_3$, respectively. With $s_1$ and $s_3$ as the leaves of the skeleton $T$, 49 (49 = 7 * 7) grid nodes can possibly be the location of $s_1$ or $s_3$. As in Eq. (8), $\phi^M_p = c^M_u(x_p) + b_p$, $p = 1, 2, ..., 49$, where $c^M_u(x_p)$ is the total distance from the grid node $x_p$ to the terminal nodes $x_1$, $x_2$, and $b_p$ is the cost of a BU at each grid node. Note that the $b_p$ value for the position of terminal nodes are 0. The same analysis is applied to Steiner nodes $s_1$. Also 49 grid nodes can possibly be the location of the root of the skeleton $T$, $s_2$, with two children, $s_1$ and $s_3$, as seen in Fig. 6. As denoted before, $w(x_q, x_p)$ is the distance between a child and its parent. If $x_p$ is the position of $s_2$, and $x_q$ is the position of $s_1$, then for this left branch, $\phi^M_p = c^M_u(x_p) + b_p + w(x_q, x_p) + b_q$. Note that $b_q = 0$, if $p = q$. Similarly, $\phi^M_p = c^M_u(x_p) + b_p + w(x_q, x_p) + b_q$, and

$$\phi^M_p = \phi^M_{s_1} + \phi^M_{s_3}. \text{ We choose the minimum from the 49 * 49 * 49 possible values of } \phi^M_p, \text{ and then trace back and determine the location of the three Steiner nodes.}$$

![Fig. 7: The DAG corresponding to the Steiner topology and skeleton tree in Fig. 5.](image)

Algorithm 1 DAG-Least-Cost-System

Input:

The graph $G = (V', E')$, $T$ with a topological order and BU cost ($b_i$) for each grid point.

Output:

Coordinates of Steiner nodes $s_i, i = 1, \ldots, M$.

1: for $i = 1, \ldots, M$ do
2:     for each node $x_p \in A_i$ do
3:         if $i$ is a leaf in $T$ then
4:             $\phi^i_p = c^i_u(x_p) + b_p$;
5:             $\pi(x_p) = \text{NIL}$;
6:         else
7:             for each child $s_j$ of $s_i$ do
8:                 $\pi(x_p, j) = \text{NIL}$;
9:             end for
10:            $\phi^i_p = 0$;
11:        end if
12:    end for
13: end for
14: for $i = 1, \ldots, M, i$ is not a leaf do
15:     for each node $x_q \in A_i$ do
16:         for each child $s_j$ of $s_i$ do
17:             $\psi = \infty$;
18:                 for each node $x_q \in A_j$ do
19:                     if $x_q = x_p$ then
20:                         $\psi = \phi^i_q + w(x_q, x_p)$
21:                     else
22:                         $\psi = \phi^i_q + w(x_q, x_p) + b_q$
23:                 end if
24:             if $\psi > \psi'$ then
25:                 $\psi = \psi'$;
26:                 $\pi(x_p, j) = q$;
27:         end if
28:     end for
29: end for
30: $\phi^i_p = \phi^i_q + \psi$;
31: end for
32: end for
33: $\hat{p}_M = \text{arg min}_x \in A_M \phi^M_p$
34: Trace back from $\hat{p}_M$ to leaves via $\pi$;
35: return $x_{p_1}, \ldots, x_{p_M}$.

B. Minimal cost Steiner tree for regional connectivity

We aim, in this subsection, to find a minimum total cost network connecting more than two regions and terminal nodes. To solve the regional connectivity problem, firstly we run FMM from each of the nodes in each region to every grid node in $G$ to derive $c_u(x_j)$: the sum of the minimum cost from each region that is a neighbour of $x_j$ to $x_j$ itself. Unlike in the terminal node connection problem, FMM needs to be applied for every nodes in the region. However, for a submarine cable
\[
\Phi^*_{BU}(\bar{x}_M, \bar{x}_{M-1}, \ldots, \bar{x}_1) = \min_{\bar{x}_M} \Phi_{BU}(\bar{x}_{M-1}, \ldots, \bar{x}_1) = (\tilde{c}_u(\bar{x}_M) + b_M) + \sum_{(j,M) \in R_k} \min_{j \in C(M)} c_{ij}(\bar{x}_j, \bar{x}_M). \tag{9}
\]

Fig. 8: A five-terminal example.

system, there are just a few locations on the coastline that need to be considered. In fact, it is clear that minimal lengths are achieved by connecting to nodes on the coastline (or at least on the boundaries of the regions). For each of these nodes, \( r_i \), in the region \( R_i \), we calculate the distance \( d_{ii}(x_j) \), where \( l \) indicates the index of a possible location on the boundary of \( R_i \). The minimal cost from each such node in this region to each grid node is written as \( D_l(x_j) \). We define the “pointer matrix”, \( P_{ij} \), to indicate the location (the value of \( l \) in region \( R_k \) corresponding to \( D_l(x_j) \).

Algorithm 1 provides the solution for minimizing the total cost for the terminal node connectivity problem whereas the procedure described below is for optimization of the cable system (including the location of BUs and CLSs) taking account of different cable design levels in the regional connectivity problem.

1) For each grid point \( x \) in \( D \), let the weighted cost, \( f^*(x) = \min_i (h(x, u) + \alpha \cdot g(x, u)) \), where \( h(x, u) \) and \( g(x, u) \) are the laying cost and the repair rate at grid point \( x \), respectively, with design level \( u \). The matrix \( F_u \) records the design level \( u \) of each grid point;

2) For each region \( R_i \), run FMM (region to points) and calculate the distance record \( D_l(x_j) \) and a corresponding pointer matrix \( P_{ij} \);

3) For each Steiner node \( s_i, i \in S \), let \( \tilde{c}_{ui} (x_j) = \sum_{j \in N} D_l(x_j) \) for each grid node \( x_j \) in \( D \), where \( E_i \) is based on the given topology \( \mathcal{T} \);

4) For each pair of grid nodes \( x_i, x_j \) in \( D \), run FMM for calculating the minimum weighted cost from \( x_i \) to \( x_j \);

5) Based on the Steiner topology \( \mathcal{T} \) and the grid nodes of \( D \), construct the DAG \( G = (V', E') \);

6) Run Algorithm 1 on the DAG and find the minimum weighted cost network. The nodes on the minimum weighted cost network \( x_{\beta_1}, \ldots, x_{\beta_M} \), are the Steiner nodes;

7) Find the geodesics \( \Gamma = \{ \gamma(e) \mid e \in E \} \) by gradient descent, as the last step in FMM, the chosen location in each area, and the cable design level \( u(.) \) in the geodesics \( \Gamma \), taking into account the corresponding pointer matrix \( P_{ij} \) and design level matrix \( F_{uij} \).

C. Computational complexity analysis

We assume \( M \) consists of \( H \) grid nodes and \( N \) terminals/regions to be connected. For the regional connectivity problem, without loss of generality, we assume every region has \( L \) nodes. In a similar way it is done in [10], the complexity of step 1 in the procedure provided above is \( O((2L - 1)NH \log H) \), and the complexity of step 2 is \( O((N - 2)H) \) (for a full Steiner topology with \( N - 2 \) Steiner nodes). Step 3 requires the cost calculation of each pair of grid nodes in \( m \), and its complexity is \( O(H^2 \log H) \). The DAG has \((N - 2)H \) vertices and \((N - 3)H^2 \) arcs. Therefore, finding the minimal cost tree of the \( N \) terminals/regions takes \( O((N - 2)H + (N - 3)H^2) \) operations. Therefore, the computational complexity of the whole algorithm is \( O(H^2 \log H + N - 3) \). If the number of areas \( N \) and the number of possible CLSs in each area \( n \), \( R_n \), are large, then the total possible combinations of the network could be as large as \( \prod_{n=1}^{N} R_n \), so we will need to run DAG-Least-Cost-System algorithm that many times. As in [10], the computational complexity of DAG-Least-Cost-System algorithm is \( O(H^2 \log H + N - 3) \), enumeration over all possibilities would require a computational resources of \( O\left(\prod_{n=1}^{N} R_n \ast H^2 (\log H + N - 3)\right) \). If \( H \) is large, the run-time will be prohibitive. The specific run-time is related to the values of \( H \) and \( N \) and we will explain it in each scenario in Section V.

V. APPLICATIONS

In this section, we present an application of our method to two realistic scenarios, region \( D_1 \) and \( D_2 \). As in [10], we use bathymetric data from the Global Multi-Resolution Topography synthesis [34]. We consider the trade-off between the laying cost and the total number of repairs and choose the appropriate cable design level for every point in the triangulated 2-dimensional manifold. We assume that the cable laying cost is based on the different cable design levels and the depth of the seabed. It is known that the cost of the cable is lower in deeper waters [10, 15, 35]. We assume that the total number of repairs is associated with earthquake-related cable damage risk and human activity related risk. The earthquake-related cable damage risk is related to the Peak Ground Acceleration (PGA) statistics while human activity is relevant for the depth of the seabed or delineated fishing areas. The PGA statistics in our example, provided by USGS, website have been measured over a period of 50 years, so the estimated repair rate is over 50 years as well. In each example, we will explain these considerations included how they fit into the weighted cost. To be realistic, we assume here that the cost of a CLS is around $10 million [3, 8] and...
the cost of BUs varies between $1-3 million. Notice that this range is somewhat wider than the industry estimate of $1-2 million [5, 6]. In our examples, we demonstrate how to choose the location and number of BUs and the location choices of CLSs for minimization of the total weighted cost of a cable system. We have run the code in Matlab R2017b on a 4.2 GHz Intel(R) Core(TM) i7-7700K CPU.

A. The first scenario

The first object region $D_1$ spans from the northwest corner (45.000°N, 0.000°E) to the southeast corner (36.000°N, 11.000°E). We will apply our method to optimize a cable system that connects the following six locations in this region: Genoa (44.407°N, 8.963°E), Palma (32.828°N, 4.310°E), Tunis (37.341°N, 9.078°E), Algiers (36.832°N, 3.052°E), Barcelona (41.399°N, 2.085°E), and Alghero (40.580°N, 8.324°E); these are denoted by A, B, C, D, E, F, respectively. See Fig. 9. There is no obvious high risk area, such as a fishing or earthquake area in this region. The cable weighted cost, then, in the first three examples is only the cable laying cost that is related to the depth of the seabed. The first example illustrates that the BU cost significantly affects the number of BUs used in the cable system. From the second example, we observe that BUs tends to be located in lower cost areas. The third example shows that the number of branches of a BU affects the number of BUs used in the cable system. The weighted cost in the fourth example also takes account of the repair rate caused by the earthquake risk and of various cable design level optimizations. This example demonstrates that our algorithm can apply to a cable system design for area connections and while selecting the appropriate resilience level of cables according to localized risk.

![Fig. 9: Region $D_1$. Source: Google Earth.](image)

1) The effect of BU cost on cable network design: In this example, we plan a submarine cable network using a trunk-and-branch topology between the following five locations: Genoa, Palma, Tunis, Algiers, Barcelona, as shown in Fig. 9 and denoted by A, B, C, D, E, respectively. The values of $H$ and $N$ are 12000 and 5, respectively; in this case, the run-times are in the range 6 – 8 hours.

As discussed in the Introduction, most existing BUs have three branches. We make this assumption in this example, consistent with the characteristics of the Steiner tree. We assume the weighted cost in this case is only related to the depth of the seabed. And the cable laying cost is considered with only one resilience level and set to $25000$ per kilometer. As the Steiner nodes represent BUs, an increase in the number of BUs (Steiner nodes) may adversely affect the total network cost. Therefore, our optimization must consider the tradeoff between the cost of BUs against the benefit in reduction of total cable length. We will observe, as expected, that an increase in BU cost leads to a decrease in their number. Figs. 10(a)-10(d) show the result of our method. Note that the result in Fig. 10(a), with the BU cost equal to 0, is consistent with the result of the algorithm proposed in [10].

![Fig. 10: Cable network result changed by the BU cost.](image)

Table I shows the details of each cable network with varying cost of BUs. From the result, we can conclude that, with increasing installation cost of BU, the number of BUs in the cable network decreases, while the total length of cable network becomes larger. Also as expected, the total cost of the network also rises with the increasing of BU cost, but the total cost is always the optimal solution under the specified BU cost.

2) An example with a high BU cost area: As we discuss in Section I, seabed conditions influence installation costs of BUs; BU installation cost in some areas is relatively higher than others. To illustrate the effects of geographically varying BU cost, we redo the previous experiment with an assumed higher installation costs for BUs of $2$ million across the area from northwest corner (40.696°N, 2.710°E) to southeast corner (38.375°N, 5.478°E), as shown in Fig. 11(a). The
TABLE I: Effect of BU cost on optimal cable network design.

<table>
<thead>
<tr>
<th>BU cost (million $)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of BUs</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Total length (km)</td>
<td>1704.76</td>
<td>1715.31</td>
<td>1777.95</td>
<td>1822.05</td>
</tr>
<tr>
<td>Total cost (million $)</td>
<td>92.60</td>
<td>94.87</td>
<td>95.45</td>
<td>95.55</td>
</tr>
<tr>
<td>Cable network</td>
<td>Fig. 10(a)</td>
<td>Fig. 10(b)</td>
<td>Fig. 10(c)</td>
<td>Fig. 10(d)</td>
</tr>
</tbody>
</table>

remains of the area have a BU installation cost of $1 million. The total cost of the derived network is $94.88 million with the total length being 1715.32 km. The results are shown in Fig. 11(b), where we can see that the locations of the two BUs tend to avoid the higher cost area.

3) Multi-branch BUs with more than three branches: As discussed, our algorithm is based on the Steiner tree algorithm used in [10] which considers all full Steiner topologies and applies Smith’s B&B method to reduce the computation. Here we use the extension of this algorithm described in Section IV, based on Theorem 1, to the case where the cost of BUs is taken into account, to provide a realistic example where the optimal solution involves a BU with more than three branches. Specifically, we consider an example where our aim is to connect the following four locations, Palma, Tunis, Algiers and Alghero, as shown in Fig. 9 and denoted by B, C, D, F, respectively.

First, we assume that the cost of a BU is $1.7 million across the entire area, the average cable cost is $25,000, and the number of branches of BUs is not limited. This results in an optimal tree that has just one BU with four branches and the total length is 1190.71 km, as Fig. 12(a) shows. Then, to illustrate the benefit of multi-branch BUs, we present in Fig. 12(b) the result of the algorithm for the situation where we restrict BUs to have only three branches and apply our algorithm while preventing combining of Steiner nodes. The result shows that its total cable length is 1127.86 km which is smaller than the result in Fig. 12(a). However, the cost of the cable network in Fig. 12(a), namely, $31.46 million is less than that of the solution presented in Fig. 12(b) which is $31.59 million. We remind the reader that the wide majority of cable systems have only 3-branch BUs. Nevertheless, we have demonstrated an example where it is beneficial to use 4-branch BUs.

4) SMT for areas connecting: In this example, we illustrate our regional connectivity method for optimizing the location of a CLS in each area to minimize total cost. To this end, we again apply our algorithm to the region shown in Fig. 9. In this example, besides the depth of the seabed, we take account of earthquake risk and cable design level, so that the weighted cost of each grid point depends on the cable cost (not including laying costs) for each chosen design level, depth of the seabed, possible risk of earthquake. For specific parameter settings, we refer to the previous article [15]. We assume that there are two seismic design levels, Level I and Level II with a low and a high level of protection, respectively. As in [7, 13, 15], we assume the cable cost per kilometer for Level I and Level II are $10000 and $22000, respectively.

We consider five areas corresponding to the five locations A, B, C, D, E. That is, the first area is in the vicinity of location A, the second in the vicinity of location B, etc. In each area, we assume there are three potential CLS locations. One of the potential CLS locations in each area is the same as the original locations A, B, C, D, E, and the other two are arbitrarily set, as shown in Fig. 13. Accordingly, there are three alternative CLS locations in each area, and we specify the original nodes as the first among the three alternatives. We assume a constant BU cost of $1 million which is same as that used in the example illustrated in Fig. 10(b). First, we assume that the construction costs of CLSs in all these fifteen locations are the same, and we set each CLS cost as $10 million. The result of our optimization algorithm are presented in Fig. 14(a) with one BU located at (40.977°N, 4.069°E). A pink cable path represents a cable with design Level II while a Level I path is colored black. The total length and the total cost of the optimal cable network are 1642.89 km and $75.37 million, respectively. Further details of this cable network optimization are shown in Table II.

As discussed in Introduction, the construction costs of CLSs is determined by many factors and may vary from area to area. In the next example, we study the situation where different CLSs have different cost, and use our method to find the optimal solution. The construction cost of each CLS is shown...
The weighted cost in this scenario comprises the depth of the seabed, the earthquake risk, the cable design levels and the cable laying cost. There is a fishing area in this region (Bengal) according to [36]. Note that the cable laying cost is also related to the depth of the seabed. For specific weighted cost parameter settings, we refer to [15]. The map of the repair rate (associated with the relevant risk factors) in this region is shown in Fig. 16. We assume that the construction costs of CLSs vary from area to area, and from location to location in the same area. The number of branches of a BU is assumed to be more than three. This is different from the case considered in the fourth example in the first scenario, where the number of branches of a BU is forced to be equal to three. As in the third example of the first scenario, we consider here two cable design levels: Level I and Level II for which the costs per kilometer are $10000 and $12500, respectively. We present two examples with BU costs of $1 million and $2 million to illustrate its influence.

### Table II: Multi-CLSs with same cost in each area of region $D_1$.

<table>
<thead>
<tr>
<th>area</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station choice</td>
<td>(2)</td>
<td>(3)</td>
<td>(3)</td>
<td>(1)</td>
<td>(3)</td>
</tr>
<tr>
<td>BU cost (million $)&amp;Number</td>
<td>1&amp;1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total length (km)</td>
<td>1642.891</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total cost (million $)</td>
<td>75.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table III: Multi-CLSs with different cost in each area of region $D_1$.

<table>
<thead>
<tr>
<th>area</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station (1) cost (million $)</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Station (2) cost (million $)</td>
<td>11</td>
<td>10</td>
<td>12</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Station (3) cost (million $)</td>
<td>10</td>
<td>9</td>
<td>11</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Station choice</td>
<td>(1)</td>
<td>(3)</td>
<td>(3)</td>
<td>(3)</td>
<td>(3)</td>
</tr>
<tr>
<td>BU cost (million $)&amp;Number</td>
<td>1&amp;1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total length (km)</td>
<td>1615.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total cost (million $)</td>
<td>74.42</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The result in Fig. 14(b) shows that the optimal CLS location chosen to minimize the total weighted cost by our algorithm is not exactly the one with the lowest construction cost in every area. As shown in the result, in area A, the first potential station is chosen as the CLS location, while in areas B, C, D and E, the third stations are chosen.

### B. The second scenario

The second region $D_2$ spans from the northwest corner (24.500°N, 97.000°E) to the southeast corner (5.500°N, 79.000°E). We apply our method to connect the following four areas in this region: Meinmagwe (18.780°N, 93.625°E), Andaman and Nicobar Islands (12.188°N, 92.526°E), Sri Lanka (8.287°N, 81.390°E), puri (19.849°N, 85.826°E), these are denoted by A, B, C, D, respectively. See Fig. 15. The specific values of $H$ and $N$ are 5621 and 4, respectively, in this case, the run-times are in the range 2.5-3 hours.

![Fig. 15: Region $D_2$ with four areas. Source: Google Earth.](image)
First, we assume that the BU cost is $1 million. According to [37], there are four target areas, so we just need to consider three full Steiner tree topologies. The results for the three topologies are shown in Figs. 17(a)-17(c), with total costs of $47.66, $50.23 and $47.19 million, respectively. Level I (Level II) cable paths are colored black (pink, respectively).

By comparing the results of these three topologies, we obtain the optimal cable network. This corresponds to topology III shown in Fig. 17(c) with one Steiner node located at (11.994°N, 91.305°E). In Table III we provide the construction cost of each CLS and the optimization results including the choice of CLS location in each area, the total length and cost of the cable system and the number of BUs. As shown in Fig. 17(c), cables in high-risk areas require a higher cable design level (pink color) to reduce the repair rate.

For a BU cost of $2 million, the optimal cable network result is shown in Fig. 18, also corresponding to the full Steiner topology (topology III) shown in Fig. 17(c), but without any BU. The construction cost of each CLS and the relevant detailed optimization results are shown in Table V.

The results show that our algorithm can be applied in designing optimal cable systems when considering cables with varying design levels, and various risk factors, BU costs and CLS costs.

VI. CONCLUSION

We have articulated a solution to the problem of submarine cable network path planning where the topology is of a trunk-and-branch network. This is done for a network on the surface of the earth, taking into consideration the cost of BUs and CLSs, cable laying costs with different design levels, and various risk factors. The resulting submarine cable network design for regional connectivity includes cable path planning as well as the choices of locations of BUs and CLSs.

We have introduced the weighted Steiner node problem, which in mathematical terms, has been couched as a variant of the SMT problem, on an irregular 2-dimensional manifold in $\mathbb{R}^3$. The resulting algorithm has polynomial-time computational complexity. The Steiner nodes in our problem can vary in number, while incurring a penalty (cost). We have proposed an algorithm that can solve this variant of the Steiner tree problem. We have proved that our algorithm is applicable to Steiner nodes with degrees of three or more, so that it can be used to optimize the total cost of a cable network taking account of the costs of BUs and CLSs.

We have applied our technique to several realistic scenarios to elucidate that our method is applicable to real-world ca-
The table shows different cost values for various network configurations. Here are the details:

<table>
<thead>
<tr>
<th>Station (1) cost (million $)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station (2) cost (million $)</td>
<td>10</td>
<td>9</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>Station (3) cost (million $)</td>
<td>14</td>
<td>10</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

**REFERENCE**


[24] M. Caleffi, I. F. Akayildiz, and L. Paura, “On the solution of the Steiner tree NP-hard problem via phasor optimization problems. Our technique can also be applied to other network design problems on irregular 2-dimensional manifolds, aside from the submarine cables. These include problems involving power distribution and gas and oil pipelines.


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